

Econ 401A Microeconomic Theory

Midterm

Attempt three (3) questions only.

In the time available it may not be possible to complete answers to every part of three questions. But I strongly encourage you to spend a significant amount of time on three questions. It is typically easier to pick up points for a partial answer than answer questions perfectly.

1. Multi-profit firm

The cost function of a two product firm is $C(q_1, q_2) = (q_1 + 2q_2)^2 + \frac{1}{2}q_1^2$. The output price vector is $p = (14, 24)$.

- (a) Write down the profit function. Is it concave?
- (b) Show that the First Order Necessary Conditions hold for some vector $(q_1, q_2) = (a, a)$. Is this profit-maximizing?

Henceforth suppose that the cost function is $C(q_1, q_2) = (q_1 + 2q_2)^2 - \frac{1}{2}q_1^2$ and the output price vector is $p = (10, 24)$

- (c) Show that again there is some $(\bar{q}_1, \bar{q}_2) = (b, b)$ satisfying the FOC.
- (d) Is this the profit maximizing output vector? If so why? If not why not?

2. Walrasian equilibrium in a two period model

The only commodity in an economy is coconuts. There are two periods and two individuals, Alex and Bev. Each individual has a utility function

$$U(c_1^h, c_2^h) = \ln c_1^h + \frac{1}{2} \ln c_2^h, \quad h = A, B$$

where c_t is consumption of coconuts in period t . Alex has 40 coconuts in period 1 and 60 in period 2. Bev has 80 in period 1 and 60 in period 2.

- (a) If coconuts are not storable, what is the WE interest rate in this economy?
- (b) Would the WE interest rate be any different if coconuts were storable with no loss in quality?

(c) Suppose that each coconut planted in period 1 yields 2 coconuts in period 2. So the production function in the economy is $q_2 = 2z_1$

What is the optimal consumption in each period? And what is the Walrasian equilibrium interest rate?

(d) Suppose that the production function is $q_2 = 4z_1$. What is the optimal consumption in this economy?

(e) What is the WE interest rate?

3. Production and cost

The production set of a firm is $Y = \{(z_1, z_2, q_3) \mid q_3^3 - 54z_1z_2^2 \leq 0\}$.

(a) Solve for the maximized output if the input price vector is $p = (p_1, p_2)$ and the manager has a budget of \bar{B} .

(b) Hence solve for the cost function of the firm.

(c) If the firm is a price taker in all markets and is producing 100 units what can you say about the equilibrium price of the output?

4. Envelope Theorem

Consider the following firm profit-maximization problem of a firm producing n products.

When the output price vector is $\bar{p} = (\bar{p}_1, \dots, \bar{p}_n)$, the profit-maximizing output vector is $\bar{q} = (\bar{q}_1, \dots, \bar{q}_n)$. Let $F(p)$ be maximized profit at price vector p , i.e.

$$F(p) = \text{Max}_q \{p \cdot q - C(q)\}.$$

(a) Prove the following proposition.

Proposition: Envelope Theorem

The rate at which maximized profit rises with the price of output j is $\frac{\partial F}{\partial p_j}(\bar{p}) = \bar{q}_j$

Note: You might begin by analyzing a firm that produces a single output.

(b) Suppose that the firm must also satisfy some production constraint of the form $g(q) \leq b$, so that

$$F(p) = \text{Max}_q \{p \cdot q - C(q) \mid g(q) \leq b\}$$

Does the proposition still hold? Explain carefully.