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Econ 401A Microeconomic Theory

Midterm answers

1. **Multiproduct firm**

(a) $p = (14, 24)$

$$\pi(q) = p_1 q_1 + p_2 q_2 - C(q_1, q_2) = 14q_1 + 24q_2 - (q_1 + 2q_2)^2 - \tfrac{1}{2} q_1^2$$

The first two terms and the 4th term have negative second derivatives so are concave.

$-x^2$ is concave and $x = q_1 + 2q_2$ is linear so the third term is concave.

The sum of concave functions is concave so $\pi(q)$ is concave.

(b) FOC

$$\frac{\partial \pi}{\partial q_1} = 14 - 2(q_1 + 2q_2) - q_1 = 0$$

$$\frac{\partial \pi}{\partial q_2} = 24 - 4(q_1 + 2q_2) = 0 .$$

$\bar{q} = (a, a)$ satisfies these conditions if

$$\frac{\partial \pi}{\partial q_1} = 14 - 2(3a) - a = 0 .$$

$$\frac{\partial \pi}{\partial q_2} = 24 - 4(3a) = 0 .$$

So $a = 2$

Given the concavity of the profit fucntion, the FOC are both necessary and sufficient for a maximum.

(c) $p = (10, 24)$

$$\pi = p_1 q_1 + p_2 q_2 - C(q_1, q_2) = 10q_1 + 24q_2 - (q_1 + 2q_2)^2 + \tfrac{1}{2} q_1^2$$

The last term is convex so the FOC may not yield the maximum

FOC

$$\frac{\partial \pi}{\partial q_1} = 10 - 2(q_1 + 2q_2) + q_1 = 0$$

$$\frac{\partial \pi}{\partial q_2} = 24 - 4(q_1 + 2q_2) = 0 .$$

Again $\bar{q} = (2,2)$ satisfies these conditions.

$$\pi(\bar{q}) = 10(2) + 24(2) - (6)^2 + \tfrac{1}{2}(2)^2 = 68 - 36 + 2 = 34$$

But if you set $\bar{\bar{q}}_1 = 0$

$$\frac{\partial \pi}{\partial q_2} = 24 - 4(0 + 2q_2) = 0 \text{ and so } \bar{\bar{q}}_2 = 3 .$$

$$\pi(\bar{\bar{q}}) = 24(3) - (6)^2 = 36$$

Profit is higher.

3. Two period model

(a) The utility function is homothetic since

$$U(\theta c) = \ln \theta c_1 + \delta \ln(\theta c_2) = 2\ln\theta + \ln c_1 + \delta \ln c_2 = 2\ln\theta + U(c)$$

Thus we can consider the representative agent.

Her lifetime budget constraint is

$$c_1 + (\frac{1}{1+r})c_2 \le y_1 + (\frac{1}{1+r})y_2$$

FOC

$$u'(c_1) = (1+r)\delta u'(c_2)$$

Market clearing: $\bar{c} = y$

$$u'(120) = (1+r)\delta u'(120)$$

Therefore $(1+r)\delta = 1$. Since $\delta = \frac{1}{2}$ it fololws that $1+r = 2$ and so $r = 1$.

(b) in the market place a consumer can save a coconut in period 1 and earn enough interest to buy 2 coconuts in period 2.

Storing yields only half as much. So storage is not used and the equilibrium outcome is the same.

(c) Now an input of one coconut yields 2 coconuts in period 2 which is the same as saving so adds no gains from trade.

(d) Now an input of one coconut yields 4 coconuts in period 2 which is strictly better.

$c_1 = 100 - z_1$, $c_2 = 120 + 4z_1$

$U = \ln(120 - z_1) + \frac{1}{2}(120 + 4z_2)$

For a maximum,

$U'(z_1) = -\dfrac{1}{120 - z_1} + \dfrac{2}{120 + 4z_1} = 0$. Then $\bar{z}_1 = 20$

$(\bar{c}_1, \bar{c}_2) = (100, 200)$

$MRS^R(\bar{c}) = \dfrac{u'(c_1)}{\delta u'(c_2)} = \dfrac{c_2}{\delta c_1} = 4$.

Thus the WE interest rate satisfies

$1 + r = 4$

3. Production and cost

$\underset{z}{Max}\{q_3^{\,3} = F(z) = 54 z_1 z_2^{\,2} \mid p_1 z_1 + p_2 z_2 \le \bar{B}\}$

Solution must be interior or $q = 0$

$\dfrac{1}{p_1}\dfrac{\partial F}{\partial z_1} = \dfrac{1}{p_2}\dfrac{\partial F}{\partial z_2}$

$\dfrac{54 z_2^{\,2}}{p_1} = \dfrac{128 z_1 z_2}{p_2}$

Then $p_2 z_2 = 2 p_1 z_1$.

$p_1 z_1 + p_2 z_2 = \bar{B}$

Then

$p_1 z_1 + (2 p_1 z_1) = \bar{B}$

Thus

$p_1 z_1 = \frac{1}{3}\bar{B}$ and so $p_2 z_2 = \frac{2}{3}\bar{B}$

$\bar{z}_1 = \frac{1}{3}\frac{\bar{B}}{p_1}$ and $\bar{z}_2 = \frac{2}{3}\frac{\bar{B}}{p_2}$

$$q_3{}^3 = F(z) = 54(\tfrac{1}{3}\frac{\bar{B}}{p_1})(\tfrac{2}{3}\frac{\bar{B}}{p_2})^2 = 8\frac{\bar{B}^3}{p_1 p_2{}^2}$$

$$q_3 = 2\frac{\bar{B}}{p_1^{1/3}p_2{}^{2/3}} .$$

Inverting, the budget needed to produce q_3 (i.e. the total cost) is

$$TC(q_3) = \bar{B} = \tfrac{1}{2}p_1^{1/3}p_2{}^{2/3}q_3$$

Then $MC = \frac{1}{2}p_1^{1/3}p_2{}^{2/3}$.

For a price-taking firm 100 is maximizing if $p_3 = MC(100)$.

Therefore

$$p_3 = \tfrac{1}{2}p_1^{1/3}p_2{}^{2/3} .$$

4. Envelope Theorem

(a) $F(p) = \underset{q}{Max}\{p \cdot q - C(q)\}$.

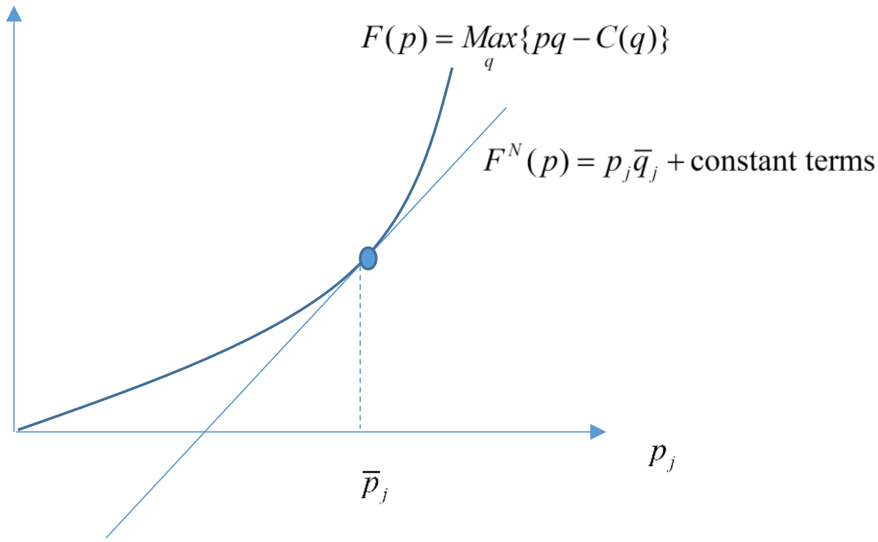
Proposition: The rate at which maximized profit rises with the price of output j is $\frac{\partial F}{\partial p_j}(\bar{p}) = \bar{q}_j$

Let $\bar{q}$ be maximizing when the price is $\bar{p}$. Suppose that the manager is naïve and does not change output when the price changes. Then

$$F^N(p) = p\bar{q} - C(\bar{q}) .$$

The graph of this function is a line of slope $\bar{q}$. This is depicted below.

The naïve manager does the same as the smart manager when the price is $\bar{p}$ but is not maximizing at other prices. Thus the graph of the maximized profit is the same at $\bar{p}$ and is higher for all other p . This is also depicted below.

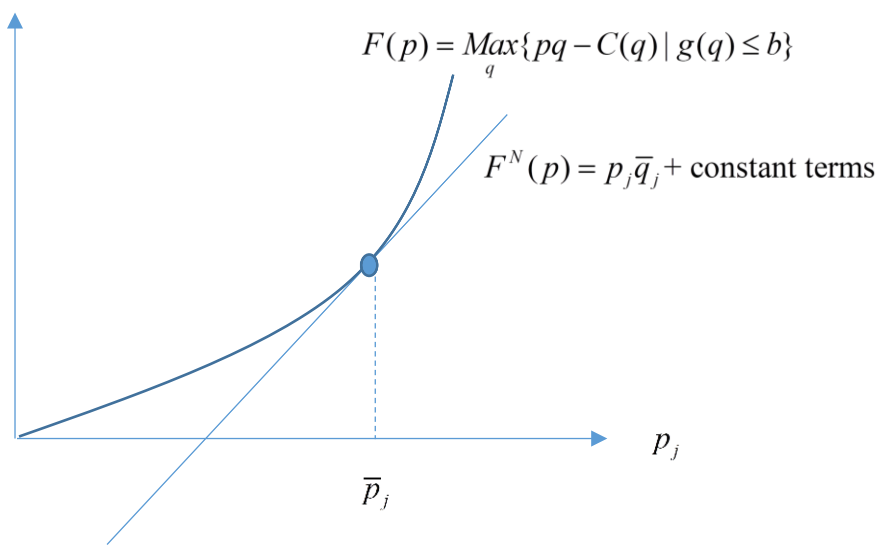


It follows that the two graphs have the same slope at $p_j = \bar{p}_j$.

Hence $\frac{\partial F}{\partial p_j}(\bar{p}) = \bar{q}_j$

(b) Suppose that the firm must also satisfy some production constraint of the form $g(q) \leq b$, so that

$\bar{q}$ solves $\underset{q}{Max}\{\bar{p} \cdot q - C(q) \mid g(q) \leq b\}$



The argument is essentially identical.