MID-TERM ANSWERS

Answer to question 1:

(a) $R(q) = 20q_1 + 48q_2 + (12 - q_3)q_3$, $C(q) = 2q_1^{\,2} + 8q_1q_2 + 8q_2^{\,2} + 2q_3^{\,2}$

$R(q) - C(q) = [20q_1 + 48q_2 - q_1^{\,2} - 8q_1q_2 - 8q_2^{\,2}] + [(12 - q_3)q_3 - 2q_3^{\,2}]$

(b) $\frac{\partial \pi}{\partial q_1}(q) = 20 - 2q_1 - 8q_2 = 0$.

$\frac{\partial \pi}{\partial q_2}(q) = 48 - 8q_1 - 16q_2 = 0$

$\frac{\partial \pi}{\partial q_3}(q) = 12 - 2q_3 - 4q_3 = 0$.

All satisfied at $\bar{q} = (2,2,2)$.

(c) Product 3 can be separated out. Since the profit on product 3 is concave the necessary condition is sufficient. The profit from producing product 3 is $\pi_3 = p_3(q_3) - 2q_3^{\,2} = 10 * 2 - 8 = 12$

If you draw the maximizer lines for x_1 and x_1, the slope of the maximizer line for commodity 1 is less steep. Hence $(2,2)$ is a saddle point not a maximum.

Then consider solutions with one of the outputs equal to zero. The profit from product 3 is unaffected so we ignore this.

(a) $\pi(q_1) = 20q_1 - q_1^{\,2}$
(b) $\pi(q_2) = 48q_2 - 8q_2^{\,2}$

The cost of producing x units is 8 times as much for product 2 and the revenue is less than three times as much so it is better to produce only product 1.

First order condition

$\pi'(q_1) = 20 - 2q_1 = 0$

so $q_1^* = 10$.

Then the profit from producing 1 is $\pi_1 =$ 100.

(d) The total profit is $\pi_1 + \pi_3 = 100 + 12 > 110$ so the solution is unaffected.

Answer to 2.

(a) There are gains from exchange unless marginal rates of substitution are equal

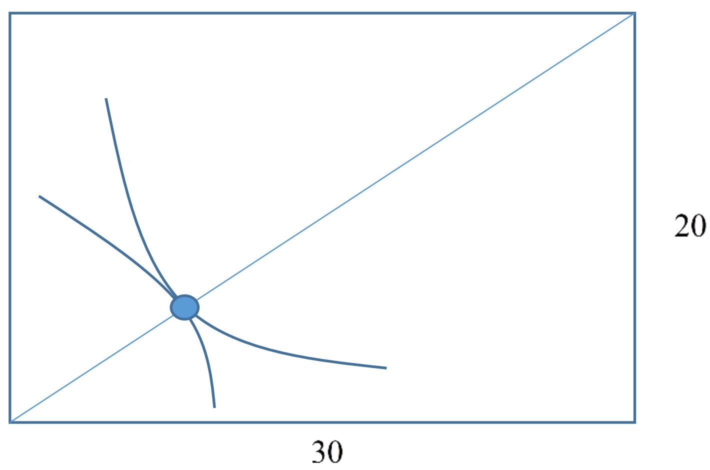
$$MRS(x_1, x_2) = \frac{\partial U}{\partial x_1} / \frac{\partial U}{\partial x_2} = \frac{3x_2}{2x_1}$$

Thus an allocation is PE if $\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B}$. Note that $\frac{4}{6} = \frac{16}{24} = \frac{2}{3}$ so the proposed allocation is a PE allocation.

(b) The consumption bundle $\hat{x}$ is supported if the price ratio is equal to the marginal rate of substitution since then the consumer will not want to trade.

$$MRS(x) = \frac{3\hat{x}_2}{2\hat{x}_1} = MRS(\hat{x}^A) = 1 = \frac{p_1}{p_2}$$

(c) The PE allocations are depicted in the Edgeworth Box below. Note that at all point on the diagonal, the consumption ratios are equal to these are the PE allocations.



Mathematically,

$$\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B} = \frac{20 - x_2^A}{30 - x_1^A}.$$

Cross multiplying,

$x_1^A(20 - x_2^A) = x_2^A(30 - x_1^A)$, therefore $20x_1^A = 30x_2^A$ and so $\frac{x_2^A}{x_1^A} = \frac{2}{3}$.

(d) The consumption bundle $\hat{x}$ is supported if the price ratio is equal to the marginal rate of substation since then the consumer will not want to trade.

$$MRS(x) = \frac{3\hat{x}_2}{2\hat{x}_1} = MRS(\hat{x}^A) = \frac{p_1}{p_2}$$

Thus the supporting price ratio is 1 so $p = (\frac{1}{2}, \frac{1}{2})$.

(e) See the diagram. The PE allocations are on the diagonal.

(f) In a WE allocation each MRS is equal to the price ratio so each person has the same MRS. Then the WE is a PE allocation. As we have just seen the supporting prices are $p = (\frac{1}{2}, \frac{1}{2})$.

Answer to 3

(a) Period budget constraints

$c_1 = \omega_1 - S_1$

$c_2 = \omega_2 + S(1+r)$

Then

$$\frac{c_2}{1+r} = \frac{\omega_2}{1+r} + S$$

Adding the constraints

$PV\{c_t\} = PV\{\omega_t\}$.

(b)

The slope on the lifetime budget line is –(1+r).

The line gets steeper as the interest rate rises so

moving around the level set c_2 raises and c_1 falls.

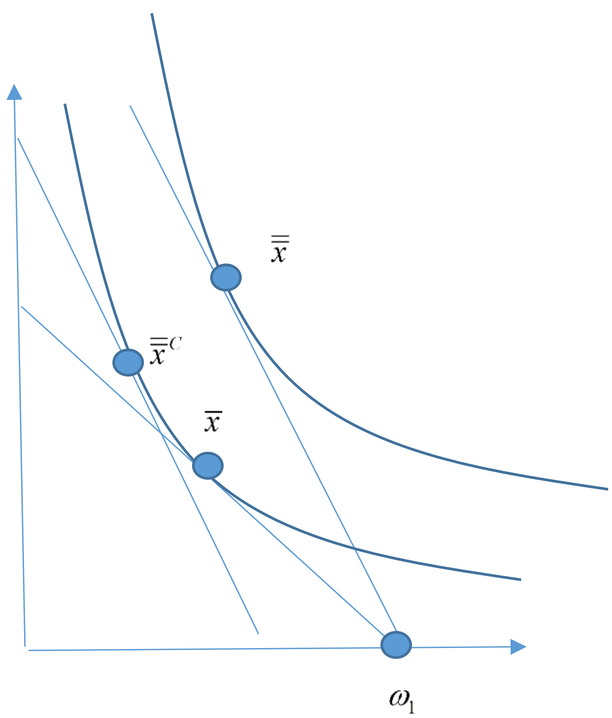
(c) The consumer is a saver so a higher interest rate

makes her better off. Thus money is taken away

to keep her on her level set. Giving the money

back increases consumption of both commodities

if they are normal.



(d) The optimum

$$U(z)=\ln(\omega_1-z_1)+\tfrac{1}{2}\ln(\omega_2+2z_1)$$

$$U'(z)=\frac{-1}{\omega_1-z_1}+\frac{\frac{1}{2}2}{\omega_2+2z_1}$$

If $\bar{z}_1>0$ the necessary condition is

$$U'(z)=\frac{-1}{\omega_1-z_1}+\frac{\frac{1}{2}2}{\omega_2+2z_1}=0$$

Therefore

$$\frac{1}{\omega_1-z_1}=\frac{1}{\omega_2+2z_1} \text{ and so } 3\bar{z}_1=\omega_1-\omega_2 \ .$$

Thus there is production if $\omega_2<\omega_1$.

Alternatively,

$$U'(0)=\frac{-1}{\omega_1}+\frac{1}{\omega_2}=\frac{\omega_1-\omega_2}{\omega_1\omega_2}.$$

Thus utility falls with production if $\omega_1<\omega_2$.

(e) With $\omega_2=24$, $\bar{z}_1=2$

$$\bar{x}=(28,28)$$

$$MRS(\bar{x}_1,\bar{x}_2)=\frac{\partial U}{\partial x_1}/\frac{\partial U}{\partial x_2}=\frac{2x_2}{x_1}=2=1+r \ .$$

So the optimum is supported by a price ratio

$$p_1/p_2=1+r=2$$

The profit of the firm is the present value:

$$-z_1+\frac{1}{1+r}q_2=-z_1+\frac{1}{1+r}2z_1=0$$

(f) In this case $(\bar{c}_1,\bar{c}_2)=(\omega_1,\omega_2)=(30,40)$.

To support the optimum

$$MRS(\bar{c})=MRS(\omega)=\frac{\omega_2}{\frac{1}{2}\omega_1}=\frac{40}{15}=1+r$$

Thus $r = \frac{40}{15} - 1 = \frac{25}{15}$.

At this high interest rate the present value of any coconut production is negative.

4. Expenditure and cost functions

(a) Solve using the Lagrangian or MRS = p1/p2

Solution: $(\bar{x}_1, \bar{x}_2) = (\frac{I}{2p_1}, \frac{I}{2p_2})$,

$$\bar{U}^A = \bar{x}_1\bar{x}_2 = (\frac{I}{2p_1})(\frac{I}{2p_2}) = (\frac{I}{2})^2 \frac{1}{p_1 p_2} .$$

(b) $M(p, \bar{U}_1)$ is the smallest income required to have a utility of $\bar{U}^A$. Appealing to (a),

$$\bar{U}^A = (\frac{M}{2})^2 \frac{1}{p_1 p_2}$$

Inverting this equation, $M^2 = 4 p_1 p_2 U^A$

And so $M(p, U^A) = 2(p_1 p_2)^{1/2} (U^A)^{1/2}$.

(c) Consumer B's preference map is the same so his choice is the same. Then

$$\bar{U}^B = (\bar{U}^A)^{1/2} = (\frac{I}{2}) \frac{1}{(p_1 p_2)^{1/2}} .$$

Therefore

$$M(p, U^B) = 2(p_1 p_2)^{1/2} U^B .$$

(d) The cost function is the minimum cost of producing any output.

The expenditure function is the cheapest way of achieving a particular utility level.

Therefore, from (c),

$$C(p, Q) = 2(p_1 p_2)^{1/2} Q .$$

(e) $Min\{p_1 z_1 + p_2 z_2 \mid (z_1 - 2)^{1/2}(z_2 - 1)^{1/2} \geq Q\}$.

Method 1: Convert to a maximization problem and write down the Lagrangian

$$Max\{-p_1 z_1 - p_2 z_2 \mid (z_1 - 2)^{1/2}(z_2 - 1)^{1/2} \geq Q\}$$

$$\mathfrak{L} = -p_1 z_1 - p_2 z_2 + \lambda((z_1 - 1)^{1/2}(z_2 - 1)^{1/2} - Q)$$

Solve in the usual way.

Method 2: Define the new variable $x_1 = z_1 - 2$ and $x_2 = z_2 - 1$

$$Min\{p_1(x_1 + 2) + p_2(x_2 + 1) \mid x_1^{1/2} x_2^{1/2} \geq Q\}.$$

The problem is already solved in (d).

Method 3: Take a fixed budget B and maximize output. i.e. solve for

$$\bar{Q} = \underset{z}{Max}\{F(z) = (z_1 - 2)^{1/2}(z_2 - 1)^{1/2} \mid p \cdot z \leq B\} .$$

Then invert to obtain the cost function.