

Midterm practice questions

Here are some practice questions for the mid-term.

Answers will be provided early next week.

1. Walrasian Equilibrium

The aggregate production function for a two commodity economy is $q = F(z) = 4z^{1/3}$ where z is the input of commodity 1 and q is the output of commodity 2. The aggregate endowment is $\omega = (32, 0)$. Consumer $h = 1, \dots, H$ has a utility function $U(x_1^h, x_2^h) = x_1^h x_2^h$.

- Solve for the utility maximizing consumption $\bar{x}(p, I^h)$.
- Hence explain why the consumption of all H consumers is the same as a single representative consumer with all of the income.
- Show that $(z^*, q^*) = (8, 8)$ maximizes the utility of the representative consumer.
- Solve for prices that would lead the profit-maximizing firms to choose (z^*, q^*) .
- Are these WE prices? Explain.

2. Utility maximization and elasticity of substitution

A consumer's preferences can be represented by the utility function $U(x) = (x_1^{-1} + x_2^{-1})^{-1}$.

- Explain why the preferences of this consumer are also represented by the utility function

$$u(x) = -x_1^{-1} - x_2^{-1}.$$

- If the price vector is p and income is I show that the FOC can be written as follows:

$$\frac{p_1^{1/2}}{p_1 x_1} = \frac{p_2^{1/2}}{p_2 x_2}.$$

- Hence solve for the utility maximizing consumption vector.
- What is the elasticity of substitution $\sigma_1 = \mathcal{E}\left(\frac{x_2^c}{x_1^c}, p_1\right)$ and $\sigma_2 = \mathcal{E}\left(\frac{x_1^c}{x_2^c}, p_2\right)$

Exercise 3: Production and cost

A factory has a production function $F(z) = (z_1^{-1} + z_2^{-1})^{-1}$. The input price vector is $r = (r_1, r_2)$

The manager is given a budget of B and told to maximize output.

(a) Show that the maximized output is a linear function of B , that is,

$$q^* = F(z(r, B)) = g(r)B . \quad (*)$$

(You only need to explain why the function $g(r)$ depends only on r . You do not need to solve explicitly for $g(r)$.)

(b) Explain why it follows from (*) that if the input price vector is r , then the cost function of the firm, $C(q, r)$, is a linear function of output.

(c) What does this imply about the equilibrium output price if the firm is a price taker in all markets?

Suppose instead that

$$F(z) = (z_1^{-1} + z_2^{-1})^{-1/2} .$$

(d) What is the new relationship between the manager's budget and maximized output?

(e) What does this imply about average and marginal cost?

4. Choice over time

A consumer lives for two periods. His utility function is

$$U(c_1, c_2) = \ln c_1 + \frac{4}{5} \ln c_2 .$$

The interest rate is $r = \frac{1}{4}$ and his endowment is $\omega = (125, 100)$.

(a) Show that the period constraints can be combined into a single life-time budget constraint.

(b) Are the income and substitution effects on period 1 saving reinforcing or opposing? Explain carefully.

(c) Solve for the consumer's best first period consumption as a function of the interest rate.

(d) For what interest rates is the consumer a saver? Note: interest rates can be negative.