A note on equilibrium strategies

A student recently asked me the following question. "If there is more than one Nash Equilibrium, how does economic theory explain what the players in the game will actually do?"

The simple answer is that when this is the case, equilibrium theory fails to explain behavior.

Another student asked "What is the difference between mutual best responses and a Nash Equilibrium?"

Definition: Strategy profile

A strategy profile is a list of strategies, one for each player.

Definition: Nash Equilibrium

A strategy profile (i.e. list of strategies) is a Nash Equilibrium if no player is strictly better off switching to another strategy. Thus strategies are mutual best responses.

Example: Rock, Scissors, Paper:

Consider the strategy profile in which each player randomizes over his strategy choice, assigning a probability of $\frac{1}{3}$ to each of the three actions.

If player 2 uses the proposed strategy, player 1 is indifferent between playing Rock, Scissors or Paper.

Note that randomizing is NOT the only best response by player 1. For example, the pure strategy of playing Rock is also a best response. But if both players follow the proposed strategy profile, neither can gain by changing to another strategy. Hence the strategies are mutual best responses.

Since this strategy profile is the only profile for which strategies are mutual best responses, it is the unique Nash Equilibrium.

Multiple Nash equilibria

Consider the following meeting game played by player 1 and player 2. If they both stand outside the Bunche Hall elevators at 10:00 AM they will both receive one million dollars. If they both stand on the quad in front of Royce Hall at 10:00 AM they will also get one million dollars. If they choose different locations they will get nothing.

The possible choices and outcomes are shown in the table below.

		Player 2	
		B	R
Player 1	B	(1,1)	(0,0)
	R	(0,0)	(1,1)

If the players know each other and collude, it is easy for them to win one million. But if neither knows who the other player is, collusion is not possible.

Economic theory tells us to look for a Nash Equilibrium. But (B,B) and (R,R) are both Nash Equilibrium strategy profiles. So economic theory does not help us in this case.[1]

To deal with this problem, economic theorists have considered ways to refine the definition of equilibrium. Each approach helps in some cases.

1. Dominated Nash Equilibrium strategy profiles

Consider the following modification of the meeting game.

		Player 2	
		B	R
Player 1	B	(2,2)	(0,0)
	R	(0,0)	(1,1)

Again (B,B) and (R,R) are both Nash Equilibrium strategy profiles. But the first NE outcome (B,B) is strictly better for both players than (R,R) . It thus seems plausible to rule out any NE strategy profile that is strictly dominated by another NE strategy profile.

2. Small trembles

Another approach is require that a NE strategy profile is still a best response for each agent if the opponents "tremble". They choose the NE strategy with very high probability but also play all of the other actions with strictly positive but very low probabilities.

[1] There is a third Nash Equilibrium strategy profile. Each randomizes setting the probability of each action equal to $\frac{1}{2}$.

Consider the sealed **second price** auction with known values $v_1 = 200$ and $v_2 = 100$. Players must bid integers. Consider the strategy profile $(b_1, b_2) = (50, 300)$. If you think about it, you will see that the strategies are mutual best responses.

Suppose that you are player 1. If player 2trembles ever so slightly and bids all possible values with positive probability, then your best response is to bid your value. If you bid above your value you simply add a chance to make a loss. And if you bid below your value you lose out on a chance to make a profit. Thus bidding your value is player 1's unique best response.

The same argument then holds for player 2. Thus while there are many Nash Equilibrium strategy profiles for this game, only one profile has the best response property when players tremble.

3. Model uncertainty

Auctions are used in environments where there is private information. The auction example discussed above is therefore not very interesting (except as a way of testing your understanding of Nash Equilibrium.)

As we shall see, for the sealed high bid auction with private information, there is a unique Nash Equilibrium strategy profile. Moreover, the best responses are unique.