On the efficiency of Bertrand and Cournot competition under incomplete information

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Abstract

Bertrand competition is generally viewed as more efficient in welfare terms than Cournot competition. This paper shows that, after introducing incomplete information about rivals’ costs, this is not always true: in a homogeneous oligopoly where costs are uniformly distributed, the Bertrand price (output) is higher (lower) than that of Cournot, if firms have sufficiently low costs. Moreover, individual firms’ ex ante expected profits as well as their actually realised profits are often higher in the Bertrand game. Finally, it is shown that, even when the Bertrand price is higher than the Cournot price, the Bertrand model may still lead to a higher level of social welfare than the Cournot model because it is more productively efficient. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The Cournot (1838) and Bertrand (1883) models are cornerstones of the modern theory of oligopoly. In the former, firms’ strategic variable is the quantity of output to produce while in the latter, firms choose price. Bertrand competition has traditionally been considered as more efficient in welfare terms than Cournot competition because it leads to lower prices and larger quantities (see for example Shubik, 1980; Vives, 1985; Singh and Vives, 1984). Indeed, if we assume that firms produce a homogeneous product at a common constant marginal cost, Bertrand competition will lead to a price equal to the marginal cost while Cournot competition will lead to a price which is intermediate...
between the competitive and the monopolistic price. If, to the contrary, we assume that firms produce differentiated products, then the Bertrand price will be above the marginal cost but it will be again lower than the corresponding Cournot price. Therefore, consumer surplus and total surplus are always higher in Bertrand competition than in Cournot competition. Furthermore, profits in Cournot competition are higher, equal or smaller than in Bertrand competition if the goods are substitutes, independent or complements.\textsuperscript{1}

However, Singh and Vives (1984) state that the conclusion that Bertrand competition is more efficient than Cournot competition is not correct ‘if one considers supergame equilibria. Price-setting supergame equilibria may support higher prices than quantity-setting equilibria for either homogeneous or differentiated products. See Brock and Scheinkman (1981) and Deneckere (1983).’ That is, Singh and Vives restrict the validity of the conclusion to the class of static games only.\textsuperscript{2}

Moreover, Vives (1984), analysing an incomplete information setting where firms receive signals about the uncertain demand, proves that the Bertrand Bayesian–Nash price (quantity) is, again, lower (higher) than the Cournot Bayesian–Nash one.\textsuperscript{3}

Finally, Amir and Jin (2001) consider a model with linear demand and a mixture of substitute and complementary products and they find support for the conventional wisdom, though with some limitations. In particular, they prove that ‘Price competition is indeed more competitive according to the following criteria: lower mark-up/output ratios, larger average output, and lower average price.’

In this paper, I show that introducing incomplete information about rivals’ costs of production brings interesting new insights to the debate and leads to results that are quite different from the traditional views. Indeed, I show that in a homogeneous oligopoly in which each firm knows the value of its own marginal cost and the distribution function of its rivals’ ones, in equilibrium, the Bertrand price (quantity) might be higher (lower) than the Cournot price (quantity). This will be the case—rather surprisingly—when all firms are relatively efficient, that is, have sufficiently low costs. The intuition for this result is that when firms have low costs, they will all produce a relatively large quantity in the Cournot game so that the price will be relatively low. To the contrary, in the Bertrand game, only one firm will produce in equilibrium and will sell at a high price-cost margin as long as its cost is low. Moreover, individual firms’ ex ante expected profits, i.e. before the game is actually played and the true costs of the rivals revealed, are often higher in the Bertrand game. This conclusion provides an interesting comparison with the results obtained by Vives (1984) in his model with uncertain demand. Vives shows that firms’ expected profits are always higher in the Cournot game. Another interesting result is that

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\textsuperscript{1} Related results can be found in Cheng (1984, 1985), Hathaway and Rickard (1979) and Okuguchi (1987).
\textsuperscript{2} The same conclusion is also implicitly obtained by Sutton (1991) who analyses a two stage game in which each firm has to decide whether or not to enter in a first stage (and pay a sunk cost in case a decision to enter is made) and then engage in either quantity or price competition. If firms compete à la Bertrand, then there is room only for one firm in the industry that will then charge its monopoly price. To the contrary, if firms compete à la Cournot, then several players will in general find it profitable to enter and the resulting market price will be lower than the monopoly one. Sutton’s conclusion is that tougher price competition is always accompanied by higher concentration of the industry but not necessarily by lower prices.
\textsuperscript{3} For extensions and generalisations, see Sakai (1985, 1986).
\end{footnotesize}
while the ex post profit of the less efficient firms is generally positive in the Cournot game and always zero in the Bertrand game, the ex post profit of the most efficient firm is very likely to be higher in the latter. Finally, I show that, even though the Bertrand price can be higher than the Cournot price when the costs of production are sufficiently low, the Bertrand model may still be preferred to the Cournot model from a social welfare point of view. This is due to the fact that while in the Bertrand model the most efficient firm satisfies the whole market demand, in the Cournot model also the inefficient firms normally produce a positive quantity but at a higher cost of production; therefore, their price-cost margins as well as their profits will be lower than the corresponding values for the most efficient firm. This implies that even when the consumer surplus generated by the Bertrand model is lower than that of Cournot because the Bertrand price is higher than the Cournot price, the Bertrand model may still lead to a higher level of social welfare, because of a higher producers’ surplus.4

The paper is organised as follows. In Section 2, I analyse the Bertrand and Cournot static games with incomplete information in an industry with n firms. In Section 3, I make comparisons between equilibrium prices, quantities, firms’ ex ante and ex post profits as well as social welfare levels implied by the two models.

2. Cournot and Bertrand models with incomplete information

In both models I will use the following assumptions.

(A1) In the industry there are n firms producing a homogeneous product.
(A2) The demand function is a linear function of the price: i.e. \( Q = 1 - p \) where \( Q = \sum_{i=1}^{n} q_i \) is the aggregate quantity and \( p \) is the price.
(A3) The cost function for firm \( i \) is \( C_i(q_i) = c_i q_i \); i.e. there are no fixed costs and the marginal cost is constant.
(A4) The marginal cost \( c_i \) is independently and uniformly distributed on \([0, 1]\).
(A5) Each firm knows the value of its own cost, but only knows the distribution function of its rivals’ unit costs.
(A6) Firms meet only once in the market and they choose the value of their strategic variable (i.e. the price in the Bertrand game and the quantity in the Cournot game) simultaneously and noncooperatively.

Given the above assumptions, both the Bertrand and the Cournot games are static games of incomplete information. Therefore, the relevant concept of equilibrium is the Bayesian–Nash equilibrium. In the next two subsections, I compute the equilibria for each of the two games under analysis.

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4 If the producers’ surplus is given a relatively larger weight in the political decision making process (see for example Peltzman, 1976), then the outcome of the Bertrand model is preferred by policy makers.
2.1. The Bertrand game

The Bertrand Bayesian–Nash equilibrium when rivals’ costs are unknown has been recently characterised by Spulber (1995). Using more general assumptions than those described above (i.e. the demand function is not necessarily linear and the costs are not necessarily drawn from a uniform distribution), he proves that the static Bertrand game of incomplete information has a unique symmetric equilibrium pricing strategy \( p^*(c) \) which is increasing, differentiable and solves a system given by a differential equation and two boundary conditions. However, because of the generality of the assumptions made, he does not actually solve for the equilibrium. Using the more specific assumptions A1–A6 and much simpler technicalities, I now show that it is possible to derive an exact solution.

Recall that in a static Bayesian game, a strategy for player \( i \) is a function from types to actions. Hence, in our game, a strategy for firm \( i \) is a function \( p_i(c_i) \) which specifies a price \( p_i \) for each possible value of the marginal cost \( c_i \). Suppose, then, that all \( i \)’s rivals adopt the same strategy \( p_j(c_j) \) with \( j=1,\ldots,n \) and \( i\neq j \). Firm \( i \)'s market demand is then

\[
q_i^B = \begin{cases} 
1 - p_i & \text{if } p_i < \hat{p}_{-i} \\
\frac{1 - p_i}{m} & \text{if } p_i = \hat{p}_{-i} \\
0 & \text{if } p_i > \hat{p}_{-i} 
\end{cases}
\]

where \( \hat{p}_{-i} \) is the smallest price in the set of the equilibrium prices chosen by all \( i \)'s rivals using function \( p_j(c_j) \) and \( m \in [2, n] \) is the number of firms that charge \( \hat{p}_{-i} \). In order to choose its optimal price, firm \( i \) will solve the following maximisation problem given the value of its cost of production \( c_i \):

\[
\max_{p_i} (1 - p_i)(p_i - c_i)\Pr(p_i < \hat{p}_{-i}) + \frac{(1 - p_i)(p_i - c_i)}{m}\Pr(p_i = \hat{p}_{-i}) + 0\Pr(p_i > \hat{p}_{-i}).
\]

(1)

Now, since the marginal costs are distributed along a continuous interval, we have

\[
\Pr(c_i = c_j) = 0 \quad \forall i, j.
\]

Supposing that the strategy \( p(\cdot) \) adopted by all \( i \)'s rivals is a strictly monotone and differentiable function of the marginal cost, then also the prices will be uniformly and independently distributed. So,

\[
\Pr(p_i = p_j) = 0 \quad \forall i, j.
\]

Hence, we can rewrite Eq. (1) in the following way:

\[
\max_{p_i} (1 - p_i)(p_i - c_i)\Pr(p_i < \hat{p}_{-i}).
\]

(2)
Now, since all firms different from \(i\) adopt the same strategy \(p(\cdot)\) and since the costs are drawn independently, we have:

\[
\Pr(p_i < \hat{\rho}_{\cdot-i}) = \Pr(p_i < p(c_1))\Pr(p_i < p(c_2)) \ldots \Pr(p_i < p(c_n)).
\]  

(3)

Let us denote with \(p^{-1}(p_i)\) the marginal cost that firm \(j\) must have in order to select price \(p_j\). Clearly, our interest is limited to the values of \(p_j\) for which

\[0 \leq p^{-1}(p_j) \leq 1.\]

(4)

Since the marginal cost \(c_j\) is uniformly distributed on \([0, 1]\),

\[\Pr(p_i < p(c_j)) = \Pr(p^{-1}(p_i) < c_j) = 1 - p^{-1}(p_i)\]

(5)

and Eq. (3) can be rewritten:

\[\Pr(p_i < \hat{\rho}_{\cdot-i}) = (1 - p^{-1}(p_i))^{n-1}.\]

(6)

The maximisation problem (Eq. (2)) becomes

\[
\begin{cases}
\max_{p_i} (1 - p_i)(p_i - c_i)(1 - p^{-1}(p_i))^{n-1} \\
\text{s.t. } 0 \leq p^{-1}(p_i) \leq 1.
\end{cases}
\]

(7)

The reader can easily verify that the unique corner solution is given by \(p_i = p^*\) where \(p^*\) is the price chosen by any rival that uses the strategy \(p(\cdot)\) when its cost of production is zero. However, this would be an equilibrium only in the particular event in which all the firms have a unit cost equal to zero. Since I have assumed that the costs are uniformly and independently distributed, the probability of this event is zero. Therefore, when looking for a Bayesian–Nash equilibrium of our game, one should ignore this corner solution.

The first order necessary condition for an interior optimum is that

\[
(-2p_i + c_i + 1)(1 - p^{-1}(p_i))^{n-1} + (-p_i^2 + p_i c_i + p_i - c_i) \frac{\partial}{\partial p_i} (1 - p^{-1}(p_i))^{n-1} = 0.
\]

(8)

The first order condition defines an implicit function of player \(i\)'s best response to the strategy \(p(\cdot)\) played by all the rival firms, given that player \(i\)'s marginal cost is \(c_i\). If the strategy \(p(\cdot)\) is to be a symmetric Bayesian–Nash equilibrium, we require that the solution to the first order condition be \(p(c_i)\): that is, for each firm \(i\)'s possible marginal costs, firm \(i\) does not wish to deviate from the strategy \(p(\cdot)\) given that \(i\)'s rivals play this strategy. To impose this requirement, we substitute \(p_i = p(c_i)\) into the first order condition, obtaining (also taking into account that \(p^{-1}[p(c_i)] = c_i\)):

\[
p'(c_i)(1 - c_i)(1 + c_i - 2p(c_i)) - (n - 1)(p(c_i) - c_i)(1 - p(c_i)) = 0.
\]

(9)
Eq. (9) can be rewritten in the following way:

\[
p_i(c_i) = \frac{(n - 1)(p(c_i) - c_i)(1 - p(c_i))}{(1 - c_i)(1 + c_i - 2p(c_i))}.
\] (10)

The following lemma is very useful for determining the unique Bayesian–Nash equilibrium of our game.

**Lemma 1.** The unique strictly monotone function which solves the differential equation (Eq. (10)) is

\[
p_i(c_i) = \frac{1}{1 + n} + \frac{n}{1 + n} c_i.
\] (11)

**Proof.** Consider the differential equation in Eq. (10). The function \( p'(c_i) \) is everywhere continuous except for all points \((c_i, p_i)\) where either \( c_i = 1 \) or \( p_i = (c_i + 1)/2 \). Representing these lines on a graph (see Fig. 1) and recalling that \( c_i \in [0, 1] \), we can infer that there exist two different regions to the left of \( c_i = 1 \) where \( p'(c_i) \) is continuous: the one below the line \( p(c_i) = (c_i + 1)/2 \) and the one above that same line. In these two regions, the conditions required by the Cauchy–Peano theorem for the existence and uniqueness of a solution of a differential equation like the one in Eq. (10) hold (see, for example Grimshaw, 1990, Chap. 1). This means that in each of the abovementioned regions there exists a unique solution.

![Fig. 1. The bold lines represent the two solutions to Eq. (10).](image-url)
Let us see now whether these solutions can be linear. By substituting in Eq. (10) in place of the generic function $p(c_i)$ the function

$$p_i = a + bc_i,$$

we obtain:

$$c_i^2(nb^2 - nb + b^2) + c_i(-2b^2 - 1 + a + b + 2nab - an - bn + n) + na^2 - na - 2ab + b - a^2 + a = 0 \tag{12}$$

The values of $a$ and $b$ that solve Eq. (12) are

$$(a, b) = (1, 0)$$

and

$$(a, b) = \left(\frac{1}{1 + n}, \frac{n}{1 + n}\right).$$

Therefore, in the region above the line $p(c_i) = (c_i + 1)/2$, the unique solution is given by the line $p_i = 1$. However, this solution is irrelevant for the solution of our problem, since the strategy $p(\cdot)$ adopted by $i$’s rivals has to be a strictly monotone function of the unit cost of production. On the contrary, in the region below the line $p(c_i) = (c_i + 1)/2$, the unique solution is given by the line $p_i = (1/(1+n)) + ((n)/(1+n))c_i$.

From Lemma 1, the following proposition follows quite easily:

**Proposition 1.** The strategy defined by Eq. (11) is the unique symmetric Bayesian–Nash equilibrium of the game defined by assumptions A1–A6.

**Proof.** From Eqs. (6) and (11) we obtain

$$\Pr(p_i < \hat{p}_{-i}) = \left[\frac{(n + 1)(1 - p_i)}{n}\right]^{n-1} \text{ with } \frac{1}{1 + n} \leq p_i \leq 1. \tag{13}$$

Therefore, if all firms except firm $i$ adopt strategy (11), then problem (2) can be rewritten in the following way:

$$\max_{p_i} (1 - p_i)^n (p_i - c_i) \left[\frac{n + 1}{n}\right]^{n-1} \text{ s.t. } \frac{1}{1 + n} \leq p_i \leq 1,$$ \tag{14}

the unique solution to which is Eq. (11).

Therefore, strategy (11) is a Bayesian–Nash equilibrium of our game. Moreover, since Lemma 1 states that there do not exist other strategies that satisfy the first
order conditions, strategy (11) is also the unique Bayesian–Nash equilibrium of the game.

Since all firms adopt the same strictly increasing function of the cost of production, the firm that can produce at the minimum cost will charge the smallest price. Moreover, since \( \Pr(c_i = c_j) = 0 \), there will be only one firm charging the minimum price. Therefore, this firm will satisfy the whole market demand. Now, supposing, without loss of generality that firm 1 has the minimum cost of production, the Bertrand equilibrium price will be

\[
p^B = \frac{1}{1+n} + \frac{n}{1+n} c_1
\]

and the equilibrium quantity will be

\[
Q^B = q^B_1 = 1 - p^B = n \frac{1 - c_1}{1+n}.
\]

Note that, even though only one firm produces in equilibrium, all firms have a positive ex ante expected profit before the game is actually played. Indeed,

\[
\Pi^B_i = (1 - p_i)(p_i - c_i)\Pr(p_i < \hat{p}_{-i}) = n \frac{(1 - c_i)^{n+1}}{(1+n)^2},
\]

which is equal to zero only in the extreme case where \( c_i = 1 \) and strictly positive elsewhere.

Note also that the price charged by each firm is always greater than the unit cost of production and, hence, the firm that ends up operating always obtains positive profits. More precisely, the profit of the firm that ends up operating is

\[
\Pi^B_1 = (1 - p^B)(p^B - c_1) = n \frac{(1 - c_1)^2}{(1+n)^2},
\]

which is equal to zero only in the extreme case where \( c_1 = 1 \), which occurs with probability zero, and strictly positive elsewhere. Clearly, all the other firms, i.e. the less efficient ones, will obtain, ex post, a profit equal to zero.

Finally, note that the price, the ex ante expected profit of every firm and the ex post profit of the winner are decreasing functions of the number of firms in the industry and if this number tends to infinity, then each firm tends to choose a price equal to its unit cost and the ex ante and ex post profits all converge to zero.

2.2. The Cournot game

The Cournot Bayesian–Nash equilibrium when rivals’ costs are unknown is more straightforward to derive than the corresponding Bertrand Bayesian–Nash equilibrium analysed in the previous subsection. In particular, Cramton and Palfrey (1990) calculate this equilibrium by making assumptions A1–A6. In the following, I give the results that they obtain. The interested reader will find the detailed proof in the original paper. In this
game, each firm maximises its expected profit given the output decisions of the other firms.

Firm $i$'s expected profit is

$$\Pi_i^C = (p - c_i)q_i = (\tilde{c} - q_i - c_i)q_i,$$

where $\tilde{c} = 1 - (n-1)\bar{q}$ and $\bar{q}$ is the expected value of $q_i$.

Cramton and Palfrey show that, given assumptions A1–A6, there exists a unique Bayesian–Nash equilibrium in which the quantity of output produced by firm $i$ with cost $c_i$ is

$$q_i^C = \begin{cases} 0 & \text{if } c_i \geq \tilde{c} \\ \frac{\tilde{c} - c_i}{2} & \text{if } c_i < \tilde{c}, \end{cases}$$

where $\tilde{c} = 1 - (n-1)\bar{q} = \frac{2}{\sqrt{n+1}}$.

Therefore, the resulting Cournot equilibrium price is

$$p^C = 1 - \sum_{i=1}^{n} q_i^C$$

and the Cournot equilibrium aggregate output is

$$Q^C = \sum_{i=1}^{n} q_i^C.$$

Firm $i$'s ex ante expected profit is

$$\Pi_i^C = \begin{cases} 0 & \text{if } c_i \geq \tilde{c} \\ \frac{(\tilde{c} - c_i)^2}{4} & \text{if } c_i < \tilde{c}, \end{cases}$$

Firm $i$'s ex post profit is

$$\Pi_i^C = \begin{cases} 0 & \text{if } c_i \geq \tilde{c} \\ \left(p^C - c_i\right) \frac{(\tilde{c} - c_i)}{2} & \text{if } c_i < \tilde{c}. \end{cases}$$

3. Comparison of the two models

We now have all the material necessary to compare the two models. In what follows, I will show that Bertrand competition can lead to a higher (lower) price (quantity) than Cournot competition. Moreover, in the Bertrand game, firm $i$'s ex ante expected profit is often higher and also the ex post profit of the most efficient firm is likely to be higher than in the Cournot game. Finally, I will prove that, even when the Bertrand price is higher than
the Cournot price, the Bertrand model may still be preferable from a social welfare point of view.

3.1. Equilibrium prices and quantities

The market price resulting from the Bertrand game can be higher than the one resulting from the Cournot game. Correspondingly, the aggregate Bertrand quantity and consumer surplus will be lower than the corresponding Cournot values.

Let the costs of the $n$ firms be $c_1, c_2, \ldots, c_n$ and suppose, without loss of generality, that $c_1 < c_2 < \ldots < c_n$. From Eq. (15), we know that if the $n$ firms engage in the Bertrand game, the resulting market price is

$$p^B = \frac{1 + nc_1}{1 + n}.$$  

On the contrary, Eqs. (20) and (21) lead to the following market price in the Cournot game:

$$p^C = \begin{cases} 
1 & \text{if } c_i \geq \frac{2}{\sqrt{n+1}}, \forall i = 1, \ldots, n \\
\frac{2c_i}{n+1} - \frac{1}{\sqrt{n+1}} & \text{if } c_1 < \frac{2}{\sqrt{n+1}}, c_j \geq \frac{2}{\sqrt{n+1}}, \forall j = 2, \ldots, n \\
\frac{2}{n+1} \sum_{s=1}^{m} c_s - \frac{m}{\sqrt{n+1}} & \text{if } c_s < \frac{2}{\sqrt{n+1}}, \forall s = 1, \ldots, m, c_r \geq \frac{2}{\sqrt{n+1}}, \forall r = m + 1, \ldots, n
\end{cases}$$

where $m$ is the number of firms that produce in equilibrium.

The reader can easily verify that the following results hold:

(i) If $c_i \geq \frac{2}{\sqrt{n+1}}, \forall i = 1, \ldots, n$, then $p^B \leq p^C \forall c_i \in [0, 1]$;
(ii) If $c_1 < \frac{2}{\sqrt{n+1}}, c_j \geq \frac{2}{\sqrt{n+1}}, \forall j = 2, \ldots, n$, then $p^B \leq p^C \forall c_i \in [0, 1]$;
(iii) If $c_s < \frac{2}{\sqrt{n+1}}, \forall s = 1, \ldots, m, c_r \geq \frac{2}{\sqrt{n+1}}, \forall r = m + 1, \ldots, n$, then $p^B \geq p^C$ whenever the producing firms’ costs of production satisfy the following condition:

$$\sum_{s=1}^{m} c_s \leq 2 \left( \frac{m}{\sqrt{n+1}} - \frac{n(1 - c_1)}{n + 1} \right).$$

In other words, result (i) indicates that if all producing firms are relatively inefficient, Cournot competition implies that none of the $n$ firms will produce in equilibrium. As under Bertrand competition the most efficient firm always produces a positive quantity, the Bertrand price will clearly be lower than the Cournot price in this special case. The same is also true if only one firm produces in equilibrium under Cournot competition (result (ii)).

However, if there is more than one producer under Cournot competition and if the producing firms are relatively efficient (i.e. if the sum of the costs of the $m$ producing firms is relatively low), the Bertrand game will lead to a higher price than the Cournot game (result (iii)). The intuition for this result is as follows. In the Cournot game, firms use Eq. (20) and, therefore, if they have low costs, they will all produce a relatively large quantity of output leading to a relatively low market price. In the Bertrand game, due to the
assumptions that goods are perfect substitutes and firms have no capacity constraints, only the firm with the lowest cost of production will produce in equilibrium and will satisfy the whole market demand given that the chosen price is the one in Eq. (25). Now, even though the price is strictly increasing in the cost of production, the price-cost margin is strictly decreasing with respect to the same variable. That is, like in the monopoly case, the market price is relatively higher when the cost of production is relatively low.

Note also that since price and aggregate quantity are inversely related (i.e. \( p = 1 - Q \)), whenever the Bertrand price is higher than the Cournot price, the Bertrand aggregate output and the consumers’ surplus will be smaller.

In order to illustrate the above result, let us consider the special case \( n=2 \), with \( c_1 < c_2 \). In this case, from result (iii) above, we know that in order for the Cournot price to be higher than the Bertrand price, it is necessary that both firms produce in equilibrium, i.e. that \( m=2 \). Moreover, from condition (27) above we have that \( p^B \geq p^C \) whenever

\[
c_1 < c_2 \leq \frac{c_1 + 4(3\sqrt{2} - 4)}{3},
\]

which implies that \( c_1 < 2(3\sqrt{2} - 4) \approx 0.485 \).

Recalling that the costs of production are uniformly and independently distributed on \([0, 1]\), we can conclude that the condition in Eq. (28) will be satisfied with a probability equal to 15.7% (see Fig. 2 where \( c' = 2(3\sqrt{2} - 4) \)).\(^5\) Due to the inverse relation between

\(^5\) Note that the result that the Bertrand price can be higher than the Cournot price is only driven by the presence of incomplete information about the rival’s cost of production and not by the fact that firms have different costs of production: in a perfect and complete information framework we have \( p^C \geq p^B \) even with heterogeneous costs. Indeed, it can be easily verified that if \( c_2 \leq (1+c_1)/2 \), we have \( p^C = (1+c_1+c_2)/3 \) and \( p^B = c_2 \) so that \( p^C \geq p^B \). To the contrary, if \( c_2 > (1+c_1)/2 \), then under both Bertrand and Cournot competition firm 2 will not produce in equilibrium, so that \( p^C = p^B = (1+c_1)/2 \).
prices and outputs, this also means that with a probability of 15.7% the static Cournot game with incomplete information will lead to an outcome which is more desirable from the consumers’ point of view than the corresponding Bertrand game.

3.2. Ex ante expected profits

We will now see that individual firms’ ex ante expected profits are often higher in the Bertrand game than in the Cournot game.

Recall from Eq. (17) that firm $i$’s ex ante Bertrand profit is

$$\Pi_i^B = n \frac{(1 - c_i)^{n+1}}{(n + 1)^2},$$

while from Eq. (23) firm $i$’s ex ante Cournot profit is

$$\Pi_i^C = \begin{cases} 0 & \text{if } c_i \geq \frac{2}{\sqrt{n+1}} \\ \left(\frac{2}{\sqrt{n+1}} - c_i\right)^2 & \text{if } c_i < \frac{2}{\sqrt{n+1}} \end{cases}.$$

Clearly, if firm $i$ is relatively inefficient, i.e. if its cost is higher than the upper bound $\frac{2}{\sqrt{n+1}}$ regardless of the costs of its rivals, this firm will not produce in equilibrium and therefore, its expected profit before the game is played is zero. On the contrary, in the Bertrand game, regardless of the value of its cost of production, there is always a positive probability that firm $i$ will be the most efficient firm and, therefore, that it will enjoy positive profits in equilibrium. This implies that, whenever the cost of firm $i$ is above the upper bound $\frac{2}{\sqrt{n+1}}$, the expected profit from the Cournot model will be lower than that from the Bertrand model.

When $c_i < \frac{2}{\sqrt{n+1}}$, due to the complexity of the expressions involved, it is not possible to obtain a simple solution to the inequality $\Pi_i^B \geq \Pi_i^C$. However, the problem can be easily overcome by comparing the Bertrand and Cournot expected profits for different number of firms. Table 1 below lists the intervals over which Bertrand competition leads to higher expected profits than Cournot competition for different values of $n$.

The following observations can be made regarding the above table:

(i) When $n=2$, $\Pi_i^B \geq \Pi_i^C \forall c_i \in [0, 1]$. That is, the ex ante expected profit from engaging in price competition is higher than the one from engaging in quantity competition and this is true whatever the value of firm $i$’s cost of production. As mentioned in the introduction, this result can be interestingly compared with proposition 9 in Vives (1984), where the opposite statement is derived in a duopoly model when firms have private information about an uncertain linear demand;

(ii) When $n=2$, Bertrand leads to a higher expected profit than Cournot whenever firm $i$’s cost is either very low (so that under Bertrand competition it has a high chance of being the lowest cost firm and the only producer in equilibrium), or very high (so that under Cournot competition it will produce a very low amount of output or nothing at all);
(iii) As $n \to \infty$, Bertrand competition tends to lead to higher expected profits than Cournot competition across the whole range of possible costs of production. This is because the cost of production, above which a firm will not produce under Cournot competition, tends to zero as the number of firms goes to infinite.

### 3.3. Ex post profits

In the Bertrand game, the ex post profit of the less efficient firms (i.e. $c_2, c_3, \ldots, c_n$) is always zero. In the Cournot game, the ex post profit will be zero if $c_i \geq \frac{2}{\sqrt{n+1}}$ and positive otherwise. Things are different for the most efficient firm. Indeed, I will now show that the ex post profit of firm 1 is very likely to be higher in the Bertrand game than in the Cournot one.

Due to the difficulty of the expressions involved, in order to prove this point, I will compare firm 1’s ex-post profits in the Cournot and Bertrand models under the following two scenarios. First, I will consider the $n$ firm case but assume that all of firm 1’s rivals are relatively inefficient (i.e. their costs are higher than $\frac{2}{\sqrt{n+1}}$), so that firm 1 is the only competitor to produce a positive quantity in the Cournot equilibrium. Second, I will remove the assumption that there is only one producer in equilibrium but will look at the special case $n=2$.

Starting with the first scenario, from Eq. (18), the ex post Bertrand profit of the winner is

$$H^B_1 = \frac{n(1 - c_1)^2}{(1 + n)^2}. \quad (31)$$

From Eq. (24), the ex post Cournot profit of the firm with the minimum cost when this is the only producer in equilibrium is

$$H^C_1 = \frac{(1 - c_1)^2 - \left(\frac{\sqrt{n} - 1}{\sqrt{n+1}}\right)^2}{4}. \quad (32)$$

### Table 1

<table>
<thead>
<tr>
<th>$n$</th>
<th>$c_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>[0, 1]</td>
</tr>
<tr>
<td>3</td>
<td>[0, 0.268] $\sqrt{0.577, 1}$</td>
</tr>
<tr>
<td>4</td>
<td>[0, 0.180] $\sqrt{0.569, 1}$</td>
</tr>
<tr>
<td>5</td>
<td>[0, 0.134] $\sqrt{0.549, 1}$</td>
</tr>
<tr>
<td>6</td>
<td>[0, 0.104] $\sqrt{0.530, 1}$</td>
</tr>
<tr>
<td>7</td>
<td>[0, 0.085] $\sqrt{0.510, 1}$</td>
</tr>
<tr>
<td>8</td>
<td>[0, 0.070] $\sqrt{0.492, 1}$</td>
</tr>
<tr>
<td>9</td>
<td>[0, 0.060] $\sqrt{0.475, 1}$</td>
</tr>
<tr>
<td>10</td>
<td>[0, 0.052] $\sqrt{0.460, 1}$</td>
</tr>
<tr>
<td>50</td>
<td>[0, 0.005] $\sqrt{0.243, 1}$</td>
</tr>
<tr>
<td>100</td>
<td>[0, 0.002] $\sqrt{0.177, 1}$</td>
</tr>
<tr>
<td>1000</td>
<td>[0, 0.001] $\sqrt{0.056, 1}$</td>
</tr>
</tbody>
</table>
A comparison of Eqs. (31) and (32) shows that $\Pi_1^B \geq \Pi_1^C$ if

$$c_1 \geq \frac{2(n - \sqrt{n})}{(\sqrt{n} + 1)(n - 1)}.$$  

(33)

As shown in Fig. 3 below, the expression on the right hand side of inequality (33) is strictly decreasing in $n$ and tends to zero as $n \to \infty$.

In other words, as the number of firms increases, Bertrand competition becomes increasingly likely to give rise to higher profits than Cournot competition.

It is worth noting that, if more than one firm produced under Cournot competition (i.e. if there were other firms as well as firm 1 with a cost of production lower than $\frac{2}{\sqrt{n+1}}$), this would affect negatively the profit of firm 1 as the market price would clearly fall due to the increase in the aggregate output. This in turn implies that the range of costs over which $\Pi_1^B \geq \Pi_1^C$ would widen. In other words, the curve in Fig. 3 would shift downwards.

Let us now look at the special case $n=2$ and remove the assumption that there is only one producer in equilibrium. From Eqs. (18) and (24) the ex post profits of the winner when $n=2$ under the two models are:

$$\Pi_1^B = \frac{2(1 - c_1)^2}{9}$$  

(34)

and

$$\Pi_1^C = \begin{cases} 
(3 - 2\sqrt{2} + \frac{c_2 - c_1}{2}) \frac{2(\sqrt{2} - 1) - c_1}{2} & \text{if } c_1 < c_2 < 2(\sqrt{2} - 1) \\
(2 - \sqrt{2} - \frac{c_1}{2}) \frac{2(\sqrt{2} - 1) - c_1}{2} & \text{if } c_1 < 2(\sqrt{2} - 1) \leq c_2 \\
0 & \text{if } 2(\sqrt{2} - 1) \leq c_1 < c_2 
\end{cases}.$$  

(35)
The reader can verify that $\Pi^B_1 \geq \Pi^C_1$ whenever the costs of the two firms satisfy the following condition:

$$c_2 \in [c_1, \hat{c}] \quad \text{if} \quad c_1 \leq c'$$

$$c_2 \in [c_1, 1] \quad \text{if} \quad c_1 > c'$$

where

$$c' = 2(3\sqrt{2} - 4),$$

$$\hat{c} = \frac{-c_1^2 + (20 - 18\sqrt{2})c_1 + 260 - 180\sqrt{2}}{9(2\sqrt{2} - 2 - c_1)}.$$

Again, since the costs of production are uniformly and independently distributed on $[0, 1]$, we can conclude that the condition in Eq. (36) will be satisfied with a probability equal to 77.9% (see Fig. 4 for a graphical representation). That is, the Bertrand game will lead with a very high probability to a higher profit for the more efficient firm than the Cournot game. This means that, under incomplete information about rivals’ cost of production, profits in Bertrand competition can be higher than in Cournot competition even when goods are substitutes and not only when they are complements as analysed by Singh and Vives (1984) in the complete information framework.

3.4. Social welfare levels

In this subsection, we will see that even when the Bertrand price is higher than the Cournot price, the Bertrand model may still be preferred to the Cournot model from a social
welfare point of view. This is due to the fact that, while in the Bertrand model the most efficient firm satisfies the whole market demand, in the Cournot model also the inefficient firms will often produce a positive quantity at a higher cost of production; when this is the case, their price-cost margins and profits will clearly be smaller than the corresponding values of the most efficient rival, i.e. firm 1. Therefore, the sum of the profits of the producing firms in the Cournot model may be lower than the profit of the efficient firm in the Bertrand model even when the Bertrand price is lower than the Cournot price. This implies that even when the Cournot model leads to a higher consumers’ surplus than the Bertrand model, the latter may still lead to a higher level of social welfare thanks to a higher producers’ surplus.

To illustrate this point and in order to keep the analysis simple, I will limit myself to the special case \( n=2 \). However, the intuition behind the result would, of course, be identical if one considered a larger number of firms.

Let us then calculate the social welfare values in the two models. Regarding the Bertrand model, we have:

\[
SW^B = CS^B + \Pi_1^B = \frac{(1 - p^B)Q^B}{2} + \frac{2(1 - c_1)^2}{9} = \frac{4(1 - c_1)^2}{9},
\]

(37)

where SW and CS represent social welfare and consumers’ surplus, respectively.

Turning to the Cournot model, we have:

\[
SW^C = CS^C + \sum_{i=1}^{2} \Pi_i^C
\]

\[
= \begin{cases} 
\frac{3(c_1^2 + c_2^2) - 2(2(c_1 + c_2) + c_1c_2 - 8(3\sqrt{2} - 4))}{8} & \text{if } c_1 < c_2 < 2(\sqrt{2} - 1) \\
\frac{(3(2 - c_1) - 2\sqrt{2})(2(\sqrt{2} - 1) - c_1)}{8} & \text{if } c_1 < 2(\sqrt{2} - 1) \leq c_2 \\
0 & \text{if } 2(\sqrt{2} - 1) \leq c_1 < c_2 
\end{cases}.
\]

(38)

The reader can easily verify that \( SW_B \leq SW_C \) whenever the costs of the two firms satisfy the following condition:

\[
c_1 < c_2 \leq \frac{3(2 + c_1) - 2\sqrt{465 + 6(c_1^2 - 2c_1 - 54\sqrt{2})}}{9},
\]

(39)

which implies that \( c_1 < 2(3\sqrt{2} - 4) = 0.485 \).

Since the costs of production are uniformly and independently distributed on \([0, 1]\), we can conclude that the condition in Eq. (39) will be satisfied with a probability equal to 4.492% (see Fig. 5 where \( c' = 2(3\sqrt{2} - 4) \)). That is, even though there exists a significant range of costs of production that leads to a Bertrand price higher than the Cournot price, only a small portion of that range guarantees that the Cournot model generates a higher level of social welfare. A quick glance at Fig. 5 shows that the Cournot model is only socially preferable to the Bertrand model when the costs of the two firms are almost identical. Indeed, in this case the loss in the producers’ surplus incurred when playing the
Cournot game is not sufficient to compensate the higher consumers’ surplus generated by the lower equilibrium price.

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