Introduction to choice over time

19 pages

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Two periods:

Utility $U(c_1, c_2)$ is strictly increasing

Interest rate on savings: r

Income sequence $\{y_t\}_{t=1}^{2}$.

Consumption sequence $\{c_t\}_{t=1}^{2}$

Financial capital sequence $\{K_t\}_{t=1}^{3}$.

Period budget constraints:

$(1+r)(K_1 + y_1 - c_1) \geq K_2$ (1)

$(1+r)(K_2 + y_2 - c_2) \geq K_3$ (2)

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G. Introduction to choice over time

Two periods:

Utility $U(c_1,c_2)$ is strictly increasing

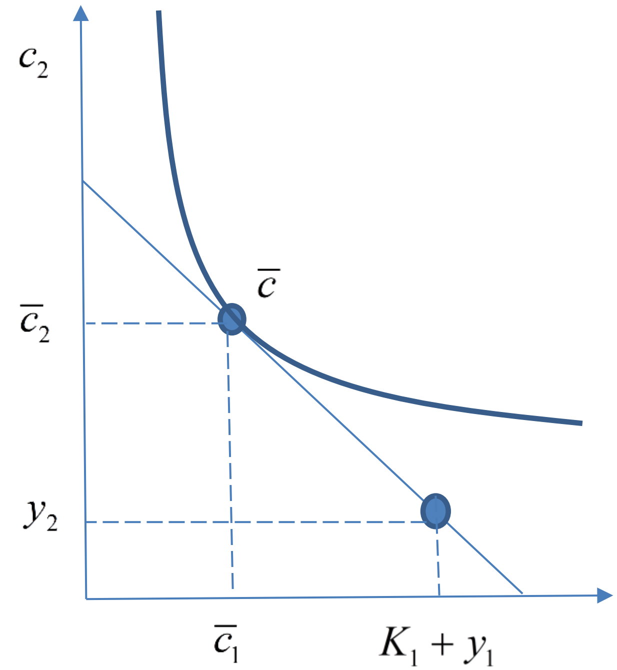
Interest rate on savings: y_2

Income sequence $\{y_t\}_{t=1}^{2}$.

Consumption sequence $\{c_t\}_{t=1}^{2}$

Financial capital sequence $\{K_t\}_{t=1}^{3}$.

Period budget constraints:



Consumer saves

$(1+r)(K_1+y_1-c_1)\geq K_2$ (1)

$(1+r)(K_2+y_2-c_2)\geq K_3$ (2)

Consumer's problem: Choose sequences $\{c_t\}$, $\{K_t\}$ to solve

$$Max\{U(c_1,c_2)\,|\,(1+r)(K_1+y_1-c_1)\geq K_2,\ (1+r)(K_2+y_2-c_2)\geq K_3\}$$

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$$Max\{U(c_1,c_2)\,|\,(1+r)(K_1+y_1-c_1)\geq K_2,\ (1+r)(K_2+y_2-c_2)\geq K_3\}$$

Step 1: Try to reduce the problem to a simpler 1 constraint problem

(Divide the first equation by $1+r$ and the second by $(1+r)^2$

$$K_1+y_1-c_1\geq\frac{K_2}{1+r} \qquad (1')$$

$$\frac{K_2+y_2-c_2}{1+r}\geq\frac{K_3}{(1+r)^2} \qquad (2')$$

To maximize utility $K_3=0$.

Then add (1') and (2')

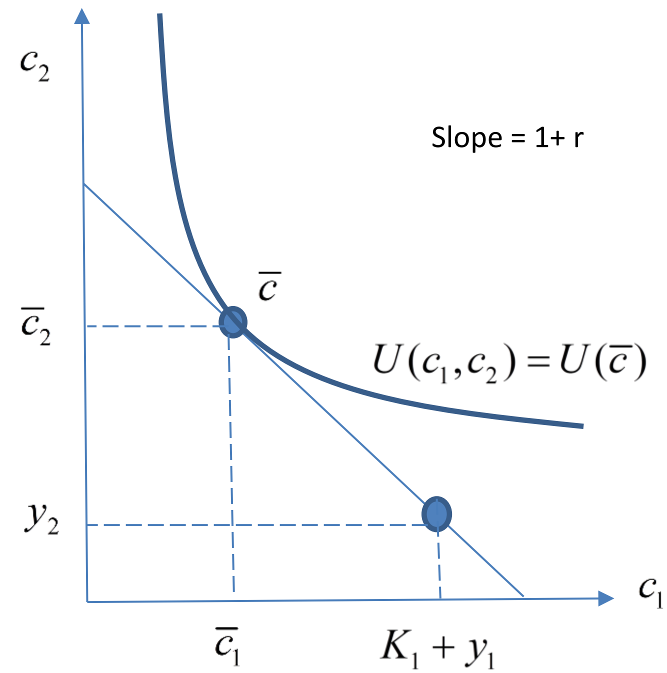
$$K_1+(y_1+\frac{y_2}{1+r})-(c_1+\frac{c_2}{1+r})\geq 0$$

$$K_1 + (y_1 + \frac{y_2}{1+r}) - (c_1 + \frac{c_2}{1+r}) \geq 0$$

Note that the higher the interest rate, the

steeper is the life-time budget line.

The relative price of future consumption has fallen.



Consumer saves

Group Exercise 1:

$U(c) = \ln c_1 + \ln c_2 \ \ r = 0.2$, $\{y_t\} = \{100, 72\}$. Solve for $\{\bar{c}_t\}$

Group Exercise 2:

$U(c) = \ln c_1 + \ln c_2 + \ln c_3 \ \ r = 0.2$, $\{y_t\} = \{100, 72, 72\}$. Solve for $\{\bar{c}_t\}$

Sketch of the answer to group exercise 2

Step 1: how that the life-time budget constraint is

$$c_1+(\frac{1}{1+r})c_2+(\frac{1}{(1+r)^2})c_3=y_1+(\frac{1}{1+r})y_2+(\frac{1}{(1+r)^2})y_3=100+\frac{72}{1.2}+\frac{72}{(1.2)^2}=100+60+50=210$$

Step 2: Obtain the FOC

Note that we can think of $\frac{1}{1+r}$ as the price of the period 2 commodity. Similarly $p_3=\frac{1}{(1+r)^2}$ is the price of the period 2 commodity.

So the prices are $(p_1,p_2,p_3)=(1,\frac{1}{1+r},\frac{1}{(1+r)^2})=(1,\frac{1}{1.2},\frac{1}{1.44})$.

The FOC are

$$\frac{1}{p_1}\frac{\partial U}{\partial c_1}=\frac{1}{p_2}\frac{\partial U}{\partial c_2}=\frac{1}{p_3}\frac{\partial U}{\partial c_3}=\lambda$$

Step 3: Solve using the FOC and budget constraint

Using these conditions you should be able to show that $\bar{c}_1=70$

Comparison with the standard budget constraint

$$c_1 + \frac{c_2}{1+r} \le K_1 + y_1 + \frac{y_2}{1+r}$$

$$p_1 x_1 + p_2 x_2 \le p_1 \omega_1 + p_2 \omega_2$$

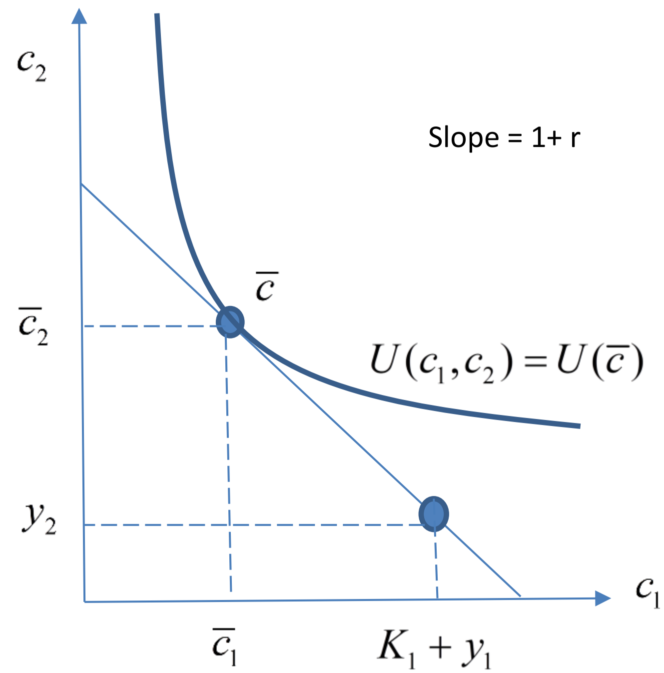
$$x_1 + \frac{p_2}{p_1} x_2 \le \omega_1 + \frac{p_2}{p_1} \omega_2$$

Thus the lifetime budget constraint is

just like the one period budget constraint.

$$\frac{p_2}{p_1} = \frac{1}{1+r}$$

is the market value of future goods relative to current goods



Consumer saves

We call this the present value of future goods.

The life-time budget constraint is

$$PV\{c_t\} \le K_1 + PV\{y_t\}$$

Decomposition into substitution and income effects of an increase in the interest rate

Substitution Effect

The relative price of future goods $1/(1+r)$ is lower so first period consumption falls and second period consumption rises.

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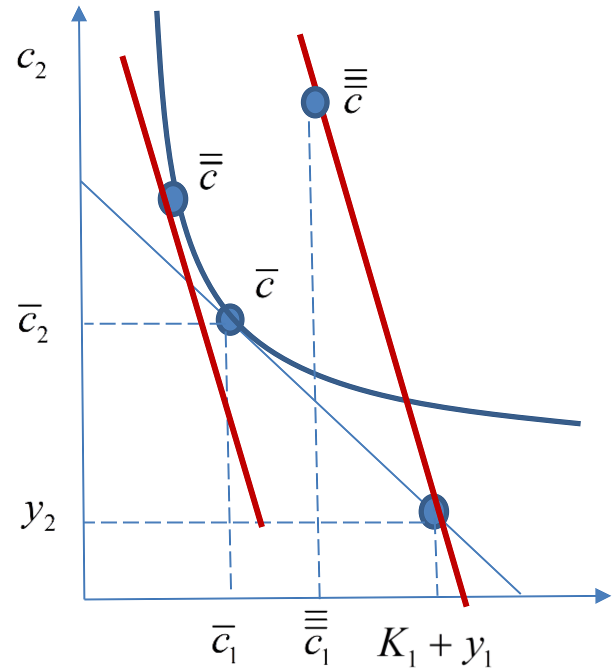


Consumer saves

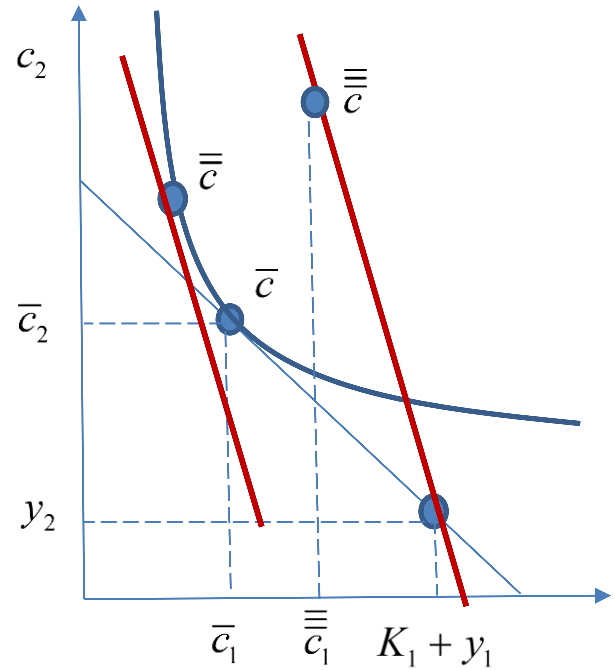
Decomposition into substitution and income effects of an increase in the interest rate

Substitution Effect

The relative price of future goods $1/(1+r)$ is lower so first period consumption falls and second period consumption rises.

Income effect

Since the consumer is a saver, a higher interest rate Makes him better off. Thus income has to be reduced to keep her on the same level set. Assuming that goods are normal, giving the income back leads to higher consumption in both periods. In particular, first period consumption rises.



Consumer saves

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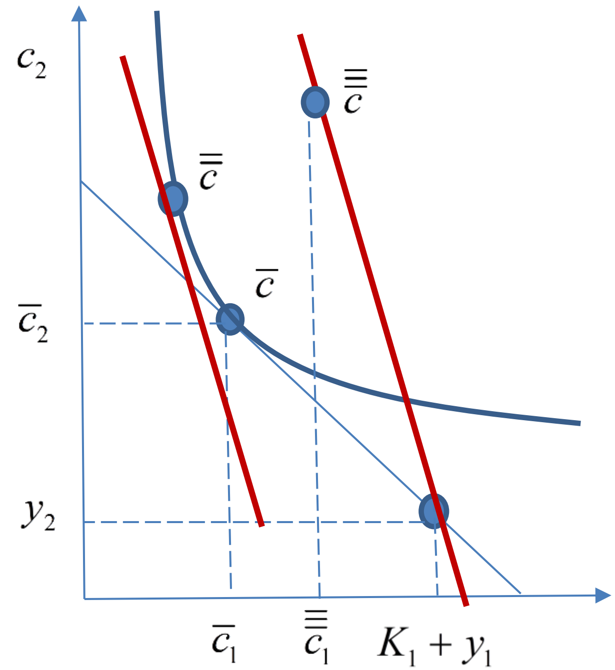


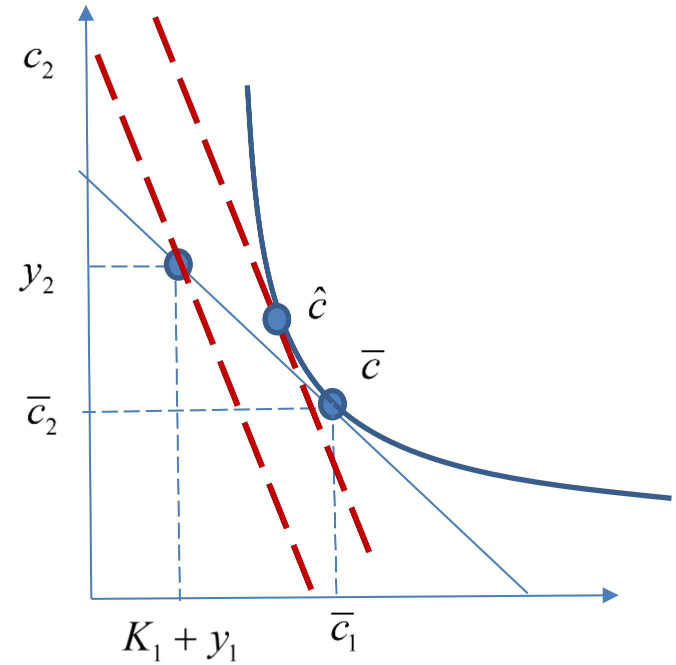
Consumer saves

Total effect

The two effects are off-setting for first period consumption and hence for first period saving.

Data analytics typically show small interest rate effects on personal saving.

Class Exercise: Are the effects reinforcing for borrowers?



Consumer borrows

If the interest rate rise the new life-time budget line

(the dashed line through (y_1, y_2) in the figure)

is steeper.

The consumer is worse off.

If the consumer is subsidized to get her back to the

original level set, the consumes the bundle $\hat{c}$ depicted.

Thus the substitution effect is to move

Around the level set reducing c_1 and increasing c_2 .

Exercise: Discuss the income effect when the subsidy is removed

Hence explain why (assuming normal goods) the two effects

are reinforcing. Both lower first period consumption hence lower borrowing.

Walrasian Equilibrium with two consumers, Alex and Bev

Example 1: Exchange economy with two periods

$\{y_t^A\}=(4,16)$ $\{y_t^B\}=(21,20)$,

No initial financial capital $K_1=0$

$U^h(c_1^h,c_2^h)=u(c_1^h)+\delta u(c_2^h)$, $h=A,B$, where $u(c)=c^{1/2}$

Period 1 goods cannot be stored.

What is the marginal rate of substitution

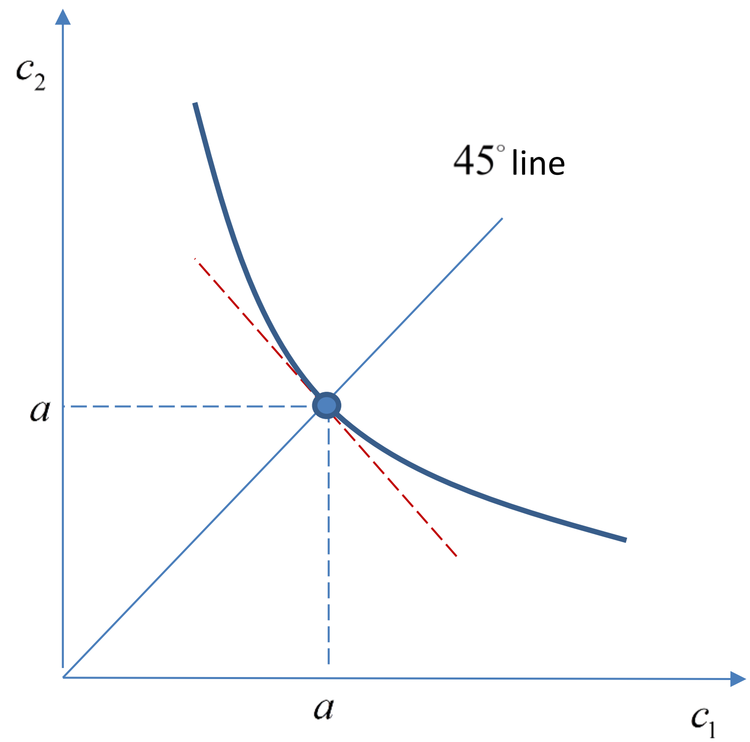
at the 45° line?

$$MRS(c_1,c_2)=\frac{\partial U}{\partial c_1}/\frac{\partial U}{\partial c_2}=\frac{u'(c_1)}{\delta u'(c_2)}\ .$$

Therefore

$$MRS(a_1,a)=\frac{\partial U}{\partial c_1}/\frac{\partial U}{\partial c_2}=\frac{u'(a)}{\delta u'(a)}=\frac{1}{\delta}$$

δ is the discount factor for future consumption



The smaller is the discount factor the smaller is the value of future consumption. Thus the consumer is willing to give up more period 2 consumption for an additional unit of current consumption

Walrasian Equilibrium with two consumers, Alex and Bev

Class example: Exchange economy with two periods

$\{\omega_t^A\}_{t=1}^2 = \{4,16\}$ $\{\omega_t^B\}_{t=1}^2 = \{21,20\}$,

$U^h(c_1^h, c_2^h) = (c_1^h)^{1/2} + \delta(c_2^h)^{1/2}$, $h = A, B$.

Period 1 goods cannot be stored.

For simplicity assume $\delta = 1$

(a) Are there potential gains from exchange?

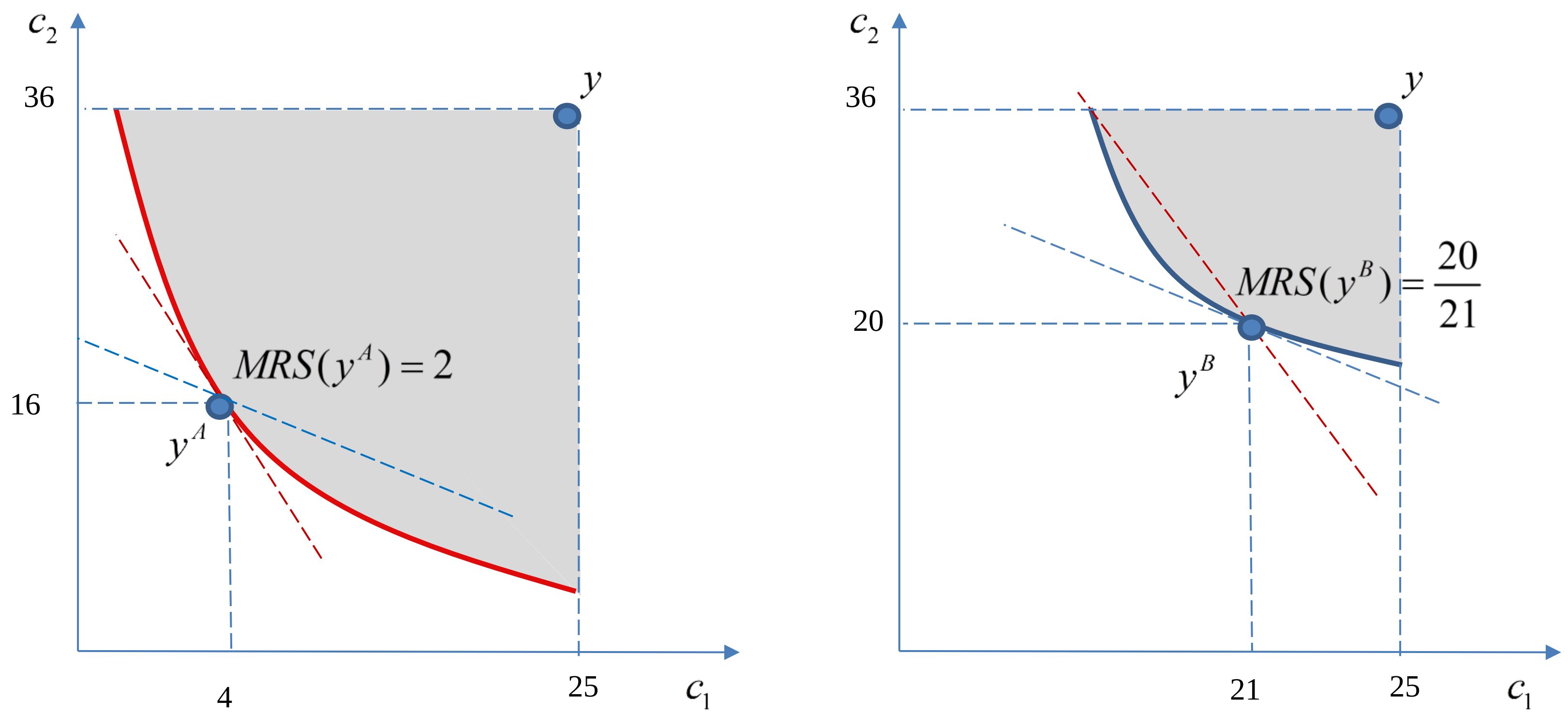
(b) Solve for the equilibrium interest rate.

HINT: Can we solve a simpler problem first?

(c) How does the outcome change if period 1 goods can be costlessly stored and consumed in period 2?

(d) If $\delta < 1$ what is the new equilibrium interest rate?

(a) Gains from exchange



(b) Walrasian equilibrium interest rate

$\{y_t^A\} = \{4, 16\}$, $\{y_t^B\} = \{21, 20\}$,

$U^h(c_1^h, c_2^h) = u(c_1^h) + \delta u(c_2^h) = (c_1^h)^{1/2} + \delta(c_2^h)^{1/2}$, $h = A, B$.

Hint: What is special about this model?

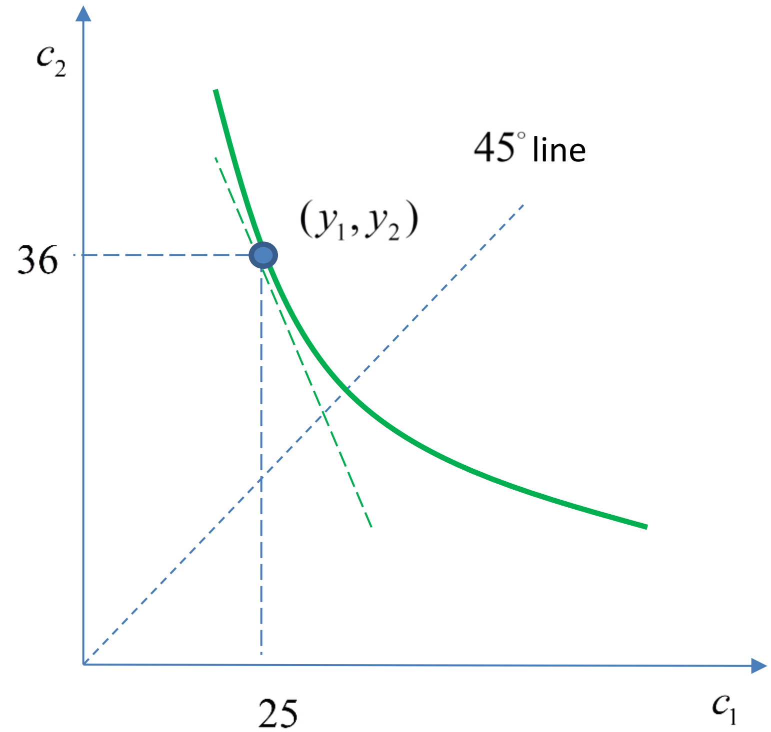
Representative agent

$$MRS(c_1,c_2)=\frac{\frac{\partial U}{\partial c_1}}{\frac{\partial U}{\partial c_2}}=\frac{u'(c_1)}{\delta u'(c_2)}=\frac{1}{\delta}\frac{\frac{1}{2}c_1^{-1/2}}{\frac{1}{2}c_2^{-1/2}}=\frac{1}{\delta}(\frac{c_2}{c_1})^{1/2}$$

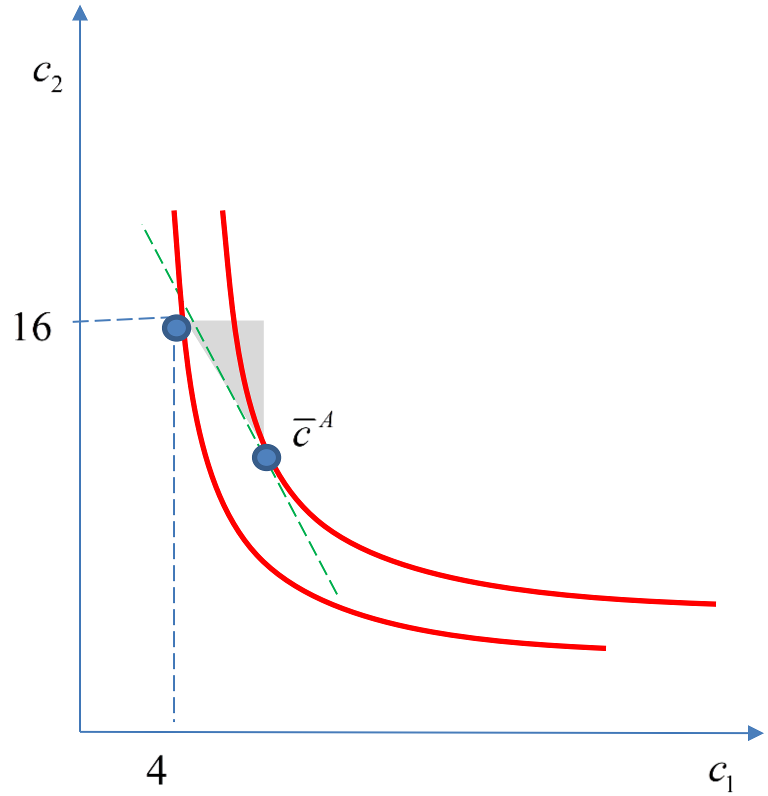
$$MRS(y_1,y_2)=\frac{1}{\delta}(\frac{y_2}{y_1})^{1/2}$$

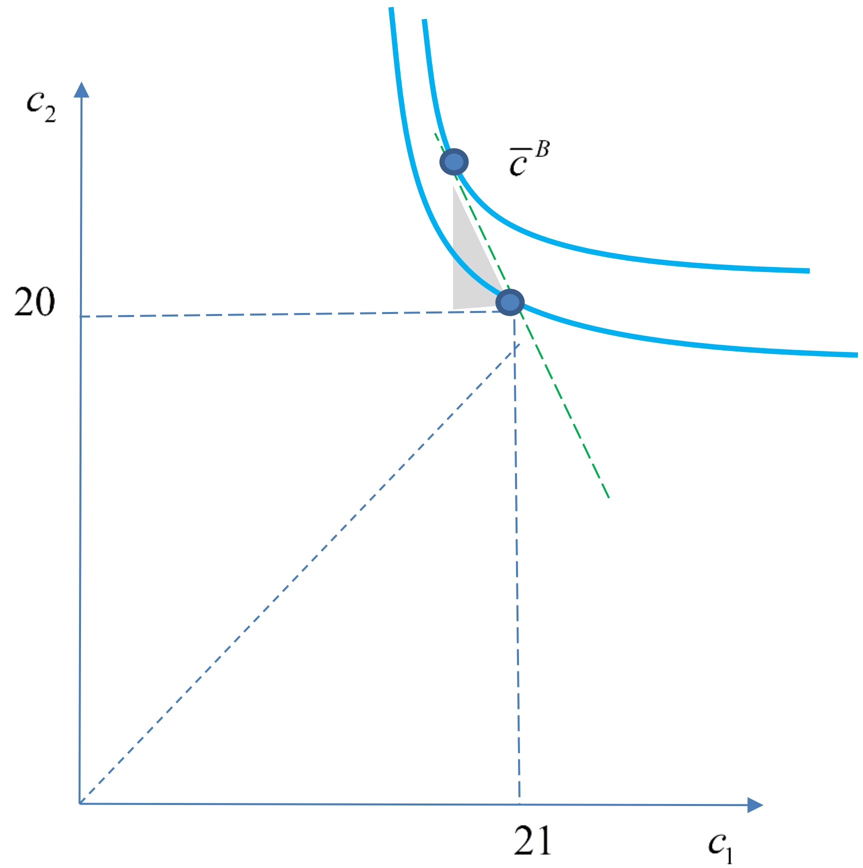
$$(y_1,y_2)=(25,36),\quad \delta=1$$

$$MRS(y_1,y_2)=\frac{1}{\delta}(\frac{y_2}{y_1})^{1/2}=\frac{6}{5}$$



Equilibrium trading





(c) How do we add storage to the model?

Hint: What is the technology?

Group problem

Example 2: Exchange economy with two periods

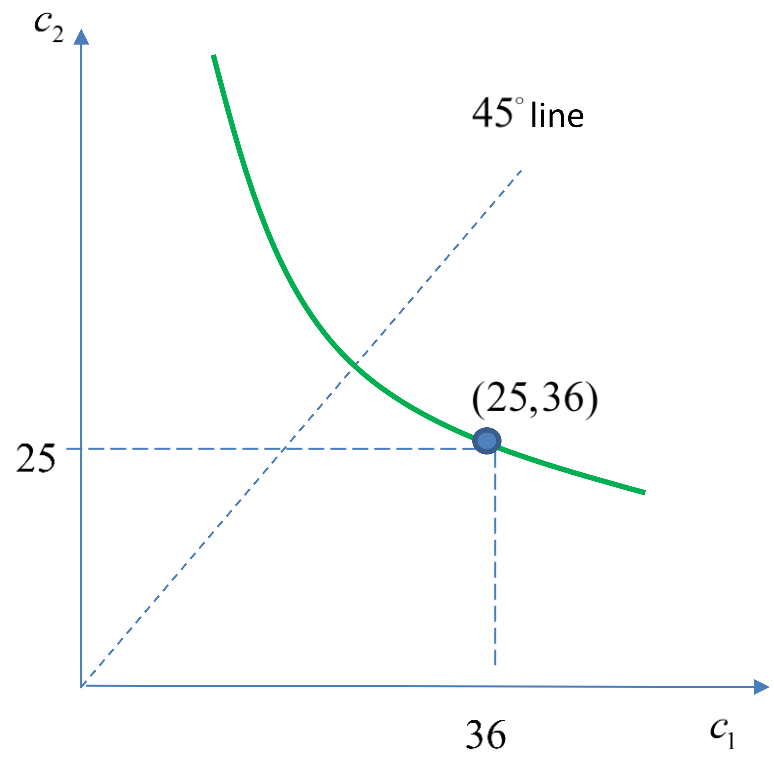
$\{y_t^A\}=\{16,4\}$ $\{y_t^B\}=\{20,21\}$,

Assume $\delta=1$ and that goods cannot be stored

(a) Are there potential gains from exchange?

(b) Solve for the equilibrium interest rate.

(c) How does the outcome change if period 1 goods can be stored at no cost and consumed in period 2?



Costless storage

A very simple example of constant returns to scale. Each unit stored to day yields one unit tomorrow.

The set of feasible plans is $Y = \{(z,q) \mid q \leq z\}$