Econ 401A Microeconomic Theory

Final Exam- Answers

ANSWER TO 1

Cost function and monopoly

(a) Maximize output with a budget of C .

$$\underset{z}{Max}\{z_1^{1/4} z_2^{1/4} \mid r \cdot z \leq C\}$$

Equivalently,

$$\underset{z}{Max}\{\ln z_1 + \ln z_2 \mid r \cdot z \leq C\}$$

FOC

$$\frac{1}{r_1 z_1} = \frac{1}{r_2 z_2}$$

Appealing to the Ratio Rule,

$$\frac{1}{r_1 z_1} = \frac{1}{r_2 z_2} = \frac{2}{C}.$$ Then $z_1 = \frac{C}{2r_1}$, $z_2 = \frac{C}{2r_2}$,

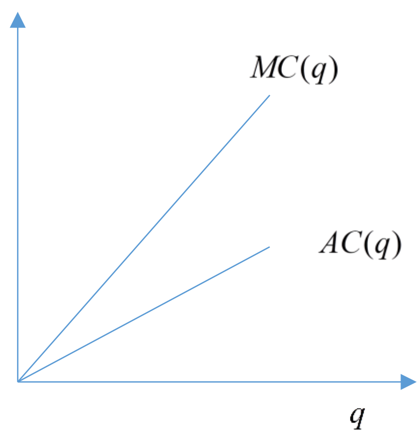
$$q = (z_1 z_2)^{1/4} \quad q^2 = (z_1 z_2)^{1/2} = \frac{C}{2r_1^{1/2} r_2^{1/2}}$$

Inverting,

$$C = 2(r_1 r_2)^{1/2} q^2 = 12q^2$$

(b)

$$AC = 12q, \quad MC = 24q$$



(c)

$$MR = 1200 - 24q, \quad MC = 24q$$

Equating, $\bar{q} = 25$. Then $\bar{p} = 1200 - 12(25) = 900$

(d) Arguing as above

$$\frac{1}{r_1(z_1-12)} = \frac{1}{r_2 z_2} = \frac{2}{C-r_1 12}$$ Then $z_1 - r_1 12 = \frac{C-r_1 12}{2r_1}$, $z_2 = \frac{C-r_1 12}{2r_2}$,

$$q^2 = ((z_1-12)z_2)^{1/2} = \frac{C-r_1 12}{2r_1^{1/2} r_2^{1/2}}.$$

Inverting,

$$C - r_1 12 = 2(r_1 r_2)^{1/2} q^2 = 12q^2$$

Therefore

$$C = r_1 12 + 12q^2 \ .$$

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(d) Sophisticated answer

$$C = r_1 z_1 + r_2 z_2 = 12r_1 + r_1(z_1 - 12) + r_2 z_2$$

Therefore

$$C - r_1 12 = r_1(z_1 - 12) + r_2 z_2$$

Define the new variable $\hat{z}_1$. Then the problem solved on (a) is identical except that the new budget is $C - 12r_1$.

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(e) MC is the same so $\bar{q} = 25$

(f) If k is sufficently small, no change. If k gets too large the monopoly cannot make a profit.

ANSWER TO 2

Waltraian Equilibrium

(a) $U(x) \geq U(u) \Rightarrow U(\theta x) \geq U(\theta y)$

(b) Since $x(p,I)$ is maximizing and $Ix(p,1)$ is feasible with income I

$U(x(p,I) \geq U(Ix(p,1))$ (i)

By an identical argument

$U(x(p,1) \geq U(\frac{1}{I}x(p,I))$ (ii)

Appealing to homotheticity, it follows from (ii) that

$U(Ix(p,1) \geq U(x(p,I))$ (iii)

(i) and (iii) imply that $U(x(p,I) = U(Ix(p,1))$

(c) $x_1^r = 32 - z_1,\ x_2^r = 32 - z_2,\ z_3^r = q_3 = z_1^{1/4} z_2^{1/4} z_4^{1/2}$

Substitute into the utility fucntion

$$U^r = \ln(32 - z_1) + \ln(32 - z_2) + 4\ln z_1^{1/4} z_2^{1/4} z_4^{1/2}$$

$$= \ln(32 - z_1) + \ln(32 - z_2) + \ln z_1 + \ln z_2 + 2\ln z_4$$

To maximize output, all input 4 is used in production so $\bar{z}_4^r = 16$

The optimim is achieved by allocating the two inputs

FOC

Input 1: $\frac{\partial U^r}{\partial z_1} = -\frac{1}{32 - z_1} + \frac{1}{z_1} = 0$. Solving, $\bar{z}_1^r = 16$.

Input 2: By a symmetrical argument, $\bar{z}_2^r = 16$.

Then output of commodity 3 is $q_3 = z_1^{1/4} z_2^{1/4} z_4^{1/2}$ = 16.

(d) For the representative agent, the FOC for utility maximization is

$$\frac{1}{p_1}\frac{\partial U}{\partial x_1} = \frac{1}{p_2}\frac{\partial U}{\partial x_2} = \frac{1}{p_3}\frac{\partial U}{\partial x_3}$$

$$\frac{1}{p_1}\frac{1}{x_1} = \frac{1}{p_2}\frac{1}{x_2} = \frac{1}{p_3}\frac{4}{x_3}.$$

SInce $\bar{x}^r = (16,16,16)$ it follows that $(p_1, p_2, p_3) = (1,1,4)$.

(e) To determine the equilibrium price of commodity 4 we must consider the profit maximizing firm.

$$\Pi = p_3 q_3 - p_1 z_1 - p_2 z_2 - p_4 z_4$$

$$= 4q_3 - 1z_1 - 1z_2 - p_4 z_4$$

$$= 4z_1^{1/4} z_2^{1/4} z_4^{1/2} - z_1 - z_2 - p_4 z_4$$

$\frac{\partial \Pi}{\partial z_4} = 2z_1^{1/4} z_2^{1/4} z_4^{-1/2} - p_4 = 2(2)(2)\frac{1}{4} - p_4 = 2 - p_4 = 0$ for profit maximization.

Remark: We could also use the FOC for profit maximization to solve for p_2 and p_3 .

ANSWER TO 3

Asset prices and state claims prices

(a) Asset returns Portfolio return Desired return

z_b z_c $q_b z_b + q_c z_c$

$$\begin{bmatrix}500\\300\end{bmatrix} \quad \begin{bmatrix}600\\300\end{bmatrix} \quad \begin{bmatrix}500q_b+500q_c\\300q_b+300q_c\end{bmatrix} \quad \begin{bmatrix}10\\0\end{bmatrix}.$$

Therefore

$$500q_b + 600q_c = 100$$
$$300q_b + 300q_c = 0$$

From the second equation $q_c = -q_b$.

Therefore

$$500q_b + 600(-q_b) = -100q_b = 10 \ .$$

Therefore $q_b = -\frac{1}{10}$ and so $q_c = \frac{1}{10}$. Then the portfolio is

$$(\overline{q}_b, \overline{q}_c) = (-\tfrac{1}{10}, \tfrac{1}{10})$$

(b) An amost identical argument establishes that the second portfolio is $(\overline{\overline{q}}_b, \overline{\overline{q}}_c) = (\frac{1}{5}, -\frac{1}{6})$.

(c) The value of the first portfolio $(\overline{q}_b, \overline{q}_c) = (-\frac{1}{10}, \frac{1}{10})$ Is the cost of buying these quantities of the two assets

$$P_b\overline{q}_b + P_c\overline{q}_c = 2100(-\tfrac{1}{10}) + 2400(\tfrac{1}{10}) = 30 .$$

The value of the second portfolio $(\overline{\overline{q}}_b, \overline{\overline{q}}_c) = (\frac{1}{5}, -\frac{1}{6})$ Is the cost of buying these quantities of the two assets

$$P_b\overline{\overline{q}}_b + P_c\overline{\overline{q}}_c = 2100(\tfrac{1}{5}) + 2400(-\tfrac{1}{6}) = 420 - 400 = 20 .$$

(d) Buying one unit in each of these funds yields 10 in both states. The cost of this is the sum of the cost of each find = 30 + 20 = 50.

(e) A state s claim pays off 1 in state s and nothing otherwise. Thus the two mutual funds are equivalent to 10 state claims. Then

$$(p_1, p_2) = (3, 2) \ .$$

(f) The value of the first asset is computed using the state claims prices so

$$P_a = (p_1, p_2)\cdot(400,100) = 1400 \ .$$

(g) Alex has a wealth of 1400. The state claims price vector is $(p_1, p_2) = (3, 2)$

His maximization problem is

$$Max\{\tfrac{2}{3}\ln(80 + x_1) + \tfrac{1}{3}\ln(80 + x_2) \,|\, 3x_1 + 2x_2 \le 1400\}$$

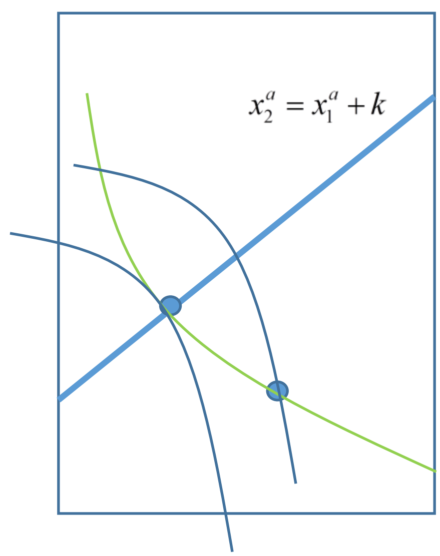
(h) If the mutual funds exist he can sell his asset and buy $\frac{1}{10}\overline{x}_1$ units of the first fund and $\frac{1}{10}\overline{x}_2$ units of the second fund. If the funds do not exist he can buy the shares compted above that are equyivalent to the mutual funds.

ANSWER TO 4

Pareto Efficiency

(a) If $MRS_a(x^a) > MRS_b(x^b)$ then Alex is will to give uop more of commodity 2 to obtain an additional unit of commodity 1. Then both are better off making such a trade at any rate between the two marginal rates of subsitution.

(b) In the Edgeworth Box the indifference curves must be tangential otherwise there is a lens shaped region of mutual gain.



(c) $\dfrac{\partial U}{\partial x_1} = e^{-x_1}$, $\dfrac{\partial U}{\partial x_2} = e^{-x_2}$. Therefore

$$MRS(x^a) = \frac{\partial U}{\partial x_1} / \frac{\partial U}{\partial x_2} = e^{x_2^a - x_1^a} \text{ and}$$

$$MRS(x^b) = \frac{\partial U}{\partial x_1} / \frac{\partial U}{\partial x_2} = e^{x_2^b - x_1^b} .$$

For effciency these must be equal. Therefore

$$x_2^a - x_1^a = x_2^b - x_1^b = (200 - x_2^a) - (100 - x_1^a) = 100 - (x_2^a - x_1^a)$$

Therefore $2(x_2^a - x_1^a) = 100$ and so $x_2^a - x_1^a = 50$

(d) $\omega^a = \omega^b = \frac{1}{2}\omega = (50,100)$ lies on this line so is Pareto Efficient. If the price ratio is equal to the MRS, i.e

$$\frac{p_1}{p_1} = MRS_a = e^{x_2^a - x_1^a} = e^{50} \ ,$$

The FOC for utility maximization are satisifed

so this is a WE allocation.

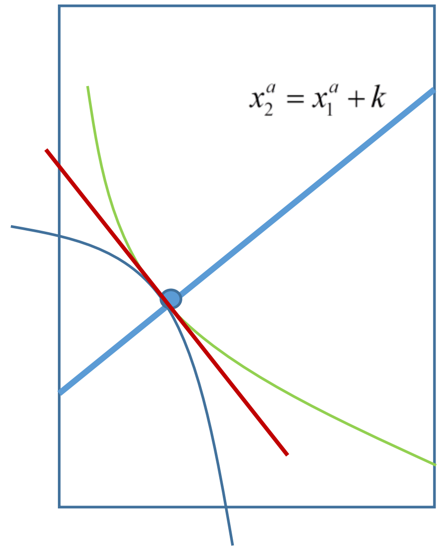
(e) Everywhere along the PE line

$$MRS_a = MRS_b = e^{x_2^a - x_1^a} = e^{50} .$$

Then regardless of the endowment,

$$\frac{p_1}{p_2} = MRS_a = MRS_b = e^{x_2^a - x_1^a} = e^{50} .$$

Thus the equilibrium price ratio does not change.



ANSWER TO 5

Strategy

(a) Mutual best responses i.e. each firm is maximizintg its profit given the choice of the other firm.

(b) FOC

$$\frac{\partial \Pi_1}{\partial q_1}(q_1, \bar{q}_2) = 0 \, , \ \frac{\partial \Pi_2}{\partial q_2}(\bar{q}_1, q_2) = 0$$

(c) Consider the last round of the game. If firm 1 believes that firm 2 will coopoerate, it has an incentive to deviate. Thus there will be no NE cooperation in the last round. Kmowing this, the firms are left with the game with $n-1$ rounds. Exactly the same argument holds for the last round of this $n-1$ round game.

Repeating this argument, there is no NE cooperation in the finitely repeated game.

(d)

$$\Pi_1 = R_1 - C_1 = p_1 q_1 - 6q_1 = (38 - q_1 - \alpha q_2) - 6q_1$$

$$\frac{\partial \Pi_1}{\partial q_1} = 38 - 2q_1 - \alpha q_2 - 6q_1 = 32 - 2q_1 - \alpha q_2 = 0$$ for a best reponse.

Then $\bar{q}_1 = 16 - \frac{1}{2}\alpha q_2$

$\Pi_2 = R_2 - C_2 = p_2 q_2 - 6q_2 = (44 - q_1 - \alpha q_2) - 6q_1$

$\frac{\partial \Pi_2}{\partial q_2} = 38 - 2q_8 - \alpha q_1 = 0$ for a best reponse.

Then $\bar{q}_2 = 19 - \frac{1}{2}\alpha q_1$

If $\alpha = 1/2$

$\bar{q}_1 + \frac{1}{4}\bar{q}_2 = 16$

$\frac{1}{4}\bar{q}_1 + \bar{q}_2 = 19$

Multiply the first equation by 4

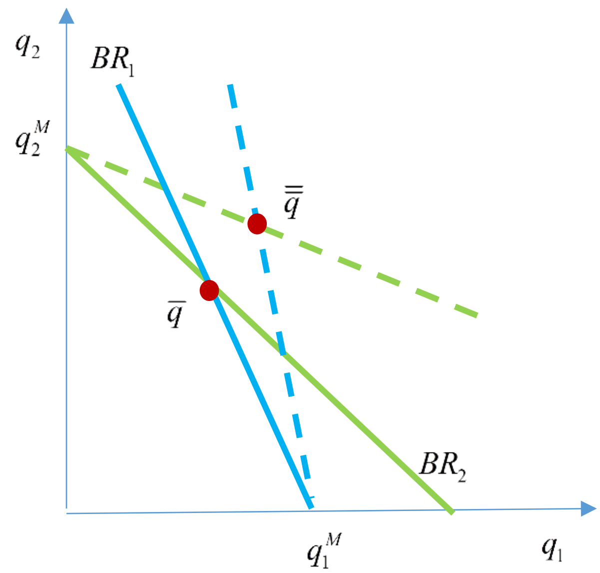
$4\bar{q}_1 + \bar{q}_2 = 64$

$\frac{1}{4}\bar{q}_1 + \bar{q}_2 = 19$

Subtract the second equation

$\frac{15}{4}\bar{q}_1 = 45$. Then $\bar{q}_1 = 12$ and so $\bar{q}_2 = 16$.

(d)



(e) The steepness of the BR_2 line is lower and the steepness of the BR_1 line is greater. The intercepts with the axes do not change. Thus both outputs rise.

In the limit the two firms are in separate markets and so both choose their monopoly outputs.

ANSWER TO 6

Sealed high-bid auctions

(a) It is natural to assume that the the equilibrium bid fucntion is symmetric and increasing. Thus in equilibrium the high value buyer wins. Then his win probability if his value is θ is the probability that the other buyer's value is lower. Then

$w(\theta) = \Pr\{v_2 \le \theta\} = F(\theta) = \theta^2$.

The equilibrium expected payoff is

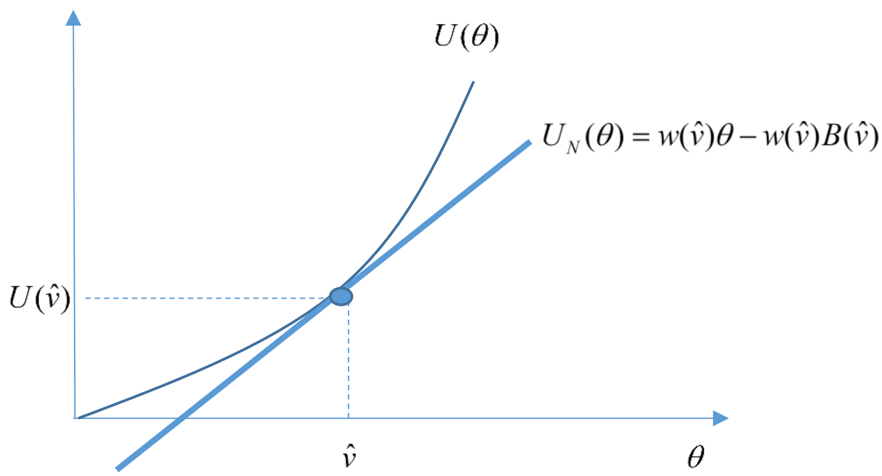
$U(\theta) = w(\theta)(\theta - B(\theta)) = w(\theta)\theta - w(\theta)B(\theta)$ (*)

We can determine the incremental payoff $U'(\theta)$ by considering the payoff of a naïve buyer who always bids $B(\hat{v})$ and so his payoff is

$$U_N(\theta) = w(\hat{v})(\theta - B(\hat{\theta})) = w(\hat{\theta})\theta - w(\hat{\theta})B(\hat{\theta}).$$

The graph of this line has a slope of $w(\hat{v})$. It has the same value at $\theta = \hat{v}$ and is lower than the maximized payoff else where. Thus the two lines touch at $\hat{v}$ and so

$U'(\hat{v}) = w(\hat{v}) = F(\hat{v})$.



This argument holds for all $\hat{v}$. Therefore

$U'(\theta) = w(\theta) = F(\theta)$.

We can then integgrate to solve for $U(\theta)$ and substiturte into (*) to obtain $B(\theta)$

(b) Then $U'(\theta) = w(\theta) = F(\theta) = \theta^2$

$U(\theta) = \frac{1}{3}\theta^3$

Also

$U(\theta) = w(\theta)\theta - w(\theta)B(\theta) = \theta^3 - \theta^2 B(\theta)$.

Therefore

$\frac{1}{3}\theta^3 = \theta^3 - \theta^2 B(\theta)$ and so $B(\theta) = \frac{2}{3}\theta$.

(c) $w(\theta) = F^2(\theta) = \theta^4$

$U'(\theta) = w(\theta) = F(\theta) = \theta^4$.

By an almist identical argument $B(\theta) = \frac{4}{5}\theta$.

(d) If you win your gross payoff is θ thus your gross expected payoff is $w(\theta)\theta$.

You pay your bid so

$U(\theta) = w(\theta)\theta - \bar{B}(\theta)$.

The argument for the Naïve buyer is the same therefore we can use the same method to solve for the new equilibrium bid.

In fact $\bar{B}(\theta) = w(\theta)B(\theta)$ where $B(\theta)$ is the equilibrium bid wghen only the winner pays.