Homework 2

1. Production and Cost

A firm has a plant with a production set

$Y = \{(z_1, z_2, q_3) \mid q_3^4 \le \frac{256}{27} z_1 z_2^3\}$, where z_1 and z_2 are inputs and q_3 is the output.

The input prices are p_1 and p_2. The manager is given a budget $\bar{B}$ and told to maximize the output of the plant (equivalently, to maximize the cube of the output.

(a) Explain why the maximizing inputs must be strictly positive.

(b) Solve for the inputs that maximize output cubed.

(c) Explain how you can use this result to solve for the cost function of the firm, i.e. the lowest cost to produce q units. .

(d) Confirm that $AC(q) = MC(q)$.

(e) If the firm is a price taker what is the maximum possible output price $\bar{p}_3$?

(f) What is the output if $p_3 < \bar{p}_3$?

2. Production and Cost

A firm has a plant with a production set

$Y = \{(z_1, z_2, q_3) \mid q_3^3 \le 4(z_1 - 4)z_2,\ z_1 \ge 4\}$, where z_1 and z_2 are inputs and q_3 is the output. Output is zero if $z_1 < 4$ The input prices are p_1 and p_2. The manager is given a budget $\bar{B}$ and told to maximize the output of the plant (equivalently, to maximize the cube of the output.

(a) Show that if output is produced, the solution is $\bar{z}_1 - 4 = \frac{\bar{C} - 4p_1}{2p_1}$ and $\bar{z}_2 = \frac{\bar{C} - 4p_1}{2p_2}$.

(b) Hence show that $\bar{q}_3^3 = \frac{(\bar{C} - 4p_1)^2}{p_1 p_2}$

Hint: It may be easier to maximize $(q_3)^3$ or $\ln q_3^3$

(c) Use this solve for the cost function $C(q_3)$

Henceforth assume that $p_1 = p_2 = 2$.

(d) Depict the average cost function $AC(q_3) = C(q_3)/q_3$ and the marginal cost function $MC(q_3)$.

(e) At what output is $AC(q_3) = MC(q_3)$?

(f) If the firm is a price taker, for what range of output prices will the firm not produce commodity 3?

3. Walrasian equilibrium with 16 identical firms.

The aggregate endowment is $\omega = (32, 0)$. The production set for firm f is

$Y^f = \{(z_1{}^f, q_2{}^f) \mid q_2{}^f \leq (z_1{}^f)^{1/2}\}$. There are 16 identical firms.

(a) Show that if the industry is allocated z units of input, then the maximized industry output can be written as follows:

$q_2 = q_2{}^1 + ... + q_2^{16} = Az^{1/2}$

(You must solve for A .)

(b) If all consumers have the same utility function $U(x_1, x_2) = x_1 x_2{}^2$, solve for the optimum using the representative consumer.

(c) Hence solve for the WE prices.

(d) Solve also for the WE profit of each firm and hence the total profit.

4. Walrasian equilibrium with 64 identical firms.

(a) Question 3 if instead there are 64 identical firms.

(b) What is the change in total profit?

(c) Do you find this result surprising? Discuss briefly.