Homework 1 Answers

Answer to 1.

$$\pi(q) = 128q_1 + 312q_2 - (2q_1 + 5q_2)^2 + q_2^{\ 2}$$

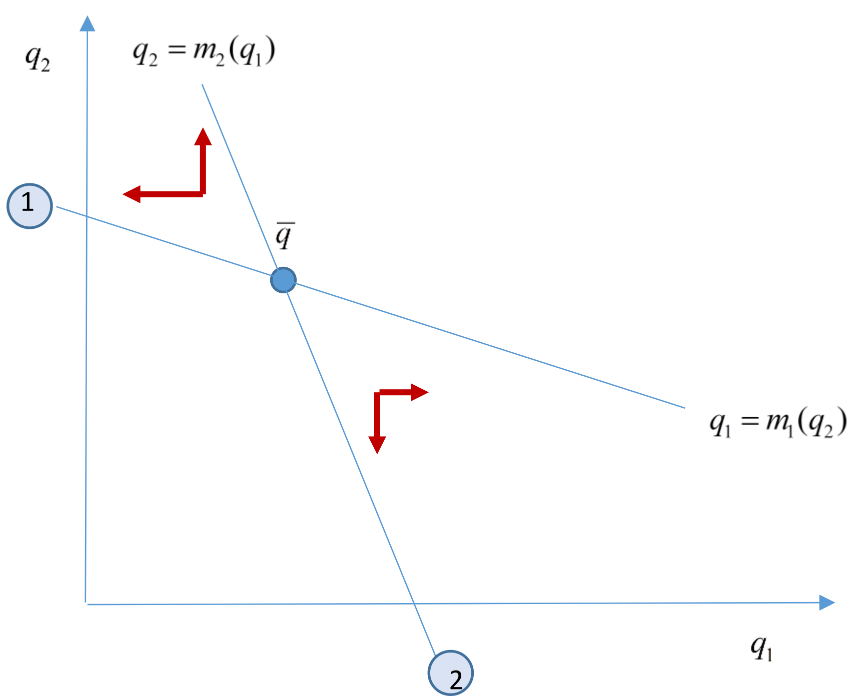
(a) Necessary conditions for a critical point.

$$\frac{\partial \pi}{\partial q_1}(q) = 128 - 4(2q_1 + 5q_2) = 0$$

$$\frac{\partial \pi}{\partial q_2}(q) = 312 - 10(2q_1 + 5q_2) + 2q_2 = 0$$

Solve two linear equations: Solution $\bar{q} = (6,4)$. Check by substitution.

(b) The maximizing lines are as depicted below.



Profit-maximizing lines.

(c) Profit is increasing in the zone indicated so $\bar{q}$ is not a local maximum.

(d) Consider solutions (i) with $q_1 = 0$ and (ii) with $q_2 = 0$.

(i) $\pi(0,q_2)=312q_2-25q_2^{\,2}+q_2^{\,2}$ (ii) $\pi(q_1,0)=128q_1-4q_1^{\,2}$

(e) $\pi(q)=128q_1+312q_2-(2q_1+5q_2)^2+bq_2^{\,2}$

E7 =40*B7+60*C7+(2*B7+5*C7)^2+D4*C7^2

	A	B	C	D	E	F	G	H
1								
2								
3		p1	p2	b				
4		168	372	-1				
5								
6		q1	q2	R(q)	C(q)	profit		
7		6	4	2496	1488	1008		
8								
9		q1	q2	R(q)	C(q)	profit		
10		16	0	2688	1664	1024		
11								
12		q1	q2	R(q)	C(q)	profit		
13		0	6.5	2418	1404	1014		
14								
15								
16		p1	p2	b				
17		168	372	5				
18								
19		q1	q2	R(q)	C(q)	profit		
20		16	0	2688	1664	1024		

(i) $b=-1$

If you choose $q=(6,4)$ Solver indicates a maximum. This is not correct! Solver only looks locally in the x1 and x2 directions and both partial derivatives are zero.

If you start anywhere else the program will climb uphill to a local maximum. The results are shown above so $q=(16,0)$ is the maximum

(ii) $b>2$

Increasing the cost of commodity 2 can only make it even less profitable so the solution is unchanged.

Remark: From the class discussion in a couple of days, you will be able to show that the profit function is concave. Thus if you find a q=(q1,q2) satisfying the necessary conditions, that vector is the maximizer.

Answer to 2:

Make $F(z)$ as big as possible so make $\dfrac{1}{F(z)} = \dfrac{1}{z_1} + \dfrac{1}{z_2}$ as small as possible. Then it is equivalent to make $G(z) = -\dfrac{1}{F(z)} = -\dfrac{1}{z_1} - \dfrac{1}{z_2}$ as big as possible.

$$L = -\frac{1}{z_1} - \frac{1}{z_2} + \lambda(B - p_1 z_1 - p_2 z_2)$$

$$\frac{\partial L}{\partial z_1} = \frac{1}{z_1^{\ 2}} - \lambda p_1 \leq 0 \text{ with equality if } z_1 > 0 \ .$$

Note that the right hand side approaches infinity as z_1 approaches zero. Therefore

$$\frac{\partial L}{\partial z_1} = \frac{1}{z_1^{\ 2}} - \lambda p_1 = 0 \text{. Hence } \frac{1}{z_1} = p_1^{\ 1/2} \lambda^{1/2} \text{ and so } \frac{1}{p_1 z_1} = \frac{p_1^{\ 1/2} \lambda^{1/2}}{p_1} = \frac{\lambda^{1/2}}{p_1^{\ 1/2}}$$

Therefore $p_1 z_1 = \dfrac{p_1^{\ 1/2}}{\lambda^{1/2}}$. By an identical argument $p_2 z_2 = \dfrac{p_2^{\ 1/2}}{\lambda^{1/2}}$. Therefore

$$p_1 z_1 + p_2 z_2 = \frac{1}{\lambda^{1/2}} (p_1^{\ 1/2} + p_2^{\ 1/2}) = B$$

So $\qquad \lambda^{1/2} = \dfrac{p_1^{\ 1/2} + p_2^{\ 1/2}}{B}$.

Hence $\dfrac{1}{z_1} = p_1^{\ 1/2} \lambda^{1/2} = (\dfrac{p_1^{\ 1/2} + p_2^{\ 1/2}}{B}) p_1^{\ 1/2}$

By an identical argument,

$$\frac{1}{z_2} = p_2^{\ 1/2} \lambda^{1/2} = (\frac{p_1^{\ 1/2} + p_2^{\ 1/2}}{B}) p_2^{\ 1/2} .$$

Therefore

$$\frac{1}{z_1} + \frac{1}{z_2} = (\frac{p_1^{\ 1/2} + p_2^{\ 1/2}}{B})(p_1^{\ 1/2} + p_2^{\ 1/2}) = \frac{(p_1^{\ 1/2} + p_2^{\ 1/2})^2}{B}$$

Therefore

$$q = F(z) = \frac{1}{\frac{1}{z_1} + \frac{1}{z_2}} = \frac{B}{(p_1^{1/2} + p_2^{1/2})^2} \ . \qquad (*)$$

Remark: We can use this result to ask how big a budget we need to produce $\bar{q}$ units. From the result above, $\bar{q}$ is feasible with a budget of $\bar{B}$ satisfying

$$\bar{q} = \frac{\bar{B}}{(p_1^{1/2} + p_2^{1/2})^2} .$$

From (*) maximum output is a strictly increasing function of the budget. Thus with any smaller budget it is not possible to produce $\bar{q}$. Thus the minimum budget is

$$\bar{B} = (p_1^{1/2} + p_2^{1/2})^2 \bar{q}$$

3. Walrasian Equilibrium

(a) The budget constraint is $p \cdot \omega - p \cdot x \geq 0$. If the price vector is scaled up to θp the budget constraint becomes

$$\theta p \cdot \omega - \theta p \cdot x = \theta(p \cdot \omega - p \cdot x) \geq 0$$

Thus the constraint on x is unaffected.

(b) Supply

Suppose that $p_1 > 4$. Bev likes commodity 1 four times as much. If she sells a unit that gives her $p_1 > 4$ dollars so she can then purchase more than 4 units of commodity 1. So she supplies all of her endowment. (7 units)

If $p_1 < 4$ the argument is reversed. Bev will not supply any units of commodity 1.

A similar argument holds for Alex. If $p_1 > 2$ he supplies all of his endowment of commodity 1 (8 units). If $p_1 < 2$ he supplies no units.

Hence the market supply function is as depicted.

Note that if the price $p_1 = 2$, Alex is indifferent as to how many units to trade.

(c) Demand.

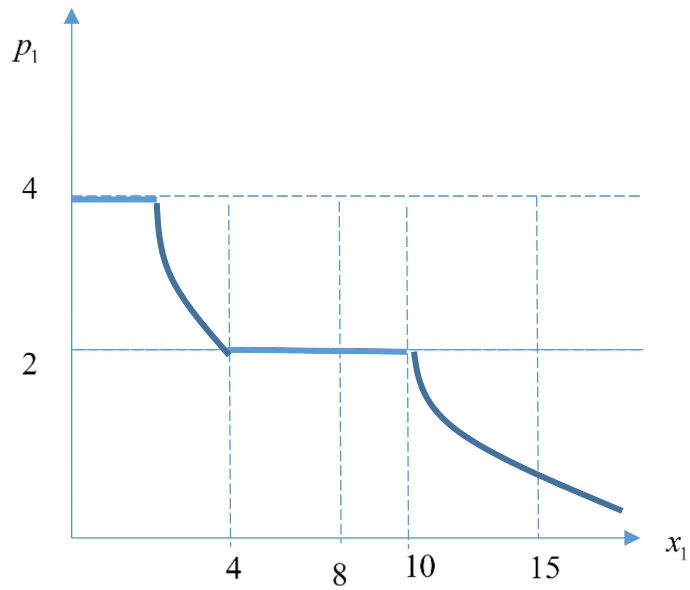
If the price is above 4 neither consumer purchases any of commodity 1. If the price is between 4 and 2 Alex sells her 8 units of commodity 2 and uses the money to purchase commodity 1.

If the price is below 2 both sells their endowment of commodity 2 and use the money to purchases commodity 1

(d) If the price is between 4 and 2 Alex sells her 8 units of commodity 2. She uses this to purchase commodity 1 so $p_1 x_1^A = 8$. Then market demand is $x_1 = x_1^A = \dfrac{8}{p_1}$.

If the price is below 2 all 20 units of commodity 2 are sold and the funds are spent on commodity 1. Therefore $p_1(x_1^A + x_1^B) = 20$ and so demand is $x_1 = x_1^A + x_1^B = \dfrac{20}{p_1}$

Market demand is therefore as depicted below.



Market demand

Since one of the consumers is indifferent as to how much is traded when $p_1 = 4$ or $p_1 = 2$ there is a range of possible demand sat these prices.

(e) From the two figure the equilibrium price is $p_1 = 2$

(f) $p_1(x_1(p) - \omega_1) + p_2(x_2(p) - \omega_2) = 0$.

If the market for commodity 1 clears so that $x_1(p) = \omega_1$, then

$p_2(x_2(p) - \omega_2) = -p_1(x_1(p) - \omega_1) = 0$. So $x_2(p) = \omega_2$.

Remark: You could also answer this question by depicting the budget line and level sets of the two consumers.

4. Equilibrium trades

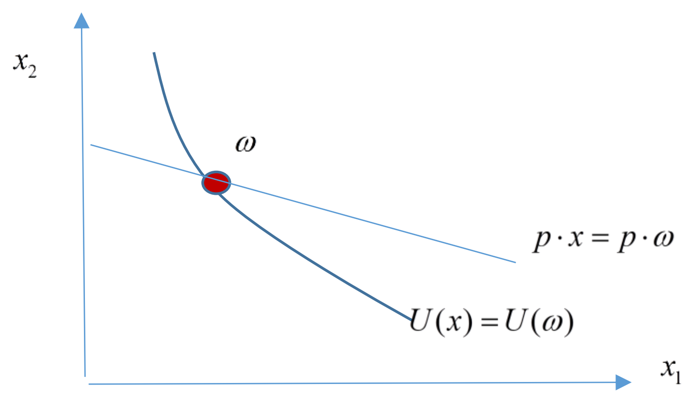
(a) If the rate at which utility rises with commodity 1 is 6 and the rate at which utility rises with commodity 2 is 2, then the consumer is willing to give up 6/2 = 3 units of commodity 2 for one more unit of commodity 1. Generally, If the rate at which utility rises with commodity 1 is MU1 and the rate at which utility rises with commodity 2 is MU2, then the consumer is willing to give up MU1/MU2 units of commodity 2 for one more unit of commodity 1.

This is the consumer's marginal willingness to substitute commodity 1 for commodity 2.

$$MRS^A(x) = \frac{x_2}{x_1} \text{ , } MRS^B(x) = (\frac{x_2}{x_1})^2$$

(b) These are both 1 on the 45° line. If $x_2 > x_1$ then $(\frac{x_2}{x_1})^2 - \frac{x_2}{x_1} = \frac{x_2}{x_1}(\frac{x_2}{x_1} - 1) > 0$. Thus Bev has a higher MRS. The argument is reversed if $x_2 < x_1$.

(c) Suppose as depicted below that $MRS(\omega) > p_1 / p_2$ as depicted below.



Then such a consumer will wish to purchase additional units of commodity 1 by selling some of his endowment of commodity 1.

On the 45° line the two consumers have a MRS =1. Thus if the price ratio $p_1 / p_2 < 1$ both will wish to buy more of commodity 1 and sell some of commodity 2.

If $p_1 = p_2$ then neither will wish to trade so markets clear.

(d) If the two consumers have the same endowment above the 45° line then Bev has a greater MRS than Alex. Both therefore gain if the exchange rate lies between the two marginal rates of substitution. Bev sells commodity 1 and Alex sells commodity 2.

If the ratios are equal and greater than 1 then Bev's MRS is higher. Then raising Alex's ratio above that of Bev's increases his MRS so it is not possible to say who trades what.

If the ratios are equal and less than 1 then Alex's MRS is higher. Raising Alex's ratio increases his MRS further so now Alex sells commodity 1 and Bev sells commodity 2