Weak and Strong Signals*

by

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1. Introduction

Asymmetric information and the implications for economic incentives are now central to microeconomic theory. It is therefore hard to think back to the revolutionary days of the 1970's when General Equilibrium analysis was dominant and our most recent Nobel Laureates wrote their early path-breaking papers (Akerlof (1970), Spence (1973) and Rothschild-Stiglitz (1976)).[1] From a modeling perspective, their analysis of participation and incentive constraints in competitive markets can be viewed as the second stage in the development of the new theory, following the foundational work on mechanism design by Vickrey (1961) and Mirrlees (1971). However, this perspective misses what I believe to be the central motivation behind the revolution: with asymmetric information competitive markets are likely to perform very poorly. In Akerlof's case, whether it was only the lemons trading, or individuals getting into an unproductive rat-race, this point is absolutely central. In Spence's case, the focus was multiple equilibria. One of his core objectives was to provide an explanation of why similarly talented pools of individuals, differentiated by race or gender, could have quite different returns to human capital accumulation. Finally, an enduring theme of Stiglitz's research is that without intervention to reduce asymmetry or regulate quality, there might not even be an equilibrium allocation.

The basic modeling issues have been very well summarized in the companion paper by Löfgren *et al* (2002) and are discussed in detail in Riley (2001). It is hard to think of

[1]The Rothschild-Stiglitz paper and the closely related papers by Riley (1975, 1979) were both stimulated by Spence's research. In writing my second paper I benefited greatly from communications with Mirrlees, Stiglitz and especially Rothschild.

any other new ideas which have led to such a revolution in theorizing or a richer set of applications. While there is also a considerable empirical literature (especially in Finance) I will argue here that there are ways of "taking the model to the data" which deserve more attention. One approach is calibration. This is illustrated in section 2 for the Rothschild-Stiglitz insurance model. For the two type example, Rothschild and Stiglitz illustrated the theoretical possibility of non-existence. For plausible degrees of risk aversion, I conclude that the existence problem is indeed severe. In addition, I show that the deductibles needed to sort different types are vastly higher than those typically observed in practice. I argue that these results obtain because risk aversion is too small to make the deductible an effective screening device.

In section 3, the screening equilibrium is further analyzed using a simple version of Spence's education model. For a continuous version of the model strong necessary and sufficient conditions for existence are derived. These yield direct testable implications.

Central to Akerlof's "lemons principle" is the idea that the seller of a higher quality product has a higher alternative opportunity. In the modern terminology, such an individual has a tighter participation constraint. Somewhat surprisingly, this possibility is simply ignored in the screening literature. Section 4 considers the implications of incorporating the Akerlovian assumption into a screening model. The core result is that there is now a minimum screening threshold. The greater the participation constraint varies with quality, the greater is this threshold. Indeed if the participation constraints for high quality workers bind too tightly, there is no equilibrium.

In more recent years, theorists have examined a variant of the sorting model in which it is the informed agent who must make the first move, rather than the uninformed employer or insurance company. In such cases, there is a continuum of equilibria if uninformed agents are sufficiently pessimistic about any agent who takes an unanticipated action. In the final section I take issue with the widespread belief that simple refinements necessarily result in a unique separating equilibrium.

2. The Insurance example

The standard analysis starts with some observable activity z which has a lower marginal cost for a supplier of higher quality items (indexed by his type t.) The implied indifference curves for higher quality types, over z and price, are then flatter thn the indifference curves of lower quality types.[2] The level of the activity z may thus be used to sort different types. A central point of this paper is that market outcomes are likely to be very different if the preferences maps of different types vary only a little, rather than a lot. In the former case, the sorting potential is weak. High types can only sort by incurring large costs. In the latter case, higher types have much lower marginal costs of z and can thus be sorted much more cheaply.

Consider the Rothschild-Stiglitz insurance example. A type t insuree, where $t \in \{L, H\}$, has a probability π_t of avoiding damage valued at C. The high quality insurees have a higher probability of loss avoidance $(\pi_H > \pi_L)$. Risk neutral firms move first, competing for customers by offering insurance contracts. Each insuree responds by selecting the most advantageous contract available. With full information, the equilibrium contracts offered by risk neutral firms are zero-profit policies to each type with full coverage. Let p_t be the premium (paid by type t in the loss and no-loss states). Then the zero-expected-profit premium is $p_t = (1-\pi_t)C$. With asymmetric information, we

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[2] This is the so called "single-crossing property".

consider the use of deductibles to sort low and high quality insurees. If type L accepts a policy with a deducible z, the expected payment by the firm is $(1-\pi_L)(C-z)$. Then if the insuree were to continue to pay p_L, the insurance company would make an expected profit of

$$V_L(z) = p_L - (1-\pi_L)(C-z).$$

Thus zero-expected-profit policies must offer a premium reduction $r = V_L(z)$. This line is depicted in Figure 1.

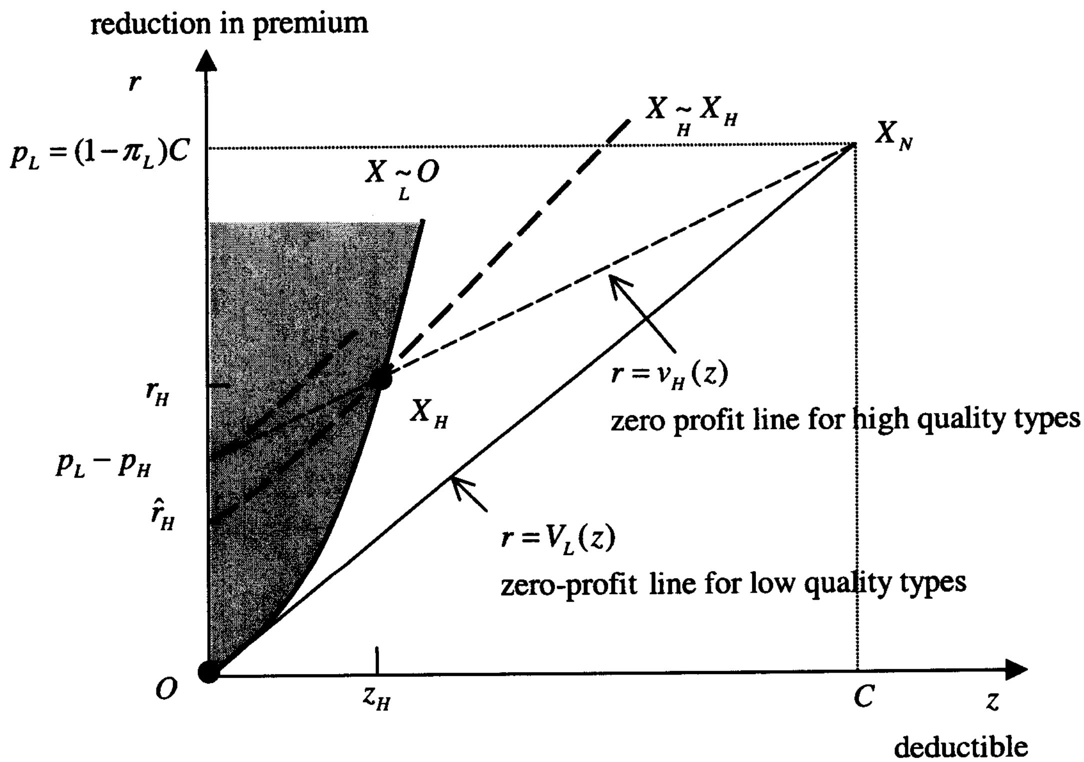

Figure 1: Rothschild-Stiglitz Insurance Model

Similarly, if a high quality type accepts a deductible z, the expected payout is $(1-\pi_H)(C-z)$. Thus, if the insurance company could charge a price p_L, the expected profit would be

$$V_H(z) = p_L - (1-\pi_H)(C-z)$$

Therefore for high quality types the zero-expected profit premium reduction is $r = V_H(z)$. This is depicted as the broken line in Figure 1.

If information about types is private, an insurance contract is fully described by the vector $X = (z, r)$ that is, the deductible z and reduction in premium r. Let $u(\cdot)$ be the von Neumann-Morgenstern utility function of a type t individual with wealth W. His expected utility is then,

$$U_t(z,r) = \pi_t u(W - (p_L - r)) + (1-\pi_t)u(W - z - (p_L - r))$$

The solid curve in Figure 1, denoted as $X \underset{L}{\sim} O$, is the set of contracts yielding the low quality insuree the same utility as the full insurance zero-profit policy O. Since high quality insurees have a higher probability of no loss, they are willing to accept a smaller reduction in premium in return taking on a greater deductible. Thus their indifference curves (depicted as broken curves) are flatter. To satisfy the zero-profit condition, the contract for the high quality insures must be on the broken line. To separate the two types, it cannot be in the interior of the shaded region, since this is the set of contracts which the low quality insuree strictly prefers. Thus the contract X_H is the Pareto efficient separating contract.

This policy yields the same expected utility as a full coverage offer with a premium reduction of $\hat{r}_H$. Thus the difference between $p_L - p_H$ and $\hat{r}_H$ is a measure of the cost of asymmetric information. Expressing this a bit differently, the presence of the low quality type imposes a negative externality on the high quality type and the size of this externality is measured by $(p_L - p_H) - \hat{r}_H$.

It is easy to see that adding any intermediate type only tightens the incentive compatibility constraints. Thus the two type case provides a lower bound on the gains to any better risk type. In Figure 2 the zero-profit premium reduction line for an intermediate risk type *M* is depicted along with the Pareto dominant contract X_M that separates the low and intermediate quality risks. The intermediate quality type has an indifference curve through X_M, denoted as $X \underset{M}{\sim} X_M$ which is lower than the indifference curve though this same contract for low quality insurees. Therefore, the best separating zero-profit contract for type *H* is X_H' to the North-East of X_H.

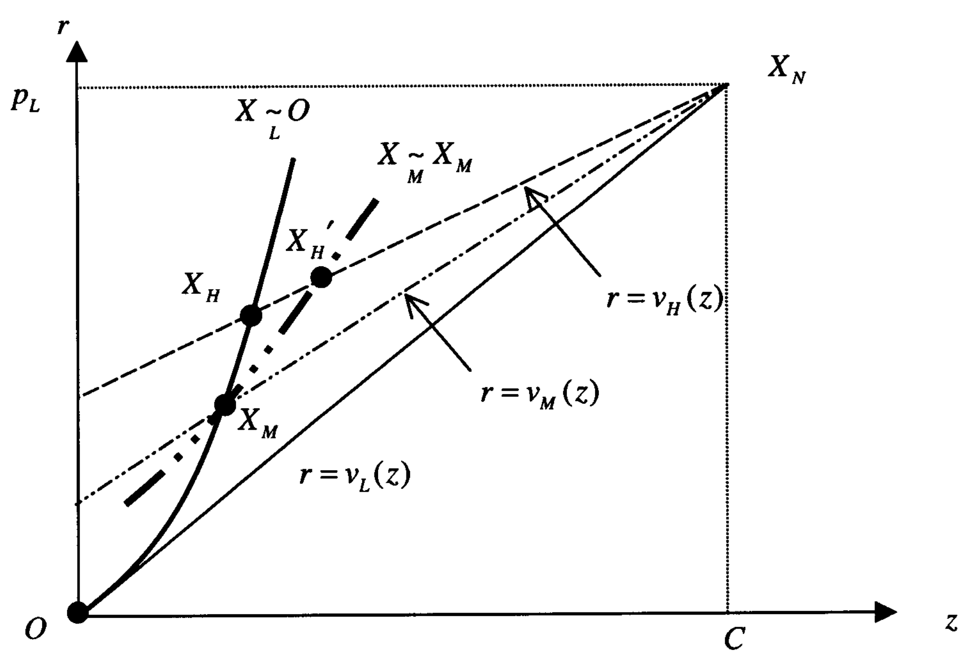

Figure 2: Additional sorting costs with 3 types.

Since adding intermediate types always increases the cost of sorting for higher types, we can use simple finite type examples to obtain lower bounds on deductible levels with larger number of types.

Consider a pool of individuals in which the mean probability of loss avoidance is 0.99. The loss is 80% of wealth. Individuals exhibit the same constant degree of relative risk aversion *R*. Suppose that a firm skims the cream from the pool by offering a policy with a deductible equal to 10% of the loss. The higher the premium offered, the smaller will be the fraction of insurees who find the new offer attractive. Consider the first row of Table 1 below.

Mean probability of loss avoidance 0.99. Deductible is 10% of loss					
R	Premium per $1000 of coverage	Critical prob. of loss avoidance	Probability of loss avoidance		
			0.991	0.993	0.995
			Profit per $1000 of coverage		
0.5	10.09	0.991	1.09	3.09	5.09
5	9.87	0.991	0.87	2.87	4.87
0.5	10.32	0.993		3.31	5.31
5	10.15	0.993		3.15	5.15
0.5	10.54	0.995			5.54
5	10.42	0.995			5.42

Table 1: Profits from skimming the cream from the pool

The degree of relative risk aversion is 0.5. If the premium per $1000 of coverage is $10.09, insurees with a probability of loss avoidance of 0.991 are indifferent between staying in the pool with full coverage and accepting a deductible equal to 10% of the loss. This type has an expected cost of $\$1000(1-0.991)=\$9$ per $1000 of coverage. Thus it generates a profit of $1.09 if it accepts the deductible. All higher types are strictly better off accepting the deductible. Since they have a higher probability of loss avoidance they generate the larger profits shown in the final two columns of the Table.[3]

Since the mean loss probability in the pool is 0.01, the pool premium per $1000 of coverage is $10.00. As the table shows, regardless of whether risk aversion is high or low, the cream can be skimmed from the pool at rates similar to the pool rate. As a result, the profit to be made from cream-skimming by offering deductibles is high relative to the pool premium.

It follows that firms have a very strong incentive to introduce deductibles to attract individuals with a higher probability of avoiding loss. Thus our simple calibration points to the economic importance of the "no pooling" result of Rothschild and Stiglitz.

We therefore consider deductible policies which separate out the different risk classes. Let π_W, π_M and π_B be the probability of loss avoidance of the worst risks, the median risks and the best risks. Let the number of types be T. We assume that for neighboring types the difference in the probability of loss avoidance is $(\pi_B - \pi_W)/(T-1)$. The Pareto efficient separating zero-profit contracts are easily computed. We then ask what premium reduction would make the median type indifferent between his separating offer and full coverage. All lower types would also prefer this offer. The offer thus attracts what I will call the "median pool." If, for this pool, the probability of loss avoidance is sufficiently low, the offer will result in losses. If it is sufficiently high, the offer is profitable. The critical probability is shown in column 7 of Table 2. For example, in the top row the worst probability is 0.989 and the median probability is 0.99. Assuming a

[3] The Mathcad files used to create the tables can be downloaded from http://www.econ.ucla.edu/riley/research/sje02

symmetric uni-modal distribution of types, the probability of loss avoidance in the median pool exceeds $(\pi_M - \pi_W)/2 = 0.9895$. Thus, for this case the median pool is profitable.

Note that for each case computed, the median pool is profitable. Our calibration thus shows that the non-existence problem emphasized by Rothschild-Stiglitz is a very serious one indeed.

Column 9 considers the same issue for a pooling offer which attracts all types. Given symmetry, the offer is profitable if the mean probability in the pool is below the median probability π_M. An asterisk indicates where this is the case.

	C/W	R	π_W	π_M	π_B	T	Critical Probabilities		Deductible as % of loss	
							Median Pool	*Pop. Pool*	*Median type*	*Best type*
1.	0.8	1.5	.989	.99	.991	3	.9894*	.9899*	29	39
2.	0.8	1.5	.989	.99	.991	9	.9892*	.9893*	30	40
3.	0.8	0.5	.989	.99	.991	3	.9895*	.9902	36	47
4.	0.8	5	.989	.99	.991	3	.9893*	.9897*	33	38
5.	0.4	1.5	.989	.99	.991	9	.9894*	.9901	22	26
6.	0.8	1.5	.9989	.99	.9991	9	.9989*	.9989*	31	41
7.	0.8	1.5	.9986	.99	.9994	9	.9988*	.9992	47	62

Table 2: Characteristics of the efficient Separating Contracts

The second row shows that adding six more types has only small effects on the results. The third and fourth rows show that changing the degree of relative risk aversion also has only small effects on profitability. Row five considers a less extreme loss and the last two rows explore the effects of narrowing and widening the range of loss probabilities.

The final two columns of the table show the separating deductibles for the median and best risks. It is striking that the deductibles are vastly higher than those observed in practice. Thus the basic insurance model does not seem to be able to explain deductibles as an equilibrium implication of asymmetric information about the probability of avoiding loss.

In this insurance application, individuals are forced to choose very high deductibles to be sorted because the differences in the marginal cost of accepting a deductible are very small. As a further implication, the gains to sorting are small. The deductible is thus a "weak" sorting device. In the next section we compare "weak" and "strong" sorting technologies using Spence's educational screening model.

3. Education as a sorting device

In the very simplest Spencian model, a worker of productivity t can accumulate educational credential z at a cost $C(t,z)$. Higher productivity types have a lower marginal cost of signaling $(\frac{\partial}{\partial t}\frac{\partial C}{\partial z}<0)$. The two type case is depicted below.

With full information the wage differential is t_2-t_1. Thus the cost of signaling, expressed as a fraction of the maximum possible gain is $C(t_2,z^*)/(t_2-t_1)$. For incentive compatibility, the difference in wages t_2-t_1 must be equal to the education cost for the low type. Thus the cost of signaling,

$$\frac{C(t_2,z^*)}{t_2-t_1}=\frac{C(t_2,z^*)}{C(t_1,z^*)}.$$

The key point is that the gains to sorting depend on the <u>rate</u> at which the cost of signaling declines with productivity.

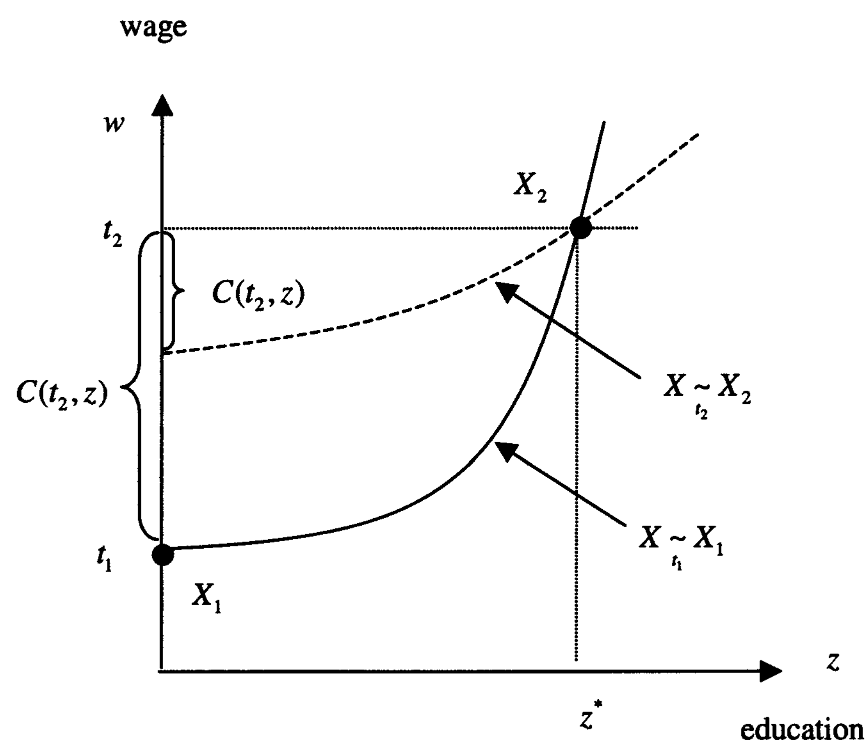

Fig. 3: Labor market Sorting

In this section we consider the continuous version of this model. As with the two type example, a type t individual has productivity $v(t) = t$. Productivity t is distributed continuously on $[\alpha, \beta]$ with c.d.f $F(\cdot) \in \mathbb{C}^3$. There are two possible sorting technologies.[1] For technology $i = 1, 2$, the sorting cost is

$$C_i(t, z) = z / A_i(t), \text{ where } A_i(\cdot) \in \mathbb{C}^3. \qquad (3.1)$$

Taking the logarithm and differentiating by t, the proportional rate of reduction in sorting cost is

[1] The analysis immediately generalizes to any multiplicatively separable cost function $c(t, z) = B(z) / A(t)$. Simply define $y = B(z)$ and $C(t, y) = c(t, B^{-1}(y)) = y / A(t)$

$$-\frac{\frac{\partial}{\partial t}\frac{\partial C_i}{\partial z}}{\frac{\partial C_i}{\partial z}} = \frac{A_i'(t)}{A_i(t)}.$$

We assume that this rate of reduction is everywhere greater for the first technology.[2]

Each type has the same outside opportunity to earn a wage $w_0(t) = t_0 \in (\alpha, \beta)$.

Let $w(z)$ be the equilibrium wage function and let $z(t)$ be the equilibrium educational choice. All types below t_0 choose their outside opportunity $w_o(t) = t_o$. Let $u(t,s)$ be the expected payoff to type $t > t_o$ if he makes the equilibrium choice of type s. That is,

$$u(t,s) = w(z(s)) - C(t, z(s)).$$

Bertrand wage competition among firms requires that the equilibrium wage is equal to equilibrium productivity. Thus $w(z(t)) = t$ and so $u(t,s) = s - C(t, z(s))$. For incentive compatibility, $u(t,s)$ takes on its maximum at $s = t$. Thus $1 - \frac{dz}{dt}\frac{\partial C}{\partial z}(t, z(t) = 0$.

Substituting from (3.1) it follows that

$$\frac{dz}{dt} = A(t). \tag{3.2}$$

Integrating, $z(t) = \int_{t_0}^{t} A(w)dw$

and hence the equilibrium cost of sorting is

[2] Define $L(\cdot)$ so that $A_1(t) = L(A_2(t))$. Then our assumption holds if and only if $L(A) < AL'(A)$. We will describe the first technology as strongly dominating if, in addition $L''(\cdot) > 0$. Then $\frac{A_1''}{A_1'} > \frac{A_2''}{A_2'}$.

$$C(t,z(t))=\frac{z(t)}{A(t)}=\frac{\int_{t_0}^{t}A(w)dw}{A(t)} \qquad (3.3)$$

The following result is derived in the Appendix.

Proposition 1: Ranking sorting Technologies

If $A_i(s)>0,\ i=1,2,\ A_i(\cdot)$ is increasing and $\frac{A_1'(x)}{A_1(x)}>\frac{A_2'(x)}{A_2(x)},\ x\in[s,t]$ then the costs of screening are lower for technology 1 than technology 2.

Thus the higher the rate at which the marginal cost of signaling declines with type, the greater are the gains from complete sorting.

Thus far we have considered a wage function that satisfies the incentive compatibility and participation constraints and also the zero-profit constraints. It remains to show that the resulting wage function is a best response. In Figure 4 we depict the indifference lines for a pair of types t_1 and $t_2>t_1$ through their respective separating contracts

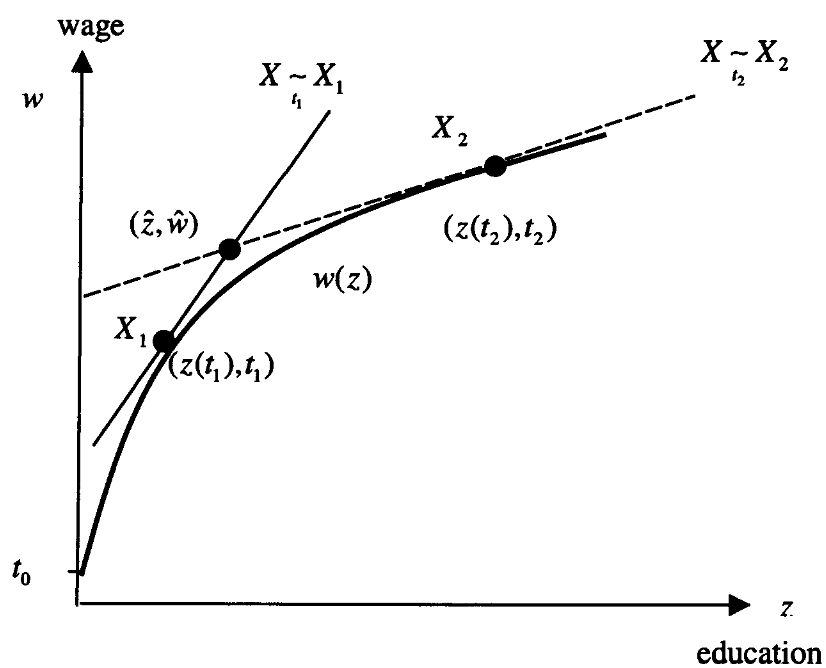

Figure 4: Attracting an interval of types

$(z(t_1),t_1)$ and $(z(t_2),t_2)$. Since the indifference line $X \underset{t_2}{\sim} X_2$ is flatter than $X \underset{t_1}{\sim} X_1$, the two lines meet at some point $(\hat{z},\hat{w})$. Suppose a firm offers the contract $\hat{X} = (\hat{z},\hat{w})$. As is clear from Figure 4, all those types on the interval (t_1,t_2), with separating contracts on the curve $w(z)$, between X_1 and X_2 strictly prefer $(\hat{z},\hat{w})$. All those types less than t_1 or greater than t_2 strictly prefer their separating contracts.

The expected profit from such an offer is the expected productivity less the wage, that is

$$\hat{\pi}(t_1,t_2) = \frac{\int_{t_1}^{t_2} tF'(t)dt}{F(t_2)-F(t_1)} - \hat{w} \tag{3.4}$$

We now seek necessary and sufficient conditions for a Nash equilibrium. Consider Figure 4 once more. Since type t_i is indifferent between $(z(t_i),t_i)$ and $(\hat{z},\hat{w})$,

$$\hat{w} - \frac{\hat{z}}{A(t_i)} = t_i - \frac{z_i}{A(t_i)}, \quad i = 1,2 \tag{3.5}$$

Solving for $\hat{w}$, we obtain

$$\hat{w}(t_1,t_2) = \frac{t_2 A(t_2) - t_1 A(t_1) - z(t_2) + z(t_1)}{A(t_2) - A(t_1)} \tag{3.6}$$

For any par of types we can then solve for the profitability of the offer $(\hat{z},\hat{w})$ by appealing to (3.4) and (3.6). By considering expected profit when $t_2 - t_1$ is small we obtain the following necessary condition for a Nash equilibrium.

Proposition 2: Necessary condition for a Nash equilibrium

For the Pareto efficient zero-profit separating contracts to be equilibrium best responses it is necessary that

$$\frac{A''(t)}{A'(t)} \geq \frac{F''(t)}{F'(t)}, \quad t \in (t_o, \beta) \tag{3.7}$$

The following further result is also derived in the Appendix.

Corollary 3: Sufficient Condition for a Nash equilibrium

If inequality (3.7) is everywhere strict, the Pareto-efficient separating zero-profit contracts are best responses.

In equilibrium the wage function satisfies $w(z(t)) = t$. Then we can rewrite inequality (3.7) as follows.

$$\frac{A''(w)}{A'(w)} \geq \frac{F''(w)}{F'(w)}.$$

If the screening technology can be estimated, this inequality provides a simple test, since the wage distribution is directly observable. Alternatively, suppose that the econometrician only observes the wage function $w(z)$. From the first order condition for incentive compatibility $w'(z) = 1 / A(w)$. Inverting this expression, $z'(w) = A(w)$. Substituting this into inequality (3.7) yields the following result.

Proposition 4: Testable implication of Nash equilibrium sorting

If a Nash equilibrium exists it is necessarily the case that

$$\frac{z'''(w)}{z''(w)} \geq \frac{F''(w)}{F'(w)}.$$

4. The Screening Threshold

Akerlof's Lemons Principle emerges because potential suppliers of higher quality products or services have higher reservation prices. However, in the basic screening model of the previous section, this feature is absent. Each type of worker has the same outside opportunity to earn a wage $w_o(t) = t_o$. In this section we reconsider equilibrium screening under the Akerlovian assumption that the reservation wage, $w_o(t)$, is an increasing concave function. We will assume that the lowest type is more productive selecting the outside opportunity, while the highest type is less productive outside, that is,

$$w_o(\alpha) > \alpha \text{ and } w_o(\beta) < \beta$$

Given these assumptions there exists a unique type $\hat{t} \in (\alpha, \beta)$ such that $w_o(\hat{t}) = \hat{t}$.

Where needed we will appeal to the following simplifying restrictions.[3]

Assumption R: (i) $A'(t)/A(t)$ is decreasing and (ii) $\dfrac{A'(t)}{A(t)} \dfrac{\int_\alpha^t A(x)dx}{A(t)}$ is increasing.

Efficient entry is achieved if all types $t > t_o$ are screened. For this to be the case, the minimum screen must be zero. As this next result shows, the minimum equilibrium type $t^* > t_o$ and the minimum "threshold" level of the screen is strictly positive.

Proposition 4: If Assumption R holds, then the minimum type that is sorted is

$$t^* = Sup\{s \mid \frac{A'(t)}{A(t)} < \frac{w_o'(t)}{t - w_o(t)},\ \forall t \in (t_o, s)\}$$

The functions $A'(t)/A(t)$ and $w_o'(t)/(t - w_o(t))$ are depicted in Figure 5 below.

From Figure 5, if the screening technology gets everywhere weaker so that

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[3] Conditions (i) and (ii) are satisfied, for example, if $A(t) = t^\theta,\ \theta > 0$.

$A'(t)/A(t)$ decreases, the heavy curve shifts down. Then the equilibrium entry level increases, as does the screening threshold. Indeed, if the screening technology is sufficiently weak, $t^* = \beta$ and there is no entry into the screening industry.

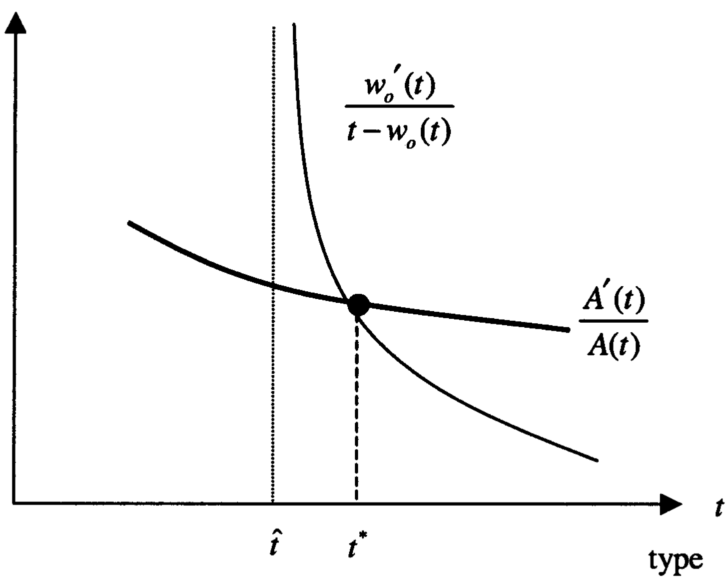

Fig. 5: Screening Threshold

Similarly, holding $w_o(\alpha)$ constant, if $w_o'(t)$ increases it follows that $t - w_o(t)$ declines, and hence that $w_o'(t)/(t - w_o(t))$ increases. Again the equilibrium entry level increases.

5. Signaling

Until now we have focused on the "screening" version of the model where the uninformed agents move first. The modern game-theoretic interpretation of Spence's multiple signaling equilibria is that they are all Nash equilibria of a model where the informed agents move first. In the labor market application, a consultant first chooses her educational credentials and then firms bid for her services based on their beliefs about her productivity. In equilibrium (a) beliefs are confirmed and (b) given these beliefs, Bertrand price competition results in a wage equal to her productivity.

This is illustrated below for the simplest 2 type case. Type t_1 has a productivity of

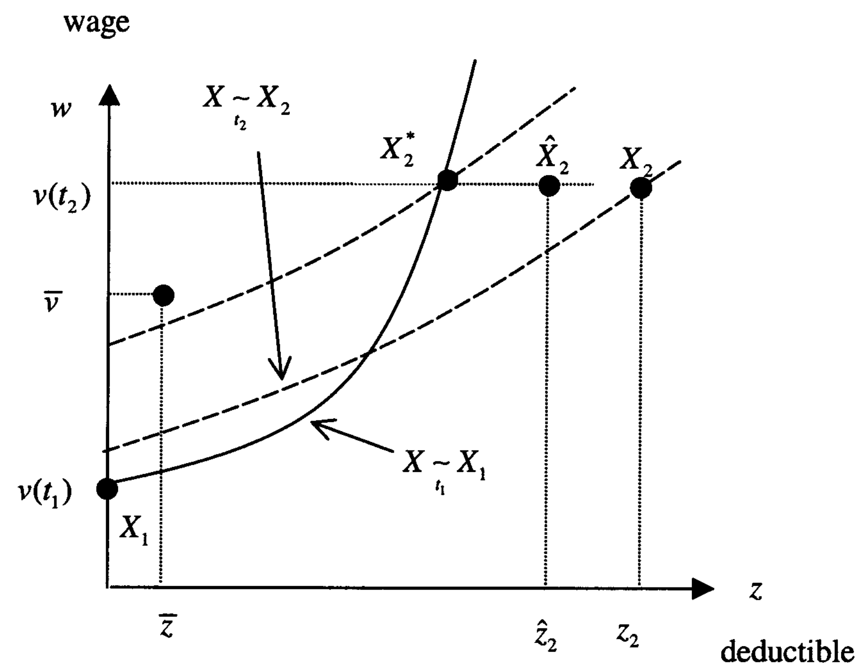

Fig. 6: Labor Market Signaling

$v(t_1)$ and type t_2 a productivity of $v(t_2) > v(t_1)$. Type t_2 has a lower signaling cost. Three possible separating equilibria are depicted in the Figure. Since firms know that the lowest productivity is $v(t_1)$, they are willing to pay this to someone who chooses $z = 0$. Thus one contract offered is $X_1 = (0, v(t_1))$. The contract X_2 is incentive compatible and satisfies the zero-profit condition. As long as firms believe that anyone choosing $z < z_2$ is much more likely to be the low type, they will respond to someone choosing $\hat{z}_2$ with a low wage offer. Given such responses, no high productivity consultant has an incentive to choose $\hat{z}_2$ rather than z_2. Of course an identical argument establishes that the contract pairs, $X_1, \hat{X}_2$ and X_1, X_2^* are also Nash signaling equilibria. Also depicted is the contract $(\bar{z}, \bar{v})$, where $\bar{v}$ is the population mean productivity. Again with sufficiently conservative beliefs about the type making an out-of-equilibrium choice of z, this is a Nash signaling equilibrium.

Since a continuum of equilibria implies that the model has little predictive power, economists typically try to reduce the number of equilibria by applying an equilibrium refinement. The simplest refinement is the Intuitive Criterion of Cho and Kreps (1987). The idea is to apply a credibility test to an out-of-equilibrium choice $\hat{z}_2$. Suppose that firms believe that it is type $\hat{t}$ which chooses $\hat{z}_2$. In this case they will bid the wage up to the type's productivity so $\hat{w} = \hat{t}$. This response is credible if (i) type $\hat{t}$ would prefer $(\hat{z}, \hat{w})$ to the Nash equilibrium contract $(z(\hat{t}), w(z(\hat{t}))$ and (ii) no lower productivity type would be better off choosing the new contract.

Consider Figure 6. Suppose that a worker chooses $\hat{z}_2$ and firms believe it is a high productivity type. The wage is thus bid up to $v(t_2)$. Type t_2 strictly prefers the contract $\hat{X}_2 = (\hat{z}_2, v(t_2))$ to X_2 while type t_1 prefers the contract $X_1 = (0, v(t_1))$ to the contract $\hat{X}_2$. Thus it is credible that $\hat{z}_2$ is selected by the high productivity type. The Nash equilibrium pair of contracts X_1, X_2 therefore fails the Intuitive Criterion. Since the same argument holds for any $z_2 > z_2^*$ it follows that the unique Nash equilibrium which satisfies the Intuitive Criterion is the contract pair X_1, X_2^*.

Now suppose we modify the model and replace type t_2 by types t_{2a} and t_{2b}. Type t_{2a} has a productivity higher than $v(t_2)$ while type t_{2b} has a productivity equal to that of type t_1. The mean productivity of types t_{2a} and t_{2b} is $v(t_2)$.

Note that we have altered the model in such a way that it is observationally equivalent to the original model. However now the Intuitive Criterion has no bite. Consider the Nash equilibrium contract pair X_1, X_2 once more. If a type chooses $\hat{z}_2$ and firms believe it is type t_{2a} they will bid the wage up to $v(t_{2a}) > v(t_2)$. But then type t_{2b} is strictly better off also, so part (ii) of the Intuitive Criterion is violated. It firms believe it is a type t_{2b} they will offer a wage of $v(t_1)$ which is strictly worse for this type and so part (i) of the Intuitive Criterion is violated. Thus X_1, X_2 survives the Cho-Kreps refinement.

An alternative resolution is to argue that since the model is essentially unchanged, the old arguments should apply. That is, since types t_{2a} and t_{2b} are observationally equivalent, make the conjecture that it is one of these types that is choosing $\hat{z}_2$. Then, applying Bayes' Rule, expected productivity is $v(t_2)$. Arguing exactly as before, the belief is credible and so the Nash equilibrium X_1, X_2 fails the strengthened refinement.

This refinement was first proposed by Grossman and Perry (1986ab). My view is that this is the appropriate approach to take. Game theorists are reluctant to employ the stronger refinement because it comes at a price. No longer is there a general existence theorem. To see this, consider the best separating Nash equilibrium X_1, X_2^* in Figure 6, and the out-of-equilibrium signal $\overline{z}$. If the belief is that any of the types might have sent it and the population mean productivity is $\overline{v}$, the wage will be bid up to $\hat{w} = \overline{v}$. As depicted the mean productivity is sufficiently high that all types are indeed better off. Then even the best Nash separating equilibrium fails the GP Criterion. It is also easy to show that no pooling equilibrium satisfies the GP criterion either.

In the example, there is no Nash signaling equilibrium which satisfies the GP Criterion precisely when there is no Nash equilibrium of the screening game. This conclusion holds very generally. Thus if the Grossman-Perry refinement is to be applied, the test proposed for the screening model is equally relevant to signaling models.

6. Concluding Remark

I have argued that there are two useful ways of testing the plausibility of screening and signaling models. First, via calibrating exercises, it is possible to find out whether the resulting sorting choices look anything like those that we observe. For insurance markets, I have suggested that the deductible is such a weak sorting mechanism that we should not expect to see it employed in practice for this purpose.[4] Second, the internal consistency of both screening and signaling stories of equilibrium sorting has important implications for empirical testing.

A large number of theoretical sorting models have been developed to explain a host of phenomena over the last two decades. The examples presented in this paper suggest that, in many cases, the models will not survive empirical scrutiny. If I am correct, it will follow that economists have not yet fully responded to the challenges laid down by this year's Nobel Laureates.

[4] It is far from clear that insurance companies do know less about insurance risks than insurees. However, where there is asymmetric information, the strong conclusions about the incentive to skim the cream from an insurance pool and the implausible separating contracts would seem to indicate that we do not yet have a satisfactory model of such markets.

APPENDIX

We begin with two technical Lemmas.

Lemma 1: If $A_i(s) > 0,\ i = 1,2,\ A_i(\cdot)$ is increasing and $\frac{A_1'(x)}{A_1(x)} > \frac{A_2'(x)}{A_2(x)},\ x \in [s,t]$ then

$$\frac{A_1(t)}{\int_s^t A_1(x)dx} > \frac{A_2(t)}{\int_s^t A_2(x)dx}.$$

Proof: Define $c_i(t) = \frac{\int_s^t A_i(w)dw}{A_i(t)}\quad i = 1,2$. Then

$$c_i'(t) = 1 - \frac{A_i'(t)}{A_i(t)} \frac{\int_{t_0}^t A_i(w)dw}{A_i(t)} = 1 - c_i(t)\frac{A_i'(t)}{A_i(t)}. \tag{A.1}$$

Given our assumptions it follows that

$$c_1(t) = c_2(t) \Rightarrow c_1'(t) < c_2'(t) \tag{A.2}$$

Since $c_i(s) = 0$, it also follows that $c_i'(s) = 1,\ i = 1,2$.

Differentiating (A.1), $c_i''(t) = -c_i'(t)\frac{A_i'(t)}{A_i(t)} - c_i(t)\frac{d}{dt}\frac{A_i'(t)}{A_i(t)}$. Then $c_i''(s) = -\frac{A_i'(s)}{A_i(s)}$.

It follows immediately that $c_1(t) < c_2(t)$ for some interval $(s, \hat{t})$. If $c_1(\hat{t}) = c_2(\hat{t})$ it follows also that $c_1'(\hat{t}) \geq c_2'(\hat{t})$. But this contradicts (A.2).

QED

Proposition 1 follows immediately

Lemma 2: If $G_i'(\cdot) > 0$ $i = 1,2$ and $\frac{G_1''(x)}{G_1'(x)} > \frac{G_2''(x)}{G_2'(x)}$, $x \in [s,t)$ then

(a) $$\frac{G_1'(t)}{G_1(t) - G_1(s)} > \frac{G_2'(t)}{G_2(t) - G_2(s)}, \; t > s$$

and

(b) $$\frac{G_1(t) - G_1(s)}{\int_s^t (G_1(x) - G_1(s))dx} > \frac{G_2(t) - G_2(s)}{\int_s^t (G_2(x) - G_2(s))dx}, \; t \geq s$$

Proof: Define $A_i(t) \equiv G_i(t) - G_i(s),\; i = 1,2$. Then $A_i(s) = 0$. We can rewrite the condition, $\frac{G_1''(x)}{G_1'(x)} > \frac{G_2''(x)}{G_2'(x)}$ as $\frac{d}{dx} \ln A_1'(x) > \frac{d}{dx} \ln A_2'(x)$, that is $\frac{d}{dx} \ln \frac{A_1'(x)}{A_2'(x)} > 0$. It follows immediately that for any t and $s < t$,

$$\frac{A_1'(t)}{A_2'(t)} > \frac{A_1'(s)}{A_2'(s)} \text{ and hence } \frac{A_2'(s)}{A_2'(t)} > \frac{A_1'(s)}{A_1'(t)}, \; s < t$$

Since $A_i(s) = 0$, we can integrate this last expression over $[s,t]$ to obtain

$$\frac{A_2(t)}{A_2'(t)} > \frac{A_1(t)}{A_1'(t)}.$$

Thus (a) holds.

To prove (b) we note that if $c_i(t) = \frac{\int_s^t A(x)dx}{A(t)}$, then by l'Hôpital's Rule, $c_i(s) = 0$ and, from the proof of Lemma 1,

$$c_i'(t) = 1 - A_i'(t) \frac{\int_s^t A_i(w)dw}{A_i(t)^2} \quad \text{(A.3)}$$

Appealing to l'Hôpital's Rule,

$$\lim_{t\downarrow s}\frac{\int_s^t A_i(w)dw}{A_i(t)^2}=\lim_{t\downarrow s}\frac{A_i(t)}{2A_i(t)A_i'(t)}=\frac{A_i'(s)}{2A_i'(s)^2}.$$

Thus $c_i'(s)=\frac{1}{2}$.

Differentiating (A.3) ,

$$c_i''(t)=-[\frac{A_iA_i''I+A_i'A_i^2-2(A_i')^2I}{A_i^3}] \text{ where } I(t)=\int_s^t A(w)dw.$$

To determine $c_i''(s)$ we apply l'Hôpital's Rule repeatedly.

$$c_i''(s)=-\frac{1}{A_i'(s)}\lim_{t\downarrow s}[\frac{A_i'A_i''I+A_iA_i'''I+' A_i''A_i^2+A_i''A_i^2+2(A_i')^2A_i-4A_i'A_i''I-2(A_i')^2A_i}{3A_i^2}]$$

$$=-\frac{1}{A_i'(s)}\lim_{t\downarrow s}[\frac{A_iA_i'''I+2A_i''A_i^2-3A_i'A_i''I}{3A_i^2}]$$

Since the first term in the numerator is $O(t-s)^3$ and A_i^2 is $O(t-s)^2$ we can ignore this term and so

$$c_i''(s)=-\frac{1}{3A_i'(s)}\lim_{t\downarrow s}[\frac{2A_i''A_i^2-3A_i'A_i''I}{A_i^2}]$$

Differentiating the numerator and denominator of the bracketed expression once more,

$$c_i''(s)=-\frac{1}{6A'(s)^2}\lim_{t\downarrow s}[\frac{4A_i''A_iA_i'+2A_i^2A_i'''-3A_i''A_iA_i'-3(A_i'')^2I-3A_i'A_i'''I}{A_i}]$$

Since $I(t)$ is $O(t-s)^2$ and $A(t)$ is $O(t-s)$, we can eliminate the 2nd , 4th and 5th terms in the numerator and so

$$c_i''(s)=-\frac{1}{6A'(s)^2}\lim_{t\downarrow s}[\frac{A_i''A_iA_i'}{A_i}]=-\frac{1}{6}\frac{A_i''(s)}{A'(s)}$$

It follows that in some right neighborhood of s, $c_1(t) < c_2(t)$. Since (a) holds we can argue exactly as in the proof of Lemma 1 that this inequality must be strict of all $t > s$.

Q.E.D.

From (1.4),

$$\hat{\pi}(t_1,t_2) = \frac{\int_{t_1}^{t_2} tF'(t)dt}{F(t_2)-F(t_1)} - \hat{w}$$

Integrating by parts,

$$\hat{\pi}(t_1,t_2) = t_2 - \frac{\int_{t_1}^{t_2} ((F(x)-F(t_1))dx}{F(t_2)-F(t_1)} - \hat{w} \tag{A.4}$$

Consider two different distributions of productivity and suppose that

$$\frac{F_1''(t)}{F_1'(t)} > \frac{F_2''(t)}{F_2'(t)}, \ t > t_0.$$

It follows from Lemma 2 that

$$\hat{\pi}_1(t_1,t_2) > \hat{\pi}_2(t_1,t_2)$$

As is readily confirmed,[1]

$$\frac{F_1''(t)}{F_1'(t)} \geq \frac{F_2''(t)}{F_2'(t)}, \ \forall t \Leftrightarrow \frac{F_1'(t)}{F_1'(s)} \geq \frac{F_2'(t)}{F_s'(t)}, \ \forall t \text{ and } \forall s < t$$

Thus our sufficient condition for ranking profit is essentially the requirement that the first distribution exhibits Monotone Likelihood dominance over the second.[2]

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[1] $\frac{d}{dt}\ln F_1'(t) \geq \frac{d}{dt}\ln F_2'(t) \Leftrightarrow \ln F_1'(x) - \ln F_1'(s) \geq \ln F_2'(x) - \ln F_2'(s), \ \forall x > s$

[2] If $A_i(0) = 0, \ i = 1,2$ it follows from Lemma 2 that $A_1'(t)/A_1(t) > A_2'(t)/A_2(t)$ thus our earlier ranking result for the two technologies continues to hold.

Lemma 3:

$$\hat{w} = t_2 - \frac{\int_{t_1}^{t_2} (A(x) - A(t_1))dx}{A(t_2) - A(t_1)}$$

Proof:

From (1.5),

$$A(t_2)(\hat{w} - t_2) = \hat{z} - z(t_2) \text{ and } A(t_1)(\hat{w} - t_1) = \hat{z} - z(t_1).$$

Subtracting the second expression from the first,

$$(A(t_2) - A(t_1))\hat{w} = t_2 A(t_2) - t_1 A(t_1) - (z(t_2) - z(t_1)).$$

From (1.2) $dz / dt = A(t)$. Hence

$$z(t_2) - z(t_1) = \int_{t_1}^{t_2} A(x)dx = \int_{t_1}^{t_2} (A(x) - A(t_1))dx + A(t_1)(t_2 - t_1).$$

Substituting into the previous expression,

$$\hat{w} = t_2 - \frac{\int_{t_1}^{t_2} (A(x) - A(t_1))dx}{A(t_2) - A(t_1)}$$

Q.E.D.

Proposition 2: Necessary and sufficient conditions for a Nash equilibrium

For the Pareto efficient zero-profit separating contracts to be equilibrium best responses it is necessary that

$$\frac{A''(t)}{A'(t)} \geq \frac{F''(t)}{F'(t)}, \; t \in (t_o, \beta)$$

It is sufficient that the inequality be everywhere strict.

Proof: Suppose $\frac{A''(t_1)}{A'(t_1)} < \frac{F''(t_1)}{F'(t_1)}$, for some $t_1 \in (t_o, \beta)$. Since $A(\cdot)$ and $F(\cdot) \in \mathbb{C}^3$, there is some interval $[t_1, t_2]$ over which this inequality continues to hold. From Lemma 3 and (A.4),

$$\hat{\pi}(t_1, t_2) = \frac{\int_{t_1}^{t_2} ((A(x) - A(t_1))dx}{A(t_2) - A(t_1)} - \frac{\int_{t_1}^{t_2} ((F(x) - F(t_1))dx}{F(t_2) - F(t_1)} \tag{A.5}$$

Then appealing to (A.5), $\hat{\pi}(t_1, t_2) > 0$ so the offer $(\hat{z}, \hat{w})$ is strictly profitable. Hence for the separating zero-profit contracts to be best replies, it cannot be the case that

$\frac{A''(t_1)}{A'(t_1)} < \frac{F''(t_1)}{F'(t_1)}$, for some $t_1 \in (t_o, \beta)$.

An appeal to Lemma 2 then establishes Corollary 3.

Q.E.D.

Proposition 4: If Assumption A holds, then the minimum type which screens is

$$t^* = Sup\{t \mid \frac{A'(s)}{A(s)} < \frac{w_o'(s)}{s - w_o(s)}, \; \forall s \in (t_o, t)\}$$

Proof: Since $\lim_{t \downarrow \hat{t}} (w_o'(t)/(t - w_o(t)) = \infty$, t^* is well defined. Integrating equation (3.2), the necessary condition for sorting, any sorting function $z(t)$, can be written as

$z(t,k) = \int_{\alpha}^{t} A(x)dx - k$. Consider first the sorting function for which all types are sorted.

Since the minimum type does not signal $z(\alpha,k)=0$ and hence $k=0$. The utility of type t is then $u(t,0)=t-z(t,0)/A(t)$.

For efficient sorting by industry, all those types $t>\hat{t}$ (for which $t>w_o(t)$) must be sorted. Type $\hat{t}$ must be indifferent between entering and staying out. Hence at $t=\hat{t}$, $u(t,\hat{k})=t-z(t,\hat{k})/A(t)=w_o(t)=t$. This is depicted in Figure 7.

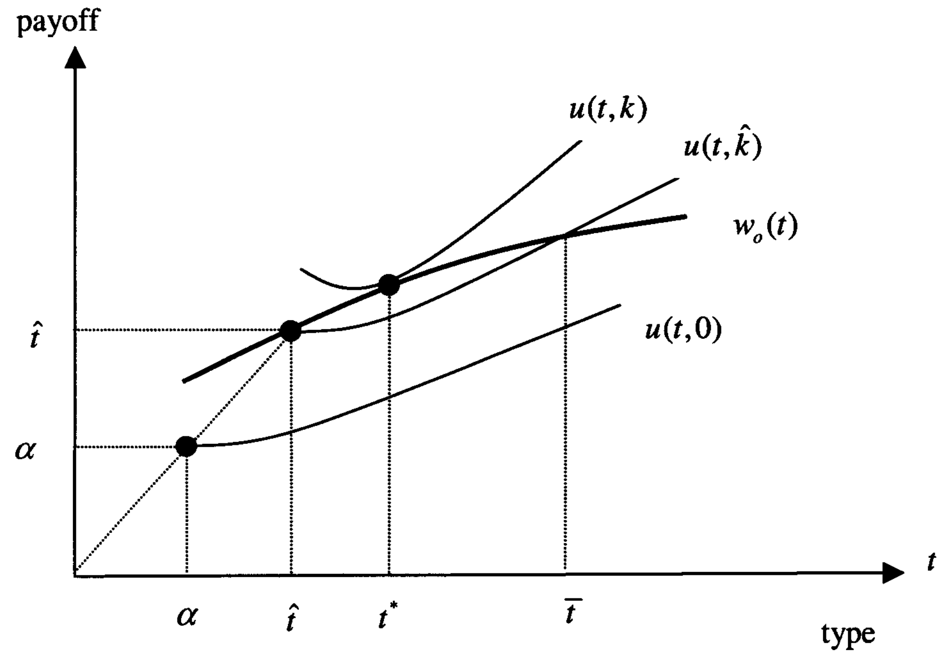

Fig. 7: The signaling threshold

It follows that $z(\hat{t},\hat{k})=0$. Since $u(\hat{t},0)<\hat{t}$, it follows that $\hat{k}>0$.

Also, $u(t,\hat{k})=t-z(t,\hat{k})/A(t)=\underset{s}{Max}\{s-z(s,\hat{k})/A(t)\}$. Thus, by the Envelope Theorem,

$$\frac{d}{dt}u(t,\hat{k})=\frac{z(t,\hat{k})}{A(t)}\frac{A'(t)}{A(t)} .$$

But we have just argued that $z(\hat{t},\hat{k})=0$ so that $\left.\frac{d}{dt}u(t,\hat{k})\right|_{t=\hat{t}}=0$.

Therefore, as depicted in Figure 7, the screening payoff is below the outside payoff for some interval of types $(\hat{t},\bar{t})$. Thus the participation constraint is violated over this interval.

It follows that the threshold entry level exceeds $\hat{t}$. In the Figure, we can satisfy the participation constraint by making type $\bar{t}$ the minimum type that is screened. However clearly there are sorting functions $z(t,k)$, with $k > \hat{k} > 0$ which are Pareto preferred. Indeed the Pareto dominant utility curve $u(t,k)$ must touch the outside opportunity curve on the interval $(\hat{t},\bar{t})$. We now argue that this occurs at t^*.

Choose k so that $u(t^*,k) = t^* - z(t^*,k)/A(t^*) = w_o(t^*)$. Then

$$z(t^*,k) = (t^* - w_o(t^*))A(t^*)$$

Arguing as above, it follows from the Envelope Theorem that

$$\frac{d}{dt}u(t,k) = \frac{z(t,k)}{A(t)}\frac{A'(t)}{A(t)}.$$

Hence,

$$\left.\frac{d}{dt}u(t,\hat{k})\right|_{t=t^*} = \frac{z(t^*,k)}{A(t^*)}\frac{A'(t^*)}{A(t^*)} = (t^* - w_o(t^*))\frac{A'(t^*)}{A(t^*)}.$$

From the definition of t^*, the right-hand side is $w_o'(t^*)$. Thus the slopes of the two functions are the same at t^*.

Also $z(t,k) = \int_\alpha^t A(x)dx - k$ and k is positive. Hence

$$\frac{d}{dt}u(t,k) = \frac{z(t,k)}{A(t)}\frac{A'(t)}{A(t)} = \frac{\int_\alpha^t A(x)dx}{A(t)}\frac{A'(t)}{A(t)} - \frac{k}{A(t)}\frac{A'(t)}{A(t)}.$$

If Assumption R holds, the right hand side is strictly increasing. Thus $u(t,k)$ is convex.

By hypothesis $w_o(t)$ is concave so the participation constraints are satisfied for all $t > t^*$.

Q.E.D.

References

Akerlof, George. (1970), "The Market for 'Lemons': Qualitative Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 84, 488-500.

Cho, In-Koo and Kreps, David M. (1987), "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics*, 102, 179-221.

Grossman, Sanford and Perry, Motty (1986a) "Sequential bargaining under Asymmetric Information" *Journal of Economic Theory* ,39, 120-154

Grossman, Sanford J. and Perry, Motty (1986b), "Perfect Sequential Equilibrium," *Journal of Economic Theory,* 39, 97-119.

Loffgren, Karl Gustav, Torsten Persson and Jorgen W. Weibull, "Markets with Asymmetric Information: The Contributions of George Akerlof, Michael Spence and Joseph Stiglitz," *Scandinavian Journal of Economics*, (2002) forthcoming.

Mirrlees, James (1971), "An exploration in the theory of optimum income taxation," *Review of Economic Studies*, 38, 175-208

Riley, John (1975), "Competitive Signalling," *Journal of Economic Theory*, 10, 174-186.

Riley, John G. (1979), "Informational Equilibrium," *Econometrica*, 47, 331-359.

Riley, John G. (2001), "Silver Signals: 25 years of Screening and Signaling," *Journal of Economic Literature*, 39, 432-478.

Rothschild, Michael and Stiglitz, Joseph (1976), "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, 90, 629-649.

Spence, A. Michael (1973), "Job Market Signaling," *Quarterly Journal of Economics*, 87, 355-379.

Vickrey William (1961) "Counterspeculation, Auctions and Competitive Sealed tenders," *Journal of Finance*, 16, 41-50