Endogenous Competitive Bargaining*

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Abstract

We analyze a bargaining game where two agents compete for the right to propose a split of the pie to the other. This captures the notion that agenda control is a valuable asset that both bargaining parties covet. In the game, time is finite and agents are endowed with bidding capital. Each period, the agents use this capital to bid for the proposal right via a first-price auction, trading off the benefit of winning recognition today against the loss of future bargaining power. We characterize how the level of competition depends on the level and distribution of capital, and show it becomes more intense over time. If agents' endowments are similar to each other and the size of the pie, bids depend on the sum of agents' capital and double each period. Under a natural tie-breaking rule, this begets alternating offers as an equilibrium result.

1 Introduction

This paper studies a bargaining game where the right to make a bargaining offer is endogenously allocated through a competitive process between agents each period. This captures the notion that the right to make an offer is a valuable asset, and that valuable assets are frequently allocated to the party that values them the most at a cost reflective of this value, whether explicitly through prices, or implicitly through some rent-seeking process.

In our model the key determinant of players' bargaining power is their relative capital which enables an agent with more resources to make more offers and obtain a larger share of the pie. The idea that bargaining power depends upon resources, size or status is not new. Within economic theory, it is often assumed that firms choose prices, employers choose wages and governments choose regulations. This pervasive sense that bargaining power increases with size

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has empirical justification. In the cable and satellite television industry, Chipty and Snyder (1999) and Crawford and Yurukoglu (2010) have found that that larger participants obtain a greater share of bilateral surplus through their contracts. In international relations, Zartman and Rubin (2000, p. 275) draw on case studies of disputes to conclude that stronger parties (based on GNP, size, military strength) dominate the exchange with smaller parties and adopt forms of a take-it-or-leave-it strategy.

By introducing resource constraints, the model allows us to explore the dynamic tradeoff between controlling the agenda today and saving capital for future negotiations. Having power today is valuable because it enables the proposer to capture the gains from ending bargaining early, avoiding competition tomorrow. However, winning today also forces the proposer to expend capital and weakens her future bargaining position. We show that together, these forces imply that competition must escalate over time. We also characterize the form of equilibria as a function of the level and distribution of capital. If agents' endowments are similar to each other and small relative to the pie, this creates a standoff with both sides bidding little initially in order to avoid yielding the advantage to their competitor. If agents' endowments are similar to each other and to the size of the pie, bids double each period, inducing alternating offers as an equilibrium result.

In the model, two agents start with initial capital endowments (k_1, k_2) and bargain over a pie of size 1. There are a finite number of periods, T, each of which is split into two stages. In the *bidding stage*, agents compete over the right to be the proposer through a first-price auction. Since this is a model of complete information, the winning bid coincides with the Bertrand or Vickrey price. The winning bid is paid to an outside party. In the *bargaining stage* the proposer offers a split of the pie. If the offer is accepted the game is over and agents' utility equals their share of the pie plus any remaining capital. If the offer is rejected the game proceeds to the next period. If an agreement is not reached in the last period then both agents get nothing from the pie.

Our model can be thought to correspond to a bargaining setting where the right to make an offer is valuable and agents compete for this right. While the model is stylized, these features are an important component of many bargaining settings. For example, bankruptcy court judges have the right to determine which creditors can formally make restructuring proposals, and parties to a bankruptcy expend considerable resources arguing for this right. More figuratively, interest groups lobby committee members to propose legislation, employees pressure their supervisor to solve intra-office disputes in their favor, and politicians expend their political capital over budget negotiations.

In Section 3, we first analyze the one- and two-period games. This is the first step in the induction process and also develops intuition for the general t-period game. While the one-period game is trivial, the two-period game is quite intricate. Today's bid depends on the

benefit of winning the proposal right (the value of avoiding competition tomorrow) and the cost of losing future bargaining power. Both of these depend upon tomorrow's capital endowments which in turn depend on today's bid, introducing a fixed point problem. As a result, the two-period game gives rise to a rich set of outcomes, depending on the size and distribution of the agents' resources. If capital stocks are small relative to the size of the pie, differences in capital are important and agents bid very little in order to avoid yielding the advantage to their competitor. Whereas, if capital is stocks are large, agents bid the entire pie, resulting in full rent dissipation.

Section 4 describes the general analysis of the model. We first show that, in equilibrium, the game generically ends in one period. While there is no discounting, the competition between agents induces an endogenous cost of remaining in the game, and provides incentives for the proposer to make an offer to end the game immediately. Second, for any given tie-breaking rule, an equilibrium exists and is unique. When bidding, an agent wins the money saved by ending the game early, which equals next period's bid. However, if she wins, she loses today's bid and enters the next period with less capital and therefore a weaker bargaining position. In equilibrium, both agents are indifferent between winning and losing, and the winner is determined by a payoff-irrelevant tie-break rule. Third, competition for the right to make an offer becomes more intense as the game proceeds. The agent who wins today experiences a loss of future bargaining power, so tomorrow's bid forms an upper bound on today's bid. On the equilibrium path, where the game ends in one period, the amount spent on bidding tends to zero at rate $O(1/T)$. Hence the rent dissipation postulate fails: agents bid very little for the pie not because of an immediate shortage of capital, but because they may be constrained later down the road if negotiations continue.

Section 5 analyzes the range of capital allocations where agents' endowments are close to each other and to the size of the pie, forming a region around $(k_1, k_2) = (1, 1)$. By induction, we show that transferring ϵ from agent 1 to agent 2 does not affect bids (the *isobid property*) and raises agent 1's bargaining power by ϵ, so the transfer is worth 2ϵ in total (the *transfer property*). Intuitively, this transfer lowers agent 1's capital in the last period by ϵ, lowering his bid in the last period, and lowering the money he can extract from agent 2 in penultimate period by avoiding the last period. The transfer lowers 1's utility by ϵ no matter who wins prior auctions, and therefore the bid at which agents are indifferent between winning and losing does not change.

Together, the isobid property and transfer property imply that equilibrium bids exactly double in each subsequent period (the *doubling property*). If agent 1 wins in period t, she wins the welfare gain from avoiding the subsequent period, b^{t-1}. However, she loses the direct cost of the bid b^t and also transfers b^t from herself to agent 2. By the isobid property, this transfer does not affect future bids, but lowers agent 1's future bargaining share by b^t. Hence both

agents are indifferent between winning and losing when $b^{t-1} = 2b^t$.

The doubling property means that, since agents start with similar endowments, whoever wins in period t has less capital in the subsequent period. Thus, under the natural tie-breaking rule where the agent with the greater capital wins any ties, equilibrium endogenously generates alternating offers. In the limit, as $T \to \infty$, the split of the pie depends on the difference between agents' capital, providing a theoretical justification for simple power functions studied by Hirshleifer (1989) among others.

Throughout the paper, we assume recognition is determined by a winner-pays auction (e.g. a first-price or second-price auction). This is appropriate in a setting where payment takes place after control has been allocated. For example, when a CEO puts forward a proposal championed by one manager, this will use up the managers "social capital". Similarly, a lobbyist may donate to a politician if their proposed amendment is discussed on the Senate floor. However, if the payment occurs before the allocation, then an all-pay auction may be more appropriate. For example, two parties in a bankruptcy litigation may spend resources to persuade the judge to allow them to make a restructuring proposal.

In Section 6 we argue that the main insights carry over to the all-pay format. In particular, the agents in period t fight over the bids that would occur in period $t-1$, if the offer were rejected. Since having an offer rejected lowers an agent's capital, weakening their bargaining position, next period's bids are an upper bound for this period's bids. This means that bids become more aggressive as the game continues, and therefore the initial bids converge to zero at rate $O(1/T)$. Unfortunately, bidding in the all-pay auction is in mixed strategies which makes the analysis significantly harder. For this reason, the analysis in this section is heuristic.

The most closely related paper is Yildirim (2007). As in our paper, each bargaining period is split into two stages: In the first stage agent i exerts effort x_i and is recognized with probability $x_i / \sum_j x_j$; In the second stage, the recognized party makes a bargaining offer to the others. Yildirim characterizes the stationary equilibria and analyzes the effect of different discount rates, cost functions and voting rules. However, since agents do not have intertemporal budgets, the paper abstracts from the dynamic tradeoff between winning recognition today and losing bargaining power tomorrow that is the centerpiece of our analysis. In a follow up paper, Yildirim (2010) compares the outcome where agents expend effort every period, to the situation where they only expend effort at the start of the game.[1]

Our model serves as an attempt to endogenize the bargaining protocol, and relates to an existing literature on endogenous bargaining. A number of papers have noted the sensitivity of standard bargaining outcomes (e.g. Rubinstein, 1982) to the specific bargaining mechanism and

[1]Evans (1997) considers a related protocol in a model of coalitional bargaining. In his game N agents bid for the right to make an offer to some subset of the other agents, who then sequentially vote whether to accept. When there is no discounting and linear costs, this the set of equilibria corresponds to the core. In our simple game, every division of the pie is in the core.

the consequent desirability of endogenizing protocol. For example, Perry and Reny (1993) suppose an agent must wait a fixed period of time before making another offer, thereby conferring some commitment power. The outcome of the bargaining can then be described as a function of the agents' commitment powers. These models retain an essential Rubinstein feature that bargaining outcomes are driven by the cost of delay. In contrast, our setting endogenizes the discount rate by examining the competition to control the agenda and therefore provides a very different foundation for bargaining power.

2 The Model

Two risk neutral agents $\{1, 2\}$, bargain over a pie normalized to size of 1. Time $t \in \{1, \ldots, T\}$ is indexed backwards. At time T agents are endowed with capital k_1 and k_2. We index agents with subscripts, and periods with superscripts, so that agent i's remaining capital at time t is given by k_i^t. There is no discounting. Agent 1 is female, while agent 2 is male.

Each period consists of two stages. In the first stage, called the *bidding stage*, agents use their capital to bid for the right to make a bargaining offer. Bidding takes the form of a first-price auction, where the (common) equilibrium bid is denoted by b^t.[2] The payment in the auction is made to an outside third party or is wasted. We discuss the tie-breaking procedure below.

In the second stage, the *bargaining stage*, the winner of the auction, agent i, proposes a split (s_i^t, s_j^t) to agent j, where $s_i^t + s_j^t = 1$. If j accepts the offer, the pie is split accordingly and the game ends. If j rejects the offer, the game moves to the next period, which proceeds just as the prior period, save for the fact that agent i's capital is lower. If the bargaining offer is rejected in the final period, the game ends and both agents receive zero.

The utility of an agent equals the sum of her split of the pie and her unused capital. Let agent i's expected *continuation utility* at the start of any period, net of capital, be given by $U_i(k_1^t, k_2^t, t)$. Formally, if an offer is eventually accepted in period τ, this is

$$U_i(k_1^t, k_2^t, t) = E^t \left[s_i^\tau - \sum_{r=t}^{\tau} b_i^r \mathbb{I}_{\{i \text{ wins in period } r\}} \right]$$

where E^t is the expectation at the start of period t, taken over tie-break rules and strategies.

We consider subgame perfect equilibrium (SPE) of this game.

[2] Because of the complete information setting, the first-price auction yields the Vickrey or Bertrand price. We could also use a second-price auction after eliminating weakly dominated strategies.

2.1 Tie-Breaking Rules

If the two agents choose the same bid b^* in the auction, then the outcome is decided by a tie-breaker. There are three types of tie. In a *classical tie* both agents strictly prefer to win at $b \leq b^*$ but neither wishes to win at a higher price. This tie occurs in region B_1 in the one-period game and B_2 in the two-period game; in these cases, the two agents both have b^* capital. We break these ties randomly.

In a *constrained tie* there exists a b such that agent i prefers to win at $b + \epsilon$ than lose at b, but the same is not true of agent j. The equilibrium price b^* is then given by the highest j is willing to bid, and we assume i wins the tie. We call this the *Bertrand tie-breaking rule* since a similar closure problem exists in heterogeneous-cost Bertrand model. These ties occur in regions A_1 and C_2.[3]

In an *unconstrained tie*, agents i and j have the same preferences over winning and losing, and are indifferent at the same bid. Such ties are typical, but the tie-break rule has no effect on equilibrium payoffs (see Proposition 2). For concreteness, the *greater-capital tie-breaking rule* awards the proposal to the agent with the most capital in the case of an unconstrained tie; if the agents have the same capital, then we pick a winner randomly. Henceforth, when we say "for all tie-break rules" we refer to unconstrained ties.

3 One- and Two-Period Games

As a first step in the induction process we first consider the one- and two-period games. This analysis highlights the importance of the scale and distribution of endowments and develops intuition for the general analysis.

3.1 One-Period Game

If the offer in the last period is rejected, both agents receive 0 utility. This means whichever agent wins the right to make a proposal keeps the entire pie and gives nothing to the other agent. This $(1, 0)$ split is then accepted. Both agents would thus be willing to pay up to 1 in capital for the right to make this offer, so agent i's bid equals $b_i^1 = \min\{1, k_1, k_2\}$, where we drop the capital superscripts for simplicity.

Equilibrium payoffs depend upon which of three regions the initial endowments fall into. Let us suppose, without loss of generality, that agent 1 has greater capital, $k_1 \geq k_2$. These regions are then illustrated in Figure 1 and summarized in Table 1.

[3] One can justify the Bertrand tie-break rule by viewing the tie-break as part of the equilibrium solution (Simon and Zame, 1990).

(A_1) Moderate Capital. Suppose agent 2 is financially constrained and has strictly less capital than agent 1, $k_2 < \min\{1, k_1\}$. Given our Bertrand tie-breaking rule, agent 1 wins with a bid of $b^1 = k_2$.

(B_1) Identical and moderate capital. Suppose agent 2 is financially constrained and has the same capital as agent 1, $k_2 = k_1 < 1$. Then both agents bid their capital, with the winner determined by an arbitrary tie-break rule; let agent 1 win with probability α.

(C_1) No capital constraints. Suppose agent 2 is unconstrained, $k_2 \geq 1$. Then each agent is willing to bid up to $b^2 = 1$, with the tie-break rule deciding the winner. They thus compete away the entire pie.

3.2 Two-Period Game

In period 2, agents face a more difficult problem. When choosing how much to bid, agent 1 must calculate how much she has to offer agent 2 so he accepts. However the offer required to satisfy agent 2 is derived from the outcome of the bargaining game in the last period. This, in turn, depends on how much agent 1 bids in the penultimate period. This means bidding aggressively early on not only costs an agent capital, but also deprives her of capital later in the game, potentially reducing her split of the bargaining pie.

As with one period, bargaining depends crucially upon the initial capital. With two periods, the endowment can be broken into five regions, as shown in Figure 2 and summarised in Table 2. Details can be found in Appendix B.1. The most interesting regions are A_2, which will become the alternating offer region, and B_2 and C_2 where small differences in capital are critical.

Looking across regions, there are several general results that we expand on in Section 4. First, bids are continuous in capital stocks everywhere, while utilities are continuous except on the diagonal

$$\mathcal{D} := \{(k_1, k_2) : k_1 = k_2, k_1 \leq 1/2\}.$$

Second, bids are zero if and only if capital stocks lie in the region

$$\mathcal{Z} := \mathcal{D} \cup \{(k_1, k_2) : k_1 = 0 \text{ or } k_2 = 0\}.$$

where the agents lie on the diagonal or one has zero capital. Third, utilities are monotone in capital, although bids are not.

(A_2) Similar and Moderate Capital. Suppose that the agents are similar enough so that whoever wins in period 2 is constrained and has least capital in period 1. This range is explicitly characterized in Appendix B.1 and contains $(1, 1)$. If agent 1 wins in period 2 at a price

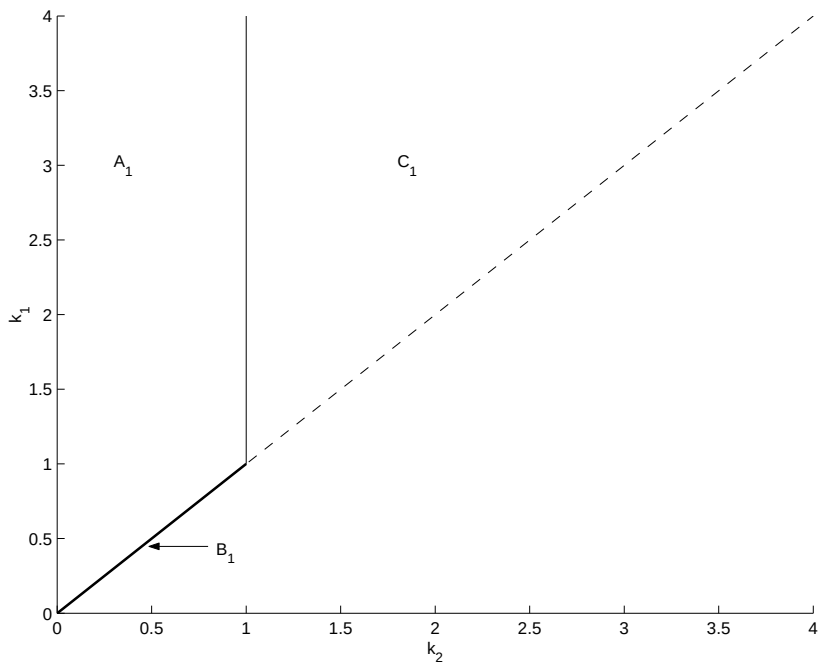

Figure 1: **Period 1 Regions.**

Region	Capital	$U_1(k_1, k_2, 1)$	$U_2(k_1, k_2, 1)$	Winning Bid	Winner
A_1	$k_2 < 1$ and $k_1 > k_2$	$1 - k_2$	0	k_2	1
B_1	$k_2 < 1$ and $k_1 = k_2$	$\alpha(1 - k_2)$	$(1 - \alpha)(1 - k_2)$	k_2	1 or 2
C_1	$k_2 \geq 1$ and $k_1 > k_2$	0	0	1	1 or 2

Table 1: **Period 1 Results.** This table shows utilities and bids for the one period game, assuming $k_1 \geq k_2$.

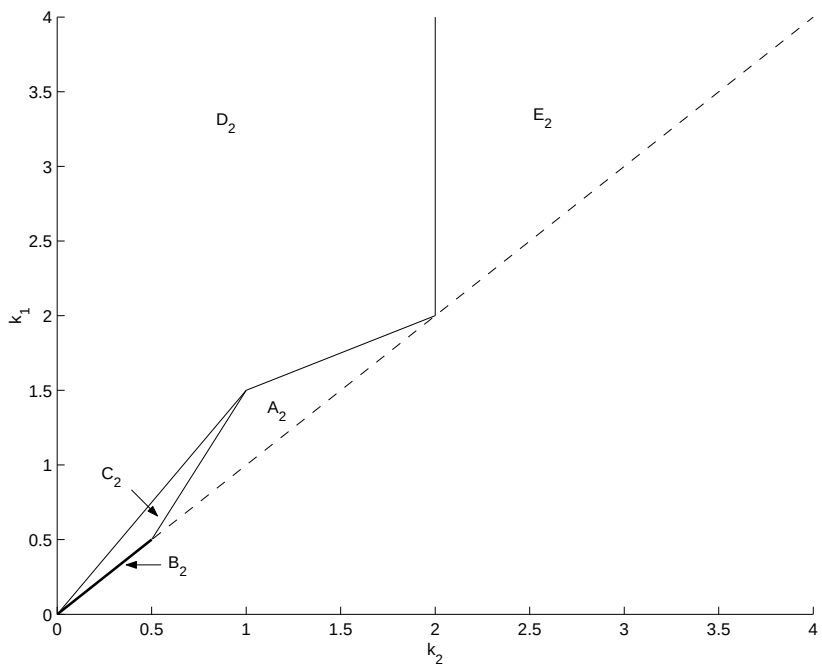

Figure 2: **Period 2 Regions.**

Region	$U_1(k_1, k_2, 2)$	$U_2(k_1, k_2, 2)$	Winning Bid	Winner
A_2 (Alternating)	$\frac{1}{3}k_1 - \frac{2}{3}k_2 + \frac{2}{3}$	$\frac{1}{3}k_2 - \frac{2}{3}k_1 + \frac{2}{3}$	$\frac{1}{3}k_1 + \frac{1}{3}k_2 - \frac{1}{3}$	$1 \to A_1'$ $2 \to A_1$
B_2 (Small Tie)	$\alpha(1-k_1) + \beta k_1$	$1 - \alpha(1-k_1) - \beta k_1$	0	$1 \to B_1$ $2 \to B_1$
C_2 (Small)	$k_1 - 2k_2 + 1$	$3k_2 - 2k_1$	$k_1 - k_2$	$2 \to A_1$
D_2 (Asymmetric)	$-\frac{1}{2}k_2 + 1$	0	$\frac{1}{2}k_2$	$1 \to C_1$ or C_1' $2 \to C_1$
E_2 (Big)	0	0	1	$1 \to C_1$ or C_1' $2 \to C_1$

Table 2: **Period 2 Results.** This table shows utilities and bids for the two period game, assuming $k_1 \geq k_2$. The last column shows which agent can win and, if they win, in which region capital stocks lie in the one period game. The notation A_1' represents the A_1 region where $k_2 \geq k_1$.

of b, Table 1 implies that utilities are

$$U_1^{\text{win}}(k_1, k_2, 2) = k_1 - 2b \quad \text{and} \quad U_2^{\text{lose}}(k_1, k_2, 2) = 1 - k_1 + b. \tag{3.1}$$

with the converse expression if agent 2 wins. Consequently, each agent is indifferent between winning and losing at price $b^2 = \frac{1}{3}(k_1 + k_2 - 1)$, with the winner determined by the tie-breaking rule. In equilibrium, agent 1 obtains $U_1(k_1, k_2, 2) = \frac{1}{3}(k_1 - 2k_2 + 2)$, with a symmetric expression for agent 2.

(B_2) Identical and Low Capital. Suppose agents have capital $k_1 = k_2 \leq 1/2$. If $b^2 > 0$ then (3.1) tells us that the loser makes the final offer and gains at least $1 - k_1$, while the winner gains at most k_1. Hence both agents prefer to lose for any positive bid. If both agents bid 0, they agent 1 gets $U_1(k_1, k_2, 2) = \alpha(1 - k_1) + \beta k_1$, where β is the probability she wins the tie. Since bidding $b > 0$ is always an option, equilibrium requires that α and β are such that $1 - k_1 \geq U_1(k_1, k_2, 2) \geq k_1$.

(C_2) Similar and Low Capital. Suppose agent 1 has a little more capital than agent 2, and both have relatively little capital. Using (3.1), agent 1 can guarantee herself at least $1 - k_2$ by waiting until the last period and beating agent 2. She is thus very reluctant to bid, and give away her advantage. Agent 2, knowing he will lose the last period, can guarantee himself $k_2 - b$ by bidding $b = k_1 - k_2$ in the penultimate period. In equilibrium, agent 2 thus wins with a bid of $b^2 = k_2 - k_1$. Intuitively, agent 1's utility, conditional on winning, jumps down at the 45^0 line, when the bid equals $b^2 = k_2 - k_1$. Agent 2 thus bids in such a way that if agent 1 outbids him, she would lose her competitive advantage.

(D_2) Dissimilar Capital. If agent 1 is much richer than agent 2 then, even if agent 1 wins in period 2, she will still have more capital in period 1. If 2 wins, utilities are given by the converse of (3.1). If 1 wins, utilities are given by

$$U_1^{\text{win}}(k_1, k_2, 2) = 1 - b \quad \text{and} \quad U_2^{\text{lose}}(k_1, k_2, 2) = 0 \tag{3.2}$$

Thus both agents are willing to bid up to $b^2 = \frac{1}{2}k_2$, with the tie–break rule deciding the winner.

(E_2) No Capital Constraints. Suppose both agents have sufficient capital so that they are unconstrained in period 1, even after winning. Then utilities are given by (3.2) if 1 wins and the converse expression if 2 wins. Hence each agent is willing to bid up to $b^2 = 1$, with the tie–break rule deciding the winner. They thus compete away the entire pie.

4 General Results

In this section we derive several general properties of the game. First, we show that in any equilibrium, the agents bargain away any future inefficiency, so their joint surplus equals the size of the pie minus the current bid. Intuitively, inefficiency arises from future bids, and acts like an endogenous discount rate. The agent who wins period t can thus hold the losing agent to her outside option, and extract the surplus from ending the game earlier.

Lemma 1. *In any SPE,*

$$U_1(k_1^t, k_2^t, t) + U_2(k_1^t, k_2^t, t) = 1 - b^t. \tag{4.1}$$

Proof. Suppose agent 1 wins in period t with a bid of b^t. Agent 2 can reject 1's offer and so is guaranteed his outside option, $U_2(k_1^t, k_2^t, t) \geq U_2(k_1^t - b^t, k_2^t, t-1)$. Conversely, agent 1 can ensure her proposal is accepted by offering 2 his outside offer plus ϵ, and can thus guarantee herself $U_1(k_1^t, k_2^t, t) \geq 1 - U_2(k_1^t - b^t, k_2^t, t-1) - b^t - \epsilon$, for any $\epsilon > 0$. Summing these two expressions and observing that feasibility implies $U_1(k_1^t, k_2^t, t) + U_2(k_1^t, k_2^t, t) \leq 1 - b^t$, yields the result. □

Lemma 1 says that after b^t has been paid, there is no more burning of surplus. Hence it has the corollary that if agent 1 wins and $b^{t-1}(k_1^t - b^t, k_2^t) > 0$, then her offer must be accepted and the game ends immediately.

We now wish to characterize the equilibrium bids. If agent 1 loses to a bid of b, then she is held to her outside option and obtains

$$U_1^{\text{lose}} = U_1(k_1^t, k_2^t - b, t-1) \tag{4.2}$$

Conversely, if 1 wins, she obtains

$$U_1^{\text{win}} = 1 - U_2(k_1^t - b, k_2^t, t-1) - b \tag{4.3}$$

Using (4.1), we can equivalently write

$$U_1^{\text{win}} = U_1(k_1^t - b, k_2^t, t-1) + b^{t-1}(k_1^t - b, k_2^t) - b \tag{4.4}$$

so agent 1 obtains her outside option minus her bid plus next period's bid, which is the surplus gained from ending the game today. Using equations (4.2) and (4.3), agent 1 prefers to win with a bid of b if

$$1 - U_2(k_1^t - b, k_2^t, t-1) - b \geq U_1(k_1^t, k_2^t - b, t-1). \tag{4.5}$$

Equation (4.5) is symmetric in 1 and 2. This means that agent 1 prefers to win at bid b if and

only if agent 2 prefers to win at bid b. Hence, if utility is continuous and the bid is feasible, then both agents will be indifferent at the optimal bid:

$$1 - U_2(k_1^t - b, k_2^t, t-1) - b = U_1(k_1^t, k_2^t - b, t-1). \tag{4.6}$$

Intuitively, for a fixed bid b, agents 1 and 2 are competing over a pie of size $1-b$. Hence if $1-b$ exceeds the sum of outside options $U_1(k_1^t - b, k_2^t, t-1) + U_2(k_1^t, k_2^t - b, t-1)$ then there is excess surplus on the table and both agents want to win; conversely, if the pie is less that the sum of outside options then neither agent wishes to win.

Lemma 2 extends several of the properties we saw in Section 3.2. There are two new results: First, we show the indifference equation (4.6) holds everywhere, despite the discontinuity along the diagonal $\mathcal{D}$. Second, the zero region $\mathcal{Z}$ is closed: capital stocks lie in $\mathcal{Z}$ in period $t-1$ if and only if they lie in $\mathcal{Z}$ in period t.

Lemma 2. *Suppose $t \geq 3$.*
(a) Bids are zero if and only if $(k_1^t, k_2^t) \in \mathcal{Z}$
(b) $(k_1^t, k_2^t) \in \mathcal{Z}$ if and only if $(k_1^{t-1}, k_2^{t-1}) \in \mathcal{Z}$.
(c) Agents are indifferent between winning and losing at the equilibrium bid, so (4.6) holds.
(d) $U_i(k_i^t, k_j^t, t)$ is increasing in k_i^t and decreasing in k_j^t.
(e) $U_i(k_i^t, k_j^t, t)$ is continuous in (k_i^t, k_j^t) except at $\mathcal{D}$, while $b^t(k_i^t, k_j^t)$ is continuous everywhere.

Proof. See Appendix A □

Part (a) says that bids are zero if capital stocks lie on the diagonal $\mathcal{D}$. Intuitively, utility is discontinuous to the right and left, and neither party wishes to lose their competitive advantage. Clearly, bids are also zero if either agent has no capital. Part (b) says that if capital stocks lie within $\mathcal{Z}$ then the agents bid zero today, and remain within $\mathcal{Z}$. Conversely, if the capital stocks lie outside $\mathcal{Z}$, then $\mathcal{Z}$ acts as a repellent: if the agents jump into $\mathcal{Z}$ then tomorrow's bid is zero and neither player would be willing to bid a positive amount today. Part (c) observes that since a game that starts away from $\mathcal{Z}$ never lands on $\mathcal{Z}$, the boundary constraints never bind and the indifference equation (4.6) holds everywhere. Part (d) shows that utilities are monotone in capital, although bids are not monotone. Hence if agent 1 has ϵ more capital, this can spur more competition, but that competition cannot burn more than ϵ. Finally part (e) shows that utilities are continuous except on the diagonal $\mathcal{D}$. Along this diagonal, agents compete tentatively so bids converge to zero and the bid function is continuous everywhere.

Proposition 1. *If $(k_1^t, k_2^t) \notin \mathcal{Z}$ then the game ends in one period in any SPE.*

Proof. If $(k_1^t, k_2^t) \notin \mathcal{Z}$ then Lemma 2(b) implies that $(k_1^{t-1}, k_2^{t-1}) \notin \mathcal{Z}$. Lemma 2(a) states that $b^{t-1} > 0$, so Lemma 1 implies that any SPE involves acceptance in period t. □

Proposition 1 elaborates on Lemma 1. It shows that if $(k_1^t, k_2^t) \notin \mathcal{Z}$ then any SPE ends in one period. This occurs because $\mathcal{Z}$ is closed: if the game starts outside $\mathcal{Z}$ then today's bid is positive and, if the offer is rejected, tomorrow's bid will also be positive. As a consequence, there is a real cost of bargaining and, on the equilibrium path, the offer will be accepted. Conversely, if $(k_1^t, k_2^t) \in \mathcal{Z}$ then there exists a SPE that ends in one period. Since $b^{t-1} = 0$, both parties are indifferent between the offer being accepted or rejected.

Proposition 2. *We have the following uniqueness results:*
(a) For a given tie-break rule, payoffs and bids are uniquely determined.
(b) The tie break rule does not affect payoffs or bids on the equilibrium path.

Proof. (a) For $t \leq 2$, Section 3 shows that the equilibrium bid and resulting utilities are determined uniquely. For $t \geq 3$, Lemma 2(c) implies that the equilibrium bid is determined by equation (4.6). Lemma 2(d) says that $U_1(k_1^t, k_2^t - b, t-1)$ and $U_2(k_1^t - b, k_2^t, t-1)$ are weakly increasing in b, implying the equilibrium bid and payoffs are unique.

(b) For $T \leq 2$, Section 3 shows that the choice of unconstrained tie-break rule does not affect utilities or bids. For $T \geq 3$, the payoffs are determined by (4.6), where agents are indifferent between winning and losing. If $(k_1^T, k_2^T) \notin \mathcal{Z}$, Proposition 1 says the game ends immediately, and (4.6) also determines the bid independent of the tie-break rule. If $(k_1^T, k_2^T) \in \mathcal{Z}$, Lemma 2(a) and (b) imply that the bids are all zero. □

Proposition 2(a) says for a given tie-break rule, bids and payoffs are uniquely determined in every subgame. Looking at the equilibrium path, part (b) says that, the choice of tie–break rule, has no impact on payoffs or bids. There may be some indeterminacy in when the game ends if $(k_1^T, k_2^T) \in \mathcal{Z}$, but the resulting bids are then equal to zero. The tie-break rule does affect the distribution of capital and thus future payoffs and bids, but these are off the equilibrium path.

The following result shows that, for any capital endowments, bids grow over time.

Proposition 3. *In any SPE profile, $b^{t-1} \geq b^t$.*

Proof. Without loss, suppose agent 1 wins in period t. In the subgame $(k_1^t - b^t, k_2^t, t-1)$, equation (4.1) implies that

$$b^{t-1} = 1 - U_1(k_1^t - b^t, k_2^t, t-1) - U_2(k_1^t - b^t, k_2^t, t-1). \tag{4.7}$$

In the subgame (k_1^t, k_2^t, t), equation (4.5) yields

$$-b^t \geq U_1(k_1^t, k_2^t - b^t, t-1) + U_2(k_1^t - b^t, k_2^t, t-1) - 1. \tag{4.8}$$

Summing (4.7) and (4.8),

$$b^{t-1} - b^t \geq U_1(k_1^t, k_2^t - b^t, t-1) - U_1(k_1^t - b^t, k_2^t, t-1) \geq 0.$$

where the last line uses the monotonicity of utility (Lemma 2(d)). □

Proposition 3 shows that, off the equilibrium path, competition becomes more intense as the game proceeds. This is despite the fact that resources fall over time. Intuitively, in period t, agents fight over the social waste in the following period, b^{t-1}. If resources are sufficiently high, agents are willing to pay b^{t-1} in period t, leading to bids that are constant over time. However, when the resource constraint binds, a high bid worsens an agent's future bargaining position and b^t is strictly less than b^{t-1}.

Proposition 4. *As $T \to \infty$, the initial bid converges to zero at rate $b^T = O(1/T)$. As a result, $U_1(k_1^T, k_2^T, T) + U_2(k_1^T, k_2^T, T) \to 1$.*

Proof. The sum of bids in any sequence of subgames must satisfy $\sum_{t=1}^T b^t \leq k_1^T + k_2^T$. Proposition 3 implies $Tb^T \leq \sum_{t=1}^T b^t$. Hence $b^T \leq (k_1^T + k_2^T)/T$.

There exists an equilibrium which ends in period 1; in such an equilibrium the initial bid converges to zero, so the sum of utilities converge to 1. By Proposition 2, all equilibria yield same utilities, so the sum always converges to 1. □

Proposition 4 shows that, as the game becomes longer, the total capital used in the competition converges to zero. It is interesting to compare this result to the rent dissipation postulate which states that "obtaining a monopoly is itself a competitive activity, so that, at the margin, the cost of obtaining a monopoly is exactly equal to the expected profit of being a monopolist." (Posner (1975, p. 809)). If agents' resources are at least as large as the pie, then our result shows that the rent dissipation postulate holds if the game is sufficiently short. However, as the number of rounds for negotiation grows, the desire to save capital for future negotiations lowers competition in earlier rounds. More broadly, this shows how the possibility that resource constraints could bind somewhere down the road mitigates rent dissipation today. This suggests that lobbying will be more fierce on a new issue with a tight time constraint than on a longstanding problem.

5 Alternating Offer Region

We now consider capital endowments that are similar to each other and the size of the pie, including $(k_1, k_2) = (1, 1)$. We show that the solution to this problem exhibits the following properties. First, equilibrium bids depend on the sum of agents' capital, so transferring ϵ from agent 1 to agent 2 does not effect equilibrium bids (the *isobid property*). Second, transferring ϵ

from agent 1 to agent 2 raises 2's equilibrium share of the pie by ϵ, in addition to the direct gain of ϵ (the *transfer property*). Third, if the bargaining offer is refused, the equilibrium bid in the following period is exactly double that of the current period (the *doubling property*). Finally, we explicitly characterize the alternating region and show that, under the greater-capital tie-breaking rule, the proposer alternates each period (hence the name of this region).

These results hold for a specific region of capital endowments, so should be taken too literally. Rather, this example cleanly illustrates the tradeoff between spending resources today and saving resources for tomorrow, and how this tradeoff leads to rapidly escalating bids. It allows us to solve for utilities in closed form, generating an endogenous power function. Finally, it shows how similarly endowed agents will make alternating offers. However, in our model, this is an implication of agents having similar levels of power, not the source of that power.

Recall that, when $t = 2$, the alternating region A_2 is defined so that, given the equilibrium bid, the agent who wins period 2 is constrained and has the least capital in period 1, if her offer is rejected. Algebraically, A_2 is defined by all capital allocations (k_1^2, k_2^2) that satisfy (B.1); that is, for $k_1^2 \geq k_2^2$,

$$k_1^2 \leq \min\left\{2k_2^2 - \frac{1}{2}, \frac{1}{2}k_2^2 + 1\right\}.$$

The *alternating offer region* A_t is defined iteratively as follows: In any period $t \geq 3$, A_t equals all capital allocations (k_1^t, k_2^t) such that if the two agents make equilibrium bids and the bargaining offer is turned down (which is off-the-equilibrium-path) then the game enters A_{t-1}, no matter which agent wins the auction. We define the range explicitly in Proposition 5.[4]

Recall from Section 3 that in period 2, both agents are indifferent between winning and losing at bid

$$b^2 = \frac{k_1^2 + k_2^2 - 1}{3}. \tag{5.1}$$

Agent 1's continuation utility is then given by

$$U_1(k_1^2, k_2^2, 2) = \frac{1}{3}k_1^2 - \frac{2}{3}k_2^2 + \frac{2}{3}, \tag{5.2}$$

with an equivalent expression for agent 2. Equations (5.1) and (5.2) exhibit both the transfer property and the isobid property. To understand why, suppose we are initially at an equilibrium so 1 and 2 are indifferent between winning and losing. Suppose we transfer ϵ from 1 to 2, and keep the equilibrium bid the same. If 2 loses in period 2, the transfer from 1 means he will be able to lower his period 1 bid by ϵ, causing 1 to offer ϵ more in period 2, and raising 2's period 2 utility by ϵ, as shown in (3.1). Conversely, if 2 wins in period 2, the extra capital means 1 will be forced to raise her period 1 bid by ϵ, causing 2 to offer ϵ less in period 2, and raising 2's

[4]Equivalently, the alternating offer region can be viewed as the capital allocations (k_1^t, k_2^t) such that if the two agents make equilibrium bids in periods $\{3, \ldots, t\}$, the game ends up in A_2 for any sequence of winners.

period 2 utility by ϵ, as shown in the converse of (3.1). Whether she wins or loses, 2's utility is ϵ higher, implying that agent 2 (and also agent 1) are still indifferent between and losing and the equilibrium bid remains unchanged.

We now turn to the $t \geq 3$ period case and show, by induction, that provided $(k_1^t, k_2^2) \in A_t$ then the equilibrium bid is given by

$$b^t(k_1, k_2) = \frac{k_1^t + k_2^t - 1}{2^t - 1}, \tag{5.3}$$

and that 1's continuation utility is

$$U_1(k_1^t, k_1^t, t) = \frac{2^{t-1} - 1}{2^t - 1} k_1^t - \frac{2^{t-1}}{2^t - 1} k_2^t + \frac{2^{t-1}}{2^t - 1}, \tag{5.4}$$

with a similar expression for agent 2. Notably, these satisfy the isobid and transfer properties.

Suppose these expressions hold for period $t-1$. Setting $\epsilon = b^t$, the transfer property implies that

$$U_1(k_1^t, k_2^t - b^t, t-1) - U_1(k_1^t - b^t, k_2^t, t-1) = b^t. \tag{5.5}$$

The isobid property means that b^{t-1} is independent of the winner of the period t auction. Hence if agent 1 wins at a bid of b^t, equation (4.4) says her utility is

$$\begin{aligned} U_1^{\text{win}}(k_1^t, k_2^t, t) &= U_1(k_1^t - b^t, k_2^t, t-1) + b^{t-1} - b^t \\ &= U_1(k_1^t, k_2^t - b^t, t-1) + b^{t-1} - 2b^t. \end{aligned}$$

where the second line uses (5.5). In contrast, if agent 1 loses, (4.2) says she obtains

$$U_1^{\text{lose}}(k_1^t, k_2^t, t) = U_1(k_1, k_2 - b^t, t-1).$$

Lemma 2(c) states that, in equilibrium, agent 1 is indifferent between winning and losing. Cancelling immediately yields the doubling property:

$$2b^t = b^{t-1} \qquad t \geq 3. \tag{5.6}$$

Intuitively, winning today costs the agent b^t but allows her to extract the welfare gain from avoiding bargaining tomorrow, b^{t-1}. In addition, winning causes the agent to have a weaker bargaining power tomorrow, effectively transferring b^t capital to her opponent, which also costs her b^t. Indifference thus requires that $2b^t = b^{t-1}$.[5]

[5]Note that the doubling property says bids double in subsequent rounds of a game, as capital is depleted. Hence if we fix capital stocks, the initial bid in game $(k_1, k_2, t-1)$ will be more than double the initial bid in game (k_1, k_2, t), assuming $(k_1, k_2) \in A_t$.

Given that b^{t-1} satisfies (5.3), bid doubling implies that

$$2b^t = \left[\frac{k_1^t + k_2^t - b^t - 1}{2^{t-1} - 1}\right]. \tag{5.7}$$

Rearranging, (5.7) implies that b^t satisfies (5.3), as required. Finally, given utility at period $t-1$ satisfies (5.4), substituting (5.7) into either (4.3) or (4.4) implies that utility at t is also given by (5.4), completing the induction proof.

In the special case where initial endowments are $(k_1^T, k_2^T) = (1, 1)$, equilibrium bids and utilities take a particularly simply form. Using (5.3) to derive the bid in period T, the doubling property implies that

$$b^t = \frac{2^{T-t}}{2^T - 1} \qquad t \geq 2.$$

Correspondingly, initial utilities are given by

$$U_1(1, 1, T) = U_2(1, 1, T) = \frac{2^{T-1} - 1}{2^T - 1}.$$

Bid doubling also implies that for $(k_1^T, k_2^T) \in A_T$, equilibrium bids go to 0 at the rate of $1/2^T$. This is much quicker than the convergence in Proposition 4 and means that, as the number of rounds grows, the sum of the two agents' continuation utility approaches one quickly. In the limit, bargaining shares converge to a particularly simple form given by

$$U_1(k_1^T, k_2^T, \infty) = \frac{1}{2}(k_1^T - k_2^T) + \frac{1}{2}.$$

According to this equation, an agent's share of the pie only depends on the difference in capital allocations, so an extra ϵ in capital yields an extra $\epsilon/2$ from the contest. Our framework thus serves as a foundation for power functions that map the resources of an agent and the proportion of the pie they obtain (e.g. Hirshleifer (1989)). Such functions have been used in applications such as tournaments (Rosen (1986)) and arms races (Skaperdas (1992)), but without much justification for any particular functional form.

We now characterise the alternating region explicitly.

Proposition 5. *Let $k_1^t \geq k_2^t$. Then $(k_1^t, k_2^t) \in A_t$ for $t \geq 2$ if and only if*

$$k_1^t \leq \min\left[\frac{2^t k_2^t - 1}{2^t - 2}, \frac{2^{t-2}}{3 \cdot 2^{t-2} - 1} k_2^t + 1\right]. \tag{5.8}$$

Hence $(1, 1) \in A_t$ for all t.

Proof. When $t = 2$, (5.8) coincides with (B.1). Continuing by induction, suppose (5.8) defines A_t up to period $t-1$ and consider period t. Without loss, suppose agent 1 wins in period t.

Using the definition of the alternating region, we must show that if 1's offer is refused then the game enters the alternating region in period $t-1$. In Proposition 6 we show that (5.8) implies that $k_1^t - b^t \leq k_2^t$. We must therefore verify that (5.8) is satisfied at t with $k_2^{t-1} \geq k_1^{t-1}$.

First, we require that the first part of (5.8) holds in period $t-1$. That is,

$$k_2^t \leq \frac{2^{t-1}(k_1^t - b^t) - 1}{2^{t-1} - 2}$$

Using equation (5.3) to substitute for b^t and rearranging yields the first term in (5.8).

Next, we require that the second part of (5.8) holds in period $t-1$. That is,

$$k_2^t \leq \frac{2^{t-3}}{3 \cdot 2^{t-3} - 1}(k_1^t - b^t) + 1$$

Using equation (5.3) to substitute for b^t and rearranging yields the second term in (5.8). □

Proposition 6. *Suppose $(k_1^t, k_2^t) \in A_t$. In any SPE the winner of period t has less capital in period $t-1$ if their offer is rejected.*

Proof. Suppose $k_1^t \geq k_2^t$ and (k_1^t, k_2^t) satisfies (5.8). If agent 2 wins in period t, the result trivially holds. If agent 1 wins,

$$k_1^t - b^t = k_1^t - \frac{k_1^t + k_2^t - 1}{2^t - 1} = \frac{1}{2^t - 1}[(2^t - 2)k_1^t - k_2^t + 1] \leq k_2^t$$

where the first equality uses the bid equation (5.3), and the inequality uses the first part of constraint (5.8). □

Proposition 6 says that the equilibrium bid (5.3) is always larger than the difference between the two agents' capital allocations. Consequently, if agent 1 has more capital in period t and wins the auction, agent 2 will have more capital in period $t-1$ if 1's bargaining offer is rejected. This means that under the greater-capital tie-breaking rule the two agents will end up making alternating offers, just as assumed in standard Rubinstein bargaining.

The fact that bid doubling results in alternating offers is no coincidence. Fixing period t, if agent 1 wins the first $t-2$ auctions, while agent 2 wins the period 2 auction, the definition of A_2 implies that agent 1 has more capital in the last period, $k_1^1 \geq k_2^1$. This puts an upper bound on the difference between k_1^t and k_2^t,

$$0 \geq k_1^1 - k_2^1 = \left(k_1^t - (2^{t-2} - 1)b^t\right) - \left(k_2^t - 2^{t-1}b^t\right) = k_1^t - k_2^t + b^t$$

Hence the difference between initial capital allocations is bounded by b^t, implying that whoever wins the auction in the period t must have less capital in period $t-1$.

6 All-Pay Auctions

So far we have assumed that the recognition right is determined by a winner-pays auction. In this section we argue that the main insights of Section 4 carry over to an all-pay format. In particular, we argue that bids become more aggressive as the game continues, and therefore that the initial bids converge to zero at rate $O(1/T)$. The intuition is the same as before: the agents in period t fight over the social waste that would occur in period $t-1$, if the offer were rejected. Since a rejected bid also lowers an agent's capital and weakens their bargaining position, next period's bids are an upper bound for this period's bids.

First, consider the proposal stage. If agent 1 wins, he will hold his opponent to her outside option. Denoting the (random) bids by β_1^t and β_2^t, this implies

$$U_1(k_1^t, k_2^t, t) + U_2(k_1^t, k_2^t, t) = 1 - E^t[\beta_1^t + \beta_2^t], \tag{6.1}$$

analogous to equation (4.1).

Turning to the bidding stage, both agents will use mixed strategies over some common support, $[0, \overline{\beta}]$. By standard arguments, if utilities are continuous, a bidding distribution can have no gaps or atoms on $(0, \overline{\beta}]$, although it may have an atom at 0 (e.g. Hillman and Riley (1989)). Fixing $\beta_1 \in (0, \overline{\beta}]$, agent 1's expected utility is

$$U_1(\beta_1) = \int_0^{\beta_1} [1 - U_2(k_1^t - \beta_1, k_2^t - \beta_2, t-1)] dF_2(\beta_2) + \int_{\beta_1}^{\overline{\beta}} U_1(k_1^t - \beta_1, k_2^t - \beta_2, t-1) dF_2(\beta_2) - \beta_1$$

where $F_2(\cdot)$ is the distribution of β_2. If an agent bids zero, they obtain

$$U_1(0) \geq \int_0^{\overline{\beta}} U_1(k_1^t, k_2^t - \beta_2, t-1) dF_2(\beta_2) \geq \int_0^{\overline{\beta}} U_1(k_1^t - \beta_1, k_2^t - \beta_2, t-1) dF_2(\beta_2)$$

where the first inequality is an equality if $F_2(\cdot)$ has no atom at 0, and the second inequality assumes that $U_1(k_1, k_2, t)$ is increasing in k_1. Since bidding zero is feasible, $U_1(\beta_1) \geq U_1(0)$ and

$$\beta_1 \leq \int_0^{\beta_1} [1 - U_2(k_1^t - \beta_1, k_2^t - \beta_2, t-1) - U_1(k_1^t - \beta_1, k_2^t - \beta_2, t-1)] dF_2(\beta_2) \tag{6.2}$$

which is the analogue of equation (4.5). Taking expectations over (6.2) and using (6.1) at period $t-1$,

$$E^t\Big[\beta_1^t(k_1, k_2)\Big] \leq E^t\Big[[\beta_1^{t-1}(k_1^t - \beta_1, k_2^t - \beta_2) + \beta_2^{t-1}(k_1^t - \beta_1, k_2^t - \beta_2)]\mathbb{I}_{\beta_2 < \beta_1}\Big]$$

which says that agent 1's bids are bounded above by the expected social waste next period, conditional on agent 1 winning. The symmetric expression holds for agent 2, and summing

these yields

$$E^t\Big[\beta_1^t(k_1,k_2)+\beta_2^t(k_1,k_2)\Big] \leq E^t\Big[\beta_1^{t-1}(k_1^t-\beta_1,k_2^t-\beta_2)+\beta_2^{t-1}(k_1^t-\beta_1,k_2^t-\beta_2)\Big] \tag{6.3}$$

Hence the expected sum of bids grows as the game progresses. Since the sum of bids is less that $k_1^T + k_2^T$, it follows that $E^T[\beta_1^T + \beta_2^T] = O(1/T)$, so the social waste on the equilibrium path converges to zero. This analysis is only heuristic, since we do not prove the monotonicity or continuity of continuation utilities. However, these equations show that the all-pay auction exhibits the same dynamic tradeoff as the winner-pays auction.

7 Conclusion

When bargaining, the ability to set the agenda is a valuable asset that the agents covet. We suppose agents are endowed with capital which is used to bid for this right. This captures the idea that the size and status of the agents can affect their bargaining power in addition to their outside options.

The game has a generically unique subgame perfect equilibrium, where agents trade off the gain of recognition against the cost of competing and the resulting loss of future bargaining power. In short games, competition is intense and full rent dissipation can result; in longer games, agents are wary of spending too early and use little of their resources on the equilibrium path. We have also explored the relationship between the size and distribution of agents' endowments and their bargaining power. When agents' endowments are much smaller than the pie, the bargaining shares are sensitive to asymmetries, and bids are initially very small with neither agent wanting to yield the advantage to the other. When the agents' endowments are similar to each other and to the size of the pie, bids depend on the sum of agents endowments and double each period. Under the greater-capital tie-breaking rule, this results in the proposer alternating between periods.

As with most perfect information models, our game has the analytically convenient but unrealistic feature that bargaining generically ends in the first period. If we add a small amount of asymmetric information we conjecture long delays occur, precisely because the discount rate is endogenous and grows over time. In early periods there is little real time cost and it is worth delaying to signal one's type; as the deadline approaches, the prospect of cut throat competition in the end-game eventually leads to agreement.

A Proof of Lemma 2 in Section 4

As shown in Section 3.2, (a), (b), (d) and (e) are true at $t = 2$, which is enough for the induction. Throughout this proof we assume, without loss, that $k_1^t \geq k_2^t$.

(a) If $k_2^t = 0$, then the equilibrium bid is clearly zero. If $(k_1^t, k_2^t) \in \mathcal{D}$ and the bid is zero both agents get $U_1(k_1^t, k_2^t, t) \geq k_1^t$ (see Section 3.2). Now suppose agent 1 deviates and bids $b > 0$. She will be disadvantaged in period $t-1$, allowing agent 2 to be able to guarantee himself $U_2(k_1^t - b, k_2^t, t-1) \geq 1 - k_1^t + b$ by simply waiting until period 1. Hence when agent 1 wins she receives at most receive

$$U_1^{\text{win}} = 1 - U_2(k_1^t - b, k_2^t, t-1) - b \leq k_1^t - 2b$$

Thus agent 1 is better off bidding $b^t = 0$ and losing, rather than bidding $b > 0$ and winning.

Next, suppose $(k_1^t, k_2^t) \notin \mathcal{Z}$. We claim that either agent would strictly prefer to win with probability 1 with a small positive bid rather than win with probability $\gamma < 1$ with a bid $b = 0$. Hence there cannot be an equilibrium with both agents bidding $b = 0$. To prove the claim observe that (a) implies that $b^{t-1}(k_1^t, k_2^t) > 0$, while (e) says utility and bids are continuous around (k_1^t, k_2^t) at $t-1$. Hence for small $b > 0$,

$$U_1(k_1^t - b, k_2^t, t-1) + b^{t-1}(k_1^t - b, k_2^t, t-1) - b > U_1(k_1^t, k_2^t, t-1) + \gamma b^{t-1}(k_1^t - b, k_2^t, t-1)$$

and (4.4) implies the agent prefers to bid $b > 0$.

(b) If $(k_1^t, k_2^t) \in \mathcal{Z}$ then (a) implies that $b^t = 0$ and $(k_1^{t-1}, k_2^{t-1}) \in \mathcal{Z}$.

Now, consider $(k_1^t, k_2^t) \notin \mathcal{Z}$. To show $(k_1^{t-1}, k_2^{t-1}) \notin \mathcal{Z}$ suppose that agent 1 bids $b^t = k_1$ or $b^t = k_1^t - k_2^t$. If she wins then $(k_1^{t-1}, k_2^{t-1}) \in \mathcal{Z}$ and all subsequent bids are zero. From (4.2) and (4.4),

$$U_1^{\text{win}} = U_1(k_1^t - b^t, k_2^t, t-1) + b^{t-1} - b^t < U_1(k_1^t, k_2^t - b^t, t-1) = U_1^{\text{lose}} \tag{A.1}$$

using $b^{t-1} = 0$, monotonicity at $t-1$ and $b^t > 0$. Thus agent 1 prefers to lose, yielding a contradiction.

(c) Indifference. If $(k_1^t, k_2^t) \in \mathcal{Z}$, then $b^t = b^{t-1} = 0$ and indifference holds trivially $U_1^{\text{win}} = U_1(k_1^t, k_2^t, t-1) = U_1^{\text{lose}}$.

Now suppose $(k_1^t, k_2^t) \notin \mathcal{Z}$. We know from parts (a) and (b) that equilibrium bids are internal, $b^t \in (0, k_2^t)$. If both agents strictly prefer to win at their bid b^t and agent 1 refuses to increase his bid then we must have $b \approx k_1^t - k_2^t$ since (e) implies that the 45^0 line is the only point of discontinuity at $t-1$. Yet the continuity of bids in period $t-1$ and equation (A.1) imply that agent 1 strictly prefers to lose when $b \approx k_1^t - k_2^t$, yielding a contradiction.

(d) Monotonicity. Let $\tilde{k}_1 \geq k_1^t$, $\tilde{k}_2 \leq k_2^t$ and $b = b^t(k_1^t, k_2^t)$. Then,

$$\begin{aligned} U_1(\tilde{k}_1, \tilde{k}_2, t) &\geq \min\{1 - U_2(\tilde{k}_1 - b, \tilde{k}_2, t-1) - b, U_1(\tilde{k}_1, \tilde{k}_2 - b, t-1)\} \\ &\geq \min\{1 - U_2(k_1^t - b, k_2^t, t-1) - b, U_1(k_1^t, k_2^t - b, t-1)\} = U_1(k_1^t, k_2^t, t) \end{aligned}$$

where the first line is the utility from winning/losing if agent 1 bids b, the second line follows from monotonicity at $t-1$, and the third line follows from indifference at t.

(e) Continuity. Fixing $(k_1^t, k_2^t) \notin \mathcal{D}$, (b) tells us $(k_1^{t-1}, k_2^{t-1}) \notin \mathcal{D}$ so (e) implies that utility and bids are continuous at time $t-1$. Parts (c) and (d) state that the equilibrium bid $b^t(k_1^t, k_2^t)$ is the unique solution to the indifference equation (4.6) and is thus continuous; equilibrium utility $U_1(k_1^t, k_2^t, t)$ is then given by either side of (4.6) when evaluated at b^t and is also continuous.

If $(k_1^t, k_2^t) \in \mathcal{D}$ then bids are zero. To show bids are close to zero for nearby capital stocks, suppose $k_1^t > k_2^t$. By part (b) and (d), agent 1 strictly prefers to lose to a bid $b \geq k_1^t - k_2^t$, implying that $b^t < k_1^t - k_2^t$. Hence $b^t \to 0$ as $|k_1^t - k_2^t| \to 0$.

B Two- and Three-Period Games

B.1 Two-Period Game

In this section we finish the characterization of the two round game. In particular, we define the regions in Figure 2.

Symmetric and Moderate Capital: Region $\mathbf{A_2}$. We derived bids and utilities in the main text. To see which endowments are consistent with this behavior, suppose agent 1 wins in period 2. She must have less capital than agent 2 in the next period and be financially constrained, so that $k_1^1 \leq \min\left\{1, k_2^1\right\}$. Given that $k_1^1 = k_1 - b^2$ and $k_2^1 = k_2$, we obtain

$$A_2 = \left\{k_1 \leq 2k_2 - \frac{1}{2}\right\} \cap \left\{k_1 \leq \frac{1}{2}k_2 + 1\right\}. \tag{B.1}$$

Identical and Low Capital: Region $\mathbf{B_2}$. Given that both agents wish to lose to any positive bid, this area is defined by

$$B_2 = \{k_1 = k_2 \leq 1/2\}.$$

Similar and Low Capital: Region $\mathbf{C_2}$. This is the most intricate region because the discontinuity in utilities at the 45^0 line means bids are determined by a boundary condition. In particular, $b^2 = k_2 - k_1$ and C_2 is defined by

$$C_2 = \left\{2k_2 - \frac{1}{2} \leq k_1\right\} \cap \left\{k_1 \leq \frac{3}{2}k_2\right\} \tag{B.2}$$

so that neither agent has enough capital to win both rounds.

If agent 2 wins, utility is given by equation

$$U_1^{\text{lose}}(k_1, k_2, 2) = 1 - k_2 + b \quad \text{and} \quad U_2^{\text{win}}(k_1, k_2, 2) = k_2 - 2b. \tag{B.3}$$

Conversely, if agent 1 wins, utility is discontinuous in b and we must consider three cases. First, if agent 1 wins with a bid $b < k_1 - k_2$ utilities are given by (3.2). Condition (B.2) states that $k_1 \leq \frac{3}{2}k_2$ so that both agents would like to win, and this cannot be an equilibrium. Second, if agent 1 wins with a bid $b > k_1 - k_2$ then utilities are given by equation (3.1). Condition (B.2) states that $2k_2 - \frac{1}{2} \leq k_1$ so that both agents would like to lose, and this cannot be an equilibrium. Finally, if agent 1 wins with a bid $b = k_1 - k_2$ then utilities are

$$U_1^{\text{win}}(k_1, k_2, 2) = k_2 - b + \alpha(1 - k_2) \quad \text{and} \quad U_2^{\text{lose}}(k_1, k_2, 2) = (1 - \alpha)(1 - k_2) \tag{B.4}$$

Given the Bertrand tie-breaking rule, essentially any equilibrium involves agent 2 winning with a bid of $b^2 = k_1 - k_2$. To see this, consider three cases.

First, if $(4 - \alpha)k_2 - 2k_1 - (1 - \alpha) > 0$ then (B.3) and (B.4) imply that both agents strictly prefer to win at a bid of $b^2 = k_1 - k_2$. Agent 2 prefers to win at $b = k_1 - k_2 + \epsilon$ than lose at $b = k_1 - k_2$, which is not true of agent 1, so the Bertrand tie-break rule implies that agent 2 wins the auction.

Second, if $(4 - \alpha)k_2 - 2k_1 - (1 - \alpha) < 0$ then both agents strictly prefer to lose at a bid of $b^2 = k_1 - k_2$. Hence both agents bid the smallest number below $b^2 = k_1 - k_2$.[6] Agent 2 prefers to win at $b = k_1 - k_2$ than lose at $b = k_1 - k_2 - \epsilon$, which is not true of agent 1, so again the Bertrand tie-break rule implies that agent 2 wins the auction.

Third, if $(4 - \alpha)k_2 - 2k_1 - (1 - \alpha) = 0$ then both agents are indifferent between winning and losing, and either can win. Hence the equilibrium bid is $b^2 = k_1 - k_2$.

Dissimilar Capital: Region $\mathbf{D_2}$. The derivation of bids is in the main text. We implicitly assumed that (a) if agent 2 wins he will be capital constrained $k_2 - b^2 \leq 1$, and (b) if agent 1 wins then her capital exceeds $\min\left\{1, k_2^1\right\}$. Hence the region is given by

$$D_2 = \left\{\frac{1}{2}k_2 \leq 1\right\} \cap \left\{k_1 - \frac{1}{2}k_2 \geq \min\{1, k_2\}\right\}.$$

No Capital Constraints: Region $\mathbf{E_2}$. The bids are derived in the main text. We assumed that, after the initial bid, both agents still have at least $k_i^1 \geq 1$ remaining. Since $b^2 = 1$ this

[6] The "smallest number below $k_1 - k_2$" is obviously not defined when bids are on a continuum, but makes sense as the limit of a grid. This closure problem only occurs at this point in the game. If the tie-break rule is part of the equilibrium solution (Simon and Zame, 1990), then α will be such that this problem does not arise.

means that in order to not be capital constrained, each agent must have at least $k_i^2 \geq 2$. This defines the region E_2 in Figure 2.

B.2 Three-Period Game

This section presents the outcome of the bargaining game for three rounds. This illustrates how the game develops outside the alternating offer region as the number of rounds grows, and also demonstrates the properties in Lemma 2. Equilibrium payoffs and bids are given in Table 3, with the regions illustrated in Figure 3. We assume $k_1^3 \geq k_2^3$ without loss.

Going through all the regions is quite lengthy, so we just consider region C_3, which lies next to the diagonal $\mathcal{D}$. Formally, we define C_3 such that if either agent wins, then we enter region C_2. When $t = 2$, we saw that there is a corner solution with agent 2 winning with a bid of $b^2 = k_1^2 - k_2^2$. When $t = 3$, there is an internal solution with both agents indifferent between winning and losing and $b^3 = (k_1^3 - k_2^3)/5$.

If agent 1 wins in period 3, equations (4.2) and (4.3), and Table 2 implies that the utilities are

$$U_1^{\text{win}} = 1 - [3k_2^3 - 2(k_1^3 - b)] - b \qquad \text{and} \qquad U_2^{\text{lose}} = 3k_2^3 - 2(k_1^3 - b)$$

Similarly, if agent 2 wins, utilities are

$$U_1^{\text{lose}} = k_1^3 - 2(k_2^3 - b) + 1 \qquad \text{and} \qquad U_2^{\text{win}} = 1 - [k_1^3 - 2(k_2^3 - b) + 1] - b$$

Thus, both agents are indifferent between winning and losing at $b^3 = (k_1^3 - k_2^3)/5$. Plugging back in, utilities are given in Table 3.

Region C_2 is characterized by

$$C_2 = \left\{k_1^2 \geq k_2^2\right\} \cap \left\{k_1^2 \leq \frac{3}{2}k_2^2\right\} \cap \left\{k_1^2 \geq 2k_2^2 - \frac{1}{2}\right\}.$$

We thus need to check two conditions (a) if agent 1 wins in period 3, $k_1^3 - b^3 > 2k_2^3 - \frac{1}{2}$, and (b) if agent 2 wins in period 3, $k_1^3 < \frac{3}{2}(k_2^3 - b^3)$. The other conditions are automatically satisfied. This implies

$$C_3 = \left\{k_1^3 \geq k_2^3\right\} \cap \left\{k_1^2 \leq \frac{18}{13}k_2^3\right\} \cap \left\{k_1^3 \geq \frac{9}{4}k_2^3 - \frac{5}{8}\right\},$$

as shown in Figure 3.

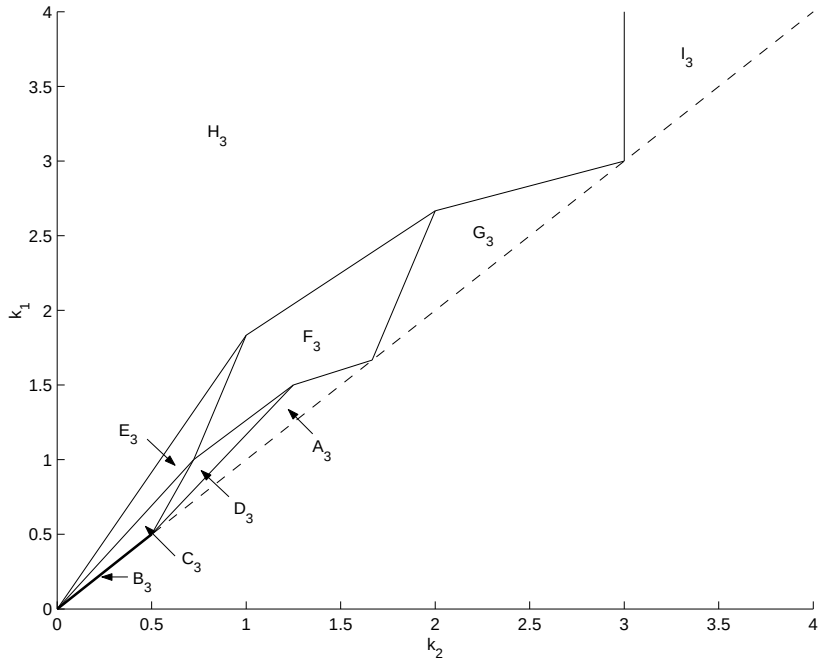

Figure 3: **Period 3 Regions.**

Region	$U_1(k_1, k_2, 3)$	$U_2(k_1, k_2, 3)$	Winning Bid	Winner
A_3 (Alternating)	$\frac{3}{7}k_1 - \frac{4}{7}k_2 + \frac{4}{7}$	$\frac{3}{7}k_2 - \frac{4}{7}k_1 + \frac{4}{7}$	$\frac{1}{7}k_1 + \frac{1}{7}k_2 - \frac{1}{7}$	$1 \to A_2'$ $2 \to A_2$
B_3 (Small Tie)	$\alpha(1-k_1) + \beta k_1$	$1 - \alpha(1-k_1) - \beta k_1$	0	$1 \to B_2$ $2 \to B_2$
C_3 (Small)	$\frac{7}{5}k_1 - \frac{12}{5}k_2 + 1$	$\frac{13}{5}k_2 - \frac{8}{5}k_1$	$\frac{1}{5}k_1 - \frac{1}{5}k_2$	$1 \to C_2$ $2 \to C_2$
D_3	$\frac{9}{11}k_1 - \frac{12}{11}k_2 + \frac{7}{11}$	$\frac{7}{11}k_2 - \frac{8}{11}k_1 + \frac{6}{11}$	$-\frac{1}{11}k_1 + \frac{5}{11}k_2 - \frac{2}{11}$	$1 \to A_2$ or A_2' $2 \to C_2$
E_3	$\frac{2}{7}k_1 - \frac{6}{7}k_2 + 1$	$\frac{11}{7}k_2 - \frac{6}{7}k_1$	$\frac{4}{7}k_1 - \frac{5}{7}k_2$	$1 \to C_2$ $2 \to D_2$
F_3	$\frac{2}{13}k_1 - \frac{6}{13}k_2 + \frac{11}{13}$	$\frac{5}{13}k_2 - \frac{6}{13}k_1 + \frac{6}{13}$	$\frac{4}{13}k_1 + \frac{1}{13}k_2 - \frac{4}{13}$	$1 \to A_2$ or A_2' $2 \to D_2$
G_3	$\frac{1}{8}k_1 - \frac{3}{8}k_2 + \frac{6}{8}$	$\frac{1}{8}k_2 - \frac{3}{8}k_1 + \frac{6}{8}$	$\frac{1}{4}k_1 + \frac{1}{4}k_2 - \frac{1}{2}$	$1 \to D_2'$ $2 \to D_2$
H_3 (Asymmetric)	$-\frac{1}{3}k_2 + 1$	0	$\frac{1}{3}k_2$	$1 \to E_2$ or E_2' or D_2 $2 \to D_2$
I_3 (Big)	0	0	1	$1 \to E_2$ or E_2' $2 \to E_2$

Table 3: **Period 3 Results.** This table shows utilities and bids for the three period game, assuming $k_1 \geq k_2$. The last column shows which agent can win and, if they win, in which region capital stocks lie in the two period game. The notation A_2' represents the A_2 region where $k_2 \geq k_1$.

References

Chipty, T. and C. Snyder, 1999, "The Role of Firm Size in Bilateral Bargaining: A Study of the Cable Television Industry," *The Review of Economics and Statistics*, Vol 81, 326-340.

Crawford, G. and A. Yurukoglu, 2012, "The Welfare Effects of Bundling in Multichannel Television Markets," *The American Economic Review*, Vol 102, 64385.

Evans, R., 1997, "Coalitional Bargaining with Competition to Make Offers," *Games and Economic Behavior*, Vol 19, 211–220.

Hillman, A., and J. Riley, 1989, "Politically Contestable Rents and Transfers," *Economics & Politics*, Vol 1, 17-39.

Hirshleifer, J., 1989, "Conflict and Rent–Seeking Success Functions: Ratio vs. Difference Models of Relative Success", *Public Choice*, Vol 63, 101–112.

Perry, M., and P. Reny, 1993, "A Non-Cooperative Bargaining Model with Strategically Timed Offers," *Journal of Economic Theory*, Vol 59, 50-77.

Posner, R., 1975, "The Social Costs of Monopoly and Regulation," *Journal of Political Economy*, Vol 83, 807–828.

Rosen, S., 1986, "Prizes and Incentives in Elimination Tournaments", *American Economic Review*, Vol 76, 701-715.

Rubinstein, A., 1982 "Perfect Equilibrium in a Bargaining Model," *Econometrica*, Vol 50, 97-110.

Simon, L. K. and W. R. Zame, 1990, "Discontinuous Games and Endogenous Sharing Rules", *Econometrica*, Vol 58, 861–872.

Skaperdas, S., 1992, "Cooperation, Conflict and Power in the Absence of Property Rights", *American Economic Review*, Vol 82, 720–739.

Yildirim, H., 2007, "Proposal Power and Majority Rule in Multilateral Bargaining with Costly Recognition," *Journal of Economic Theory*, Vol 136, pp. 167-96.

Yildirim, H., 2010, "Distribution of Surplus in Sequential Bargaining with Endogenous Recognition," *Public Choice*, Vol 142, 41–57.

Zartman, I. W. and J. Z. Rubin, 2000, "Symmetry and Asymmetri in Negotiation" in *Power and Negotiation* eds I. W. Zartman and J. Z. Rubin, University of Michigan Press, pp. 271–294.