A Reputational Theory of Firm Dynamics

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Motivation

Models of firm dynamics

- Wish to generate dispersion in productivity, profitability etc.
- Some invest in assets and grow; others disinvest and shrink.

A firm's reputation is one of its most important assets

- Kotler: "In marketing, brand reputation is everything".
- Interbrand: Apple brand worth $98b; Coca-Cola $79b.
- EisnerAmper: Reputation risk is directors' primary concern.

Reputation a special asset

- Reputation is market *belief* about quality.
- Reputation can be volatile even if underlying quality constant.

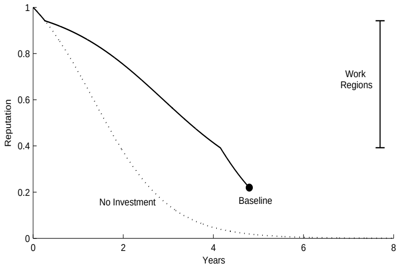
This Paper

Firm dynamics with reputation

- Firm invests in quality.
- Firm & mkt. learn about quality.
- Firm exits if unsuccessful.

Optimal investment

- Firm shirks near end.
- Incentives are hump-shaped.



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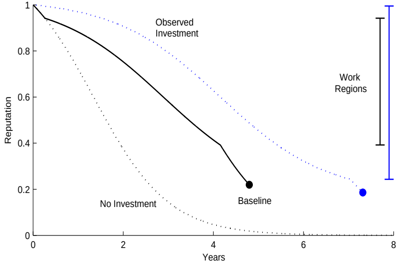
Benchmarks

- Consumers observe investment.



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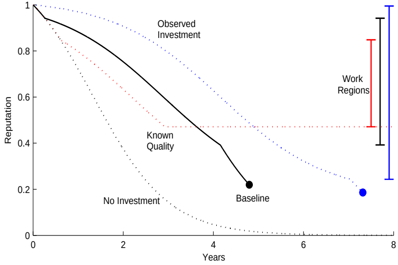
- Firm privately knows quality.



Literature

Reputation Models

- Bar-Isaac (2003)
- Kovrijnykh (2007)
- Board and Meyer-ter-Vehn (2013)

Firm Dynamics

- Jovanovic (1982)
- Hopenhayn (1992)
- Ericson and Pakes (1995)

Moral hazard and learning

- Holmstrom (1982)
- Bonatti and Horner (2011, 2013)
- Cisternas (2014)

Model

Model, Part I

Long-lived firm sells to short-lived consumers.

- Continuous time $t \in [0, \infty)$, discount rate r.
- Firm invests $A_t \in [0, \overline{a}]$, $\overline{a} < 1$, and exits at time T.

Technology

- Quality $\theta_t \in \{L, H\}$ where $L = 0$ and $H = 1$.
- Technology shocks arrive with Poisson rate λ.
- Quality given by $\Pr(\theta_s = H) = A_s$ at last shock $s \le t$.

Information

- Breakthroughs arrive with Poisson rate μ iff $\theta_t = H$.
- Consumers observe history of breakthroughs, h^t.
- Firm additionally recalls past actions.

Model, Part II

Reputation and Self-Esteem

- Consumers' beliefs over strategy of firm, $F = F(\{\tilde{A}_t\}, \tilde{T})$.
- *Self-esteem* $Z_t = \mathbb{E}^{\{A_t\}}[\theta_t | h^t]$.
- *Reputation*, $X_t = \mathbb{E}^F[\theta_t | h^t, t < \tilde{T}]$.

Payoffs

- Consumers obtain flow utility X_t.
- Firm value

$$V = \max_{\{A_t\},T} \mathbb{E}^{\{A_t\}} \left[\int_0^T e^{-rt}(X_t - cA_t - k)dt \right].$$

Recursive Strategies

Game resets at breakthrough, X=Z=1.

- $\{A_t\}, T$ is recursive if only depend on time since breakthrough.
- F is recursive if only puts weight on recursive strategies.
- If F recursive, then optimal strategies are recursive.
- Notation: $\{a_t\}, \tau, \{x_t\}, \{z_t\}, V(t, z_t)$ etc.

Self-esteem

- Jumps to $z_t = 1$ at breakthrough.
- Else, drift is $\dot{z}_t = \lambda(a_t - z_t)dt - \mu z_t \left(1 - z_t\right) dt =: g(a_t, z_t)$.

Assumption: A failing firm eventually exits

- Negative drift at top, $z^\dagger := \lambda/\mu < 1$.
- Exit before $z^\dagger$ reached, $z^\dagger - k + \mu z^\dagger (1-k)/r < 0$.

Optimal Investment & Exit

Optimal Strategies Exist

Lemma 1. *Given $\{x_t\}$, an optimal $\{a_t^*\}, \tau^*$ exists with $\tau^* \leq \overline{\tau}$.*

Idea

- Drift $g(a, z)$ is strictly negative for $z \in [z^\dagger, 1]$.
- $V(t, z^\dagger) < 0$ for any strategy, so τ^* bounded.
- Action space compact in weak topology by Alaoglu's theorem.
- Payoffs are continuous in $\{z_t\}$, and hence in $\{a_t\}, \tau$.

Notation

- Optimal strategies $\{a_t^*\}, \tau^*$.
- Optimal self-esteem $\{z_t^*\}$.

Optimal Investment

Lemma 2. *Given $\{x_t\}$, optimal investment $\{a_t^*\}$ satisfies*

$$a_t^* = \begin{cases} 0 & \textit{if} \quad \lambda V_z(t, z_t^*) < c, \\ \overline{a} & \textit{if} \quad \lambda V_z(t, z_t^*) > c. \end{cases}$$

Investment pays off by

- Raising self-esteem immediately.
- Raising reputation via breakthroughs.

Dynamic complementarity

- $V(t, z)$ is convex; strictly so if $\{x_t\}$ continuous.
- Raising a_t raises z_{t+dt} and incentives $V_z(t, z_{t+dt})$.
- Optimal strategies ordered: $z_t^* > z_t^{**} \Rightarrow z_{t'}^* > z_{t'}^{**}$ for $t' > t$.

Marginal Value of Self-Esteem

Lemma 3. *Given $\{x_t\}$, if $V_z(t, z_t^*)$ exists it equals*

$$\Gamma(t) = \int_t^{\tau^*} e^{-\int_t^s r+\lambda+\mu(1-z_u^*)du} \mu(V(0,1) - V(s, z_s^*))ds.$$

Value of self-esteem over dt

- dz raises breakthrough by $\mu dz dt$.
- Value of breakthrough is $V(0,1) - V(s, z_t^*)$.

Discounting the dividends

- Payoffs discounted at rate r.
- dz disappears with prob. $\mu z_t^* dt$, if breakthrough arrives.
- dz changes by $g_z(a, z_t) = -(\lambda + \mu(1 - 2z_t^*))$.

Derivation of Investment Incentives

- Give firm cash value of any breakthrough,

$$V(t, z_t^*) = \int_t^{\tau^*} e^{-r(s-t)}(x_s - ca_s^* - k + \mu z_s^*(V(0,1) - V(s, z_s^*))ds.$$

- Apply the envelope theorem,

$$V_z(t, z_t^*) = \int_t^{\tau^*} e^{-r(s-t)} \frac{\partial z_s^*}{\partial z_t^*} \Big(\mu(V(0,1) - V(s, z_s^*)) - \mu z_s^* V_z(s, z_s^*) \Big) ds.$$

- The partial derivative equals,

$$\partial z_s^* / \partial z_t^* = \exp\left(- \int_t^s (\lambda + \mu(1 - 2z_u^*))du \right).$$

- Placing $\mu z_s^* V_z(s, z_s^*)$ into the exponent,

$$V_z(t, z_t^*) = \Gamma(t) := \int_t^{\tau^*} e^{-\int_t^s (r+\lambda+\mu(1-z_u^*))du} \mu(V(0,1) - V(s, z_s^*))ds.$$

Property 1: Shirk at the End

Theorem 1. *Given* $\{x_t\}$*, any optimal strategy* $\{a_t^*\}, \tau^*$*, exhibits shirking* $a_t^* = 0$ *on* $[\tau^* - \epsilon, \tau^*]$*.*

Idea

- At $t \to \tau^*$, so $\Gamma(t) \to 0$.
- Need technology shock *and* breakthrough before τ^* for investment to pay off.
- Shirking accelerates the demise of the firm.

Property 2: Incentives are Single-Peaked

Theorem 2. *If $\{x_t\}$ decreases, investment incentives $\Gamma(t)$ are single-peaked with $\Gamma(0) > 0$, $\dot{\Gamma}(0) > 0$ and $\Gamma(\tau^*) = 0$.*

Proof

- Differentiating $\Gamma(t)$ with $\rho(t) := r + \lambda + \mu(1 - z_t^*)$,

$$\dot{\Gamma}(t) = \rho(t)\Gamma(t) - \mu(V(0,1) - V(t, z_t^*)).$$

- Differentiating again,

$$\begin{aligned}\ddot{\Gamma}(t) &= \rho(t)\dot{\Gamma}(t) + \dot{\rho}(t)\Gamma(t) + \mu\dot{z}_t^*\Gamma(t) + \mu V_t(t, z_t^*) \\ &= \rho(t)\dot{\Gamma}(t) + \mu V_t(t, z_t^*).\end{aligned}$$

- If $\{x_t\}$ is decreasing $V_t < 0$, and $\dot{\Gamma}(t) = 0$ implies $\ddot{\Gamma}(t) < 0$.

Countervailing forces: As t rises,

- Dividends $V(0,1) - V(t, z_t^*)$ grow, and incentives increase.
- Get close to exit and incentives decrease.

Property 3: Exit Condition

Theorem 3. *If* $\{x_t\}$ *is continuous, then* τ^* *satisfies*

$$V(\tau^*, z_{\tau^*}) = \underbrace{(x_{\tau^*} - k)}_{\textit{flow profit}} + \underbrace{\mu z_{\tau^*} V(0,1)}_{\textit{option value}} = 0.$$

Equilibrium

Definition

Equilibrium beliefs

- Reputation $x_t = E^F[\theta^t | h^t = \varnothing, t \leq \tilde{\tau}]$ given by Bayes' rule.
- Under point beliefs, $\dot{x} = \lambda(\tilde{a} - x)dt - \mu x(1-x)dt$.
- Can hold any beliefs after $\tau(F) := \min\{t : F(\tilde{\tau} \leq t) = 1\}$.

Recursive equilibrium

- Given $\{x_t\}$, any strategy $\big(\{a_t\}, \tau\big) \in \text{supp}(F)$ is optimal.
- Reputation $\{x_t\}$ derived from F via Bayes' rule for $t < \tau(F)$.

Existence

Theorem 4. *An equilibrium exists.*

Idea

- Strategy space compact in weak topology.
- Bayes rule, best response correspondences u.h.c.
- Apply Kakutani-Fan-Glicksberg Theorem.

Pure Strategy Equilibria

In a pure strategy equilibrium, $x_t = z_t^*$.

- $\{x_t\}$ decreases and incentives are single-peaked (Theorem 2).

Changes in costs

- High costs: Full shirk equilibrium.
- Intermediate costs: Shirk-work-shirk equilibrium.
- Low costs: Work-shirk equilibrium.

Simulation Parameters

Restaurant accounting

- Revenues: $\$x$ million.
- Capital cost: \$500k.
- Investment cost: \$125k.
- Interest rate: 20%.

Arrival rates

- Breakthroughs arrive once a year.
- Technology shocks arrive every 5 years.

Value Function and Firm Distribution

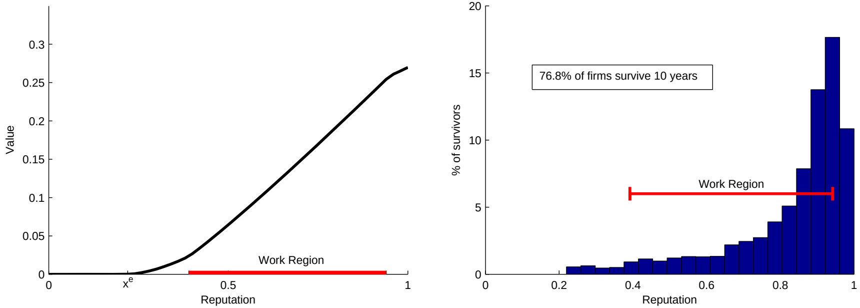
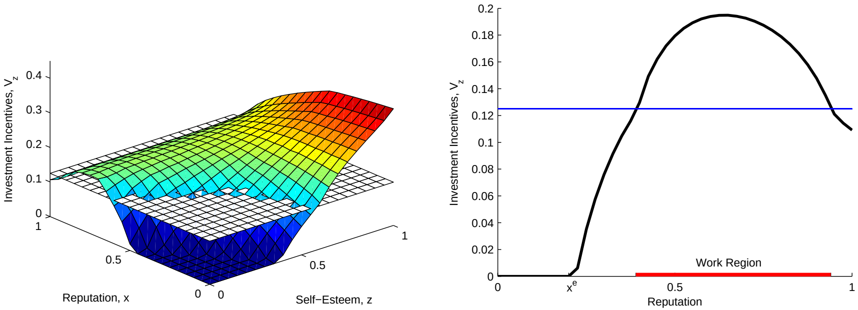


Figure: Capital cost $k = 0.5$, investment cost $c = 0.1$, interest rate $r = 0.2$, max. effort $\overline{a} = 0.99$, breakthroughs $\mu = 1$, technology shocks $\lambda = 0.1$.

Investment Incentives



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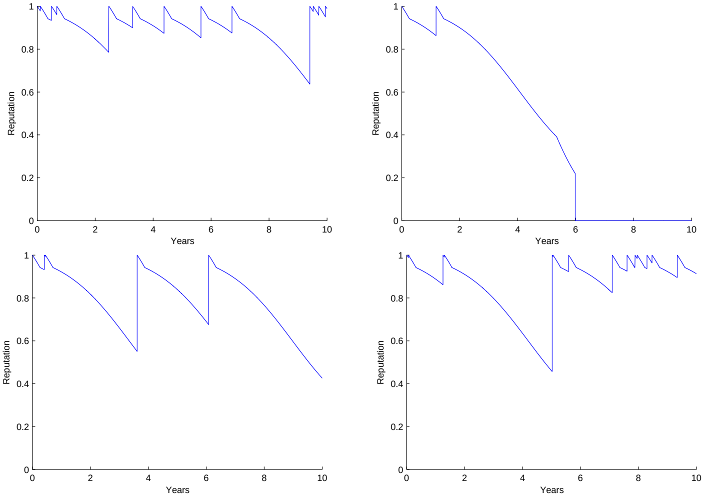
Typical Life-cycles



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Mixed Strategy Equilibria

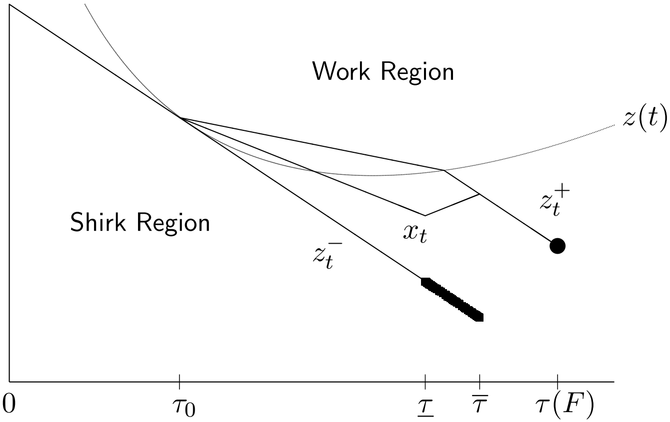
Exit

- Firm shirks near exit point (Theorem 1).
- Firms with less self-esteem exits gradually.
- Firm with most self-esteem exits suddenly.

Reputational dynamics

- $\{x_t\}$ decreases until firms start to exit.
- $\{x_t\}$ increases when firms gradually exit.

Illustration of Mixed Strategy Equilibrium



Competitive Equilibrium

Agent's preferences

- Firm i has expected output $x_{t,i}$
- Total output of experience good is $X_t = \int_i x_{t,i} di$.
- Consumers have utility $U(X_t) + N_t$.

Equilibrium

- Competitive equilibrium yields price $P_t = U'(X_t)$.
- Stationary equilibrium: P_t independent of t.
- Firm i's revenue is $x_{t,i} P$ and value is $V_i(t, z_t; P)$.

Entry

- Firm pays ξ to enter and is high quality with probability $\check{x}$.
- Given a pure equilibrium, let $z_{\check{t}} = \check{x}$.
- Free entry determines price level: $V(\check{t}, z_{\check{t}}; P) = \xi$.

Model Variation: Observable Investment

Observable Investment

Investment a_t is publicly observed

- Reputation and self-esteem coincide, $x_t = z_t$.

Optimal strategies

- Optimal investment

$$\hat{a}_t = \begin{cases} 0 & \text{if } \lambda \hat{V}_z(\hat{z}_t) < c \\ 1 & \text{if } \lambda \hat{V}_z(\hat{z}_t) > c \end{cases}$$

- Investment incentives

$$\hat{\Gamma}(t) = \int_t^{\hat{T}} e^{-\int_t^s r+\lambda+\mu(1-\hat{z}_u)du} \left[1 + \mu(\hat{V}(1) - \hat{V}(\hat{z}_s))\right] ds.$$

- Optimal exit

$$\hat{V}(\hat{z}_t) = \underbrace{\hat{z}_{\hat{T}} - k}_{\text{flow profit}} + \underbrace{\mu \hat{z}_{\hat{T}} \hat{V}(1)}_{\text{option value}} = 0.$$

Characterizing Equilibrium

Theorem 5. *If investment is observed, investment incentives* $\hat{\Gamma}(t)$ *are decreasing with* $\hat{\Gamma}(0) > 0$ *and* $\hat{\Gamma}(\hat{\tau}) = 0$.

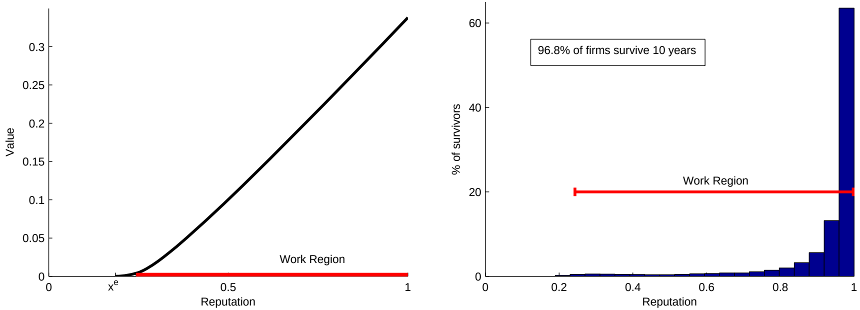
Proof

- Value $\hat{V}(\cdot)$ is strictly convex.
- Self-esteem $\hat{z}_t$ strictly decreases over time.
- Hence $\hat{V}_z(z_t)$ strictly decreases with $\hat{V}_z(z_{\hat{\tau}}) = 0$.

Idea: Investment is beneficial if

- There is a technology shock.
- There is a resulting breakthrough prior to exit time.

Value Function and Firm Distribution



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Impact of Moral Hazard

Theorem 6. *If investment is observed, the firm works longer than in any baseline equilibrium.*

Idea

- When observed firm increases investment, belief also rises.
- Such favorable beliefs are good for the firm.
- Optimal investment choice higher for observed firm.

With observable investment,

- No shirk region at the top.
- Work until lower reputation.
- Value higher, so exit later.

Model Variation: Privately Known Quality

Privately Known Quality

Firm knows θ_t

- Investment a_t still unknown, so there is moral hazard.

Recursive strategies

- Firm knows quality and time since breakthrough.
- Chooses investment a_t and exit time τ.
- Value function $V(t, \theta_t)$.

Optimal investment $a_t(\theta)$

- Independent of quality $a_t(\theta) = a_t$ and given by:

$$a_t = \begin{cases} 1 & \text{if } \lambda\Delta(t) > c \\ 0 & \text{if } \lambda\Delta(t) < c \end{cases}$$

  where $\Delta(t) := V(t,1) - V(t,0)$ is value of quality.
- Tech. shock has probability λdt, yielding benefit $\Delta(t)$ of work.

Exit Choice

Assuming $\{x_t\}$ continuously decreases

- Low quality firm exits gradually when $t > \tau^L$.
- In equilibrium, high quality firm never exits.

Assuming firm works at end,

- Exit condition becomes

$$V(t,0) = \underbrace{(x_t - c - k)}_{\text{flow profit}} + \underbrace{\lambda V(t,1)}_{\text{option value}} = 0.$$

Equilibrium Characterization

Theorem 7. *If quality is privately observed, investment incentives $\Delta(t)$ are increasing with $\Delta(0) > 0$.*

Proof

- The value of quality is present value of dividends:

$$\Delta(t) = \int_t^{\infty} e^{-(r+\lambda)(s-t)} \mu[V(0,1) - V(s,1)]ds.$$

- Investment incentives $\lambda\Delta(t)$ increase in t.

Impact of Private Information

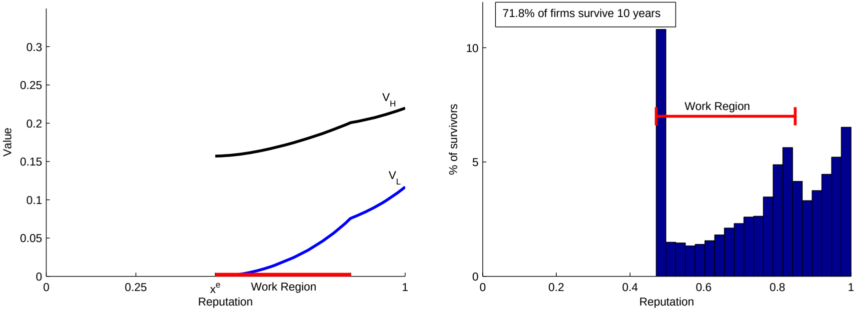
Known quality

- Work pays off if tech. shock (prob. λdt).
- Fight to bitter end.
- Low firm gradually exits; high never does.

Unknown quality

- Work pays off if tech. shock & breakthrough ($\lambda dt \times \mu dt$).
- Coast into liquidation.
- Firm exits after τ periods without breakthrough.

Value Function and Firm Distribution



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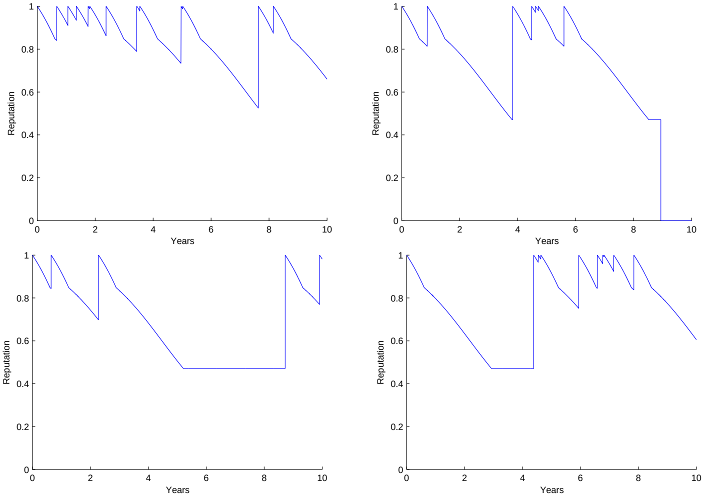

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Conclusion

Model

- Firm dynamics in which main asset is firm's reputation.
- Characterize investment and exit dynamics over life-cycle.

Equilibrium characterization

- Incentives depend on reputation and self-esteem.
- Shirk-work-shirk equilibrium.

Benchmarks

- Observed investment: Work-shirk equilibrium.
- Privately known quality: Shirk-work equilibrium.

Appendix

Imperfect Private Information

Private good news signals arrive at rate ν

- Self-esteem jumps to 1 when private/public signal arrive.
- Else, drift is $\dot{z}_t = \lambda(a_t - z_t) - (\mu + \nu)z_t(1 - z_t)$.

Equilibrium

- Model is recursive since time of last public breakthrough.
- Investment incentives equal

$$\Gamma(t) = \int_t^{\tau^*} e^{-\int_t^s \rho(u)du} \left[\mu(V(0,1) - V(s, z_s^*)) + \nu(V(s,1) - V(s, z_s^*))\right] ds$$

where $\rho(u) = r + \lambda + (\mu + \nu)(1 - z_u^*)$.

- Shirk at the end, $t \to \tau^*$.

Brownian Motion

Market observes signal Y_t

- Y_t evolves according to $dY = \mu_B \theta_t dt + dW$.
- Investment incentives are

$$V_z(x_t, z_t) = \mathbb{E}\left[\int_t^{\tau^*} e^{-\int_t^s \rho_u du + \int_t^s (1-2z_u)\mu_B dW_u} D(x_s, z_s) ds\right]$$

where $\rho_u = r + \lambda + \frac{1}{2}\mu_B^2(1-2z_u)^2$
and $D(x,z) = \mu_B\big(x(1-x)V_x(x,z) + z(1-z)V_z(x,z)\big)$.

Results similar to good news case

- Shirk at end, as $t \to \tau^*$.
- Shirk at start if $\overline{a} \approx 1$.
- Work in the middle if c not too large.