A Reputational Theory of Firm Dynamics

Simon Board        Moritz Meyer-ter-Vehn

UCLA

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Motivation

Models of firm dynamics

- Wish to generate dispersion in productivity, profitability etc.
- Some invest in assets and grow; others disinvest and shrink.

A firm’s reputation is one of its most important assets

- Kotler: “In marketing, brand reputation is everything”.
- Interbrand: Apple brand worth $98b; Coca-Cola $79b.
- EisnerAmper: Reputation risk is directors’ primary concern.

Reputation a special asset

- Reputation is market belief about quality.
- Reputation can be volatile even if underlying quality constant.
This Paper

Firm dynamics with reputation

- Firm invests in quality.
- Firm & mkt. learn about quality.
- Firm exits if unsuccessful.

Optimal investment

- Firm shirks near end.
- Incentives are hump-shaped.
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Benchmarks

- Consumers observe investment.
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Benchmarks

- Consumers observe investment.
- Firm privately knows quality.
Literature

Reputation Models
- Kovrijnykh (2007)
- Board and Meyer-ter-Vehn (2013)

Firm Dynamics
- Jovanovic (1982)
- Hopenhayn (1992)

Moral hazard and learning
- Holmstrom (1982)
- Bonatti and Horner (2011, 2013)
- Cisternas (2014)
Model
Model, Part I

Long-lived firm sells to short-lived consumers.

- Continuous time $t \in [0, \infty)$, discount rate $r$.
- Firm invests $A_t \in [0, \bar{a}]$, $\bar{a} < 1$, and exits at time $T$.

Technology

- Quality $\theta_t \in \{L, H\}$ where $L = 0$ and $H = 1$.
- Technology shocks arrive with Poisson rate $\lambda$.
- Quality given by $\Pr(\theta_s = H) = A_s$ at last shock $s \leq t$.

Information

- Breakthroughs arrive with Poisson rate $\mu$ iff $\theta_t = H$.
- Consumers observe history of breakthroughs, $h^t$.
- Firm additionally recalls past actions.
Model, Part II

Reputation and Self-Esteem

- Consumers’ beliefs over strategy of firm, \( F = F(\{\tilde{A}_t\}, \tilde{T}) \).
- Self-esteem \( Z_t = \mathbb{E}\{A_t\}[\theta_t|h^t] \).
- Reputation, \( X_t = \mathbb{E}^F[\theta_t|h^t, t < \tilde{T}] \).

Payoffs

- Consumers obtain flow utility \( X_t \).
- Firm value

\[
V = \max_{\{A_t\},T} \mathbb{E}\{A_t\} \left[ \int_0^T e^{-rt}(X_t - cA_t - k)dt \right].
\]
Recursive Strategies

Game resets at breakthrough, $X=Z=1$.

- $\{A_t\}, T$ is recursive if only depend on time since breakthrough.
- $F$ is recursive if only puts weight on recursive strategies.
- If $F$ recursive, then optimal strategies are recursive.
- Notation: $\{a_t\}, \tau, \{x_t\}, \{z_t\}, V(t, z_t)$ etc.

Self-esteem

- Jumps to $z_t = 1$ at breakthrough.
- Else, drift is $\dot{z}_t = \lambda (a_t - z_t) dt - \mu z_t (1 - z_t) dt =: g(a_t, z_t)$.

Assumption: A failing firm eventually exits

- Negative drift at top, $z^\dagger := \lambda/\mu < 1$.
- Exit before $z^\dagger$ reached, $z^\dagger - k + \mu z^\dagger (1 - k)/r < 0$. 
Optimal Investment & Exit
Optimal Strategies Exist

**Lemma 1.** Given \( \{x_t\} \), an optimal \( \{a_t^*\} \), \( \tau^* \) exists with \( \tau^* \leq \bar{\tau} \).

**Idea**
- Drift \( g(a, z) \) is strictly negative for \( z \in [z^\dagger, 1] \).
- \( V(t, z^\dagger) < 0 \) for any strategy, so \( \tau^* \) bounded.
- Action space compact in weak topology by Alaoglu’s theorem.
- Payoffs are continuous in \( \{z_t\} \), and hence in \( \{a_t\}, \tau \).

**Notation**
- Optimal strategies \( \{a_t^*\}, \tau^* \).
- Optimal self-esteem \( \{z_t^*\} \).
Optimal Investment

Lemma 2. Given \( \{x_t\} \), optimal investment \( \{a_t^*\} \) satisfies

\[
a_t^* = \begin{cases} 
0 & \text{if } \lambda V_z(t, z_t^*) < c, \\
\overline{a} & \text{if } \lambda V_z(t, z_t^*) > c.
\end{cases}
\]

Investment pays off by

- Raising self-esteem immediately.
- Raising reputation via breakthroughs.

Dynamic complementarity

- \( V(t, z) \) is convex; strictly so if \( \{x_t\} \) continuous.
- Raising \( a_t \) raises \( z_{t+dt} \) and incentives \( V_z(t, z_{t+dt}) \).
- Optimal strategies ordered: \( z_t^* > z_t^{**} \Rightarrow z_{t'}^* > z_{t'}^{**} \) for \( t' > t \).
Marginal Value of Self-Esteem

Lemma 3. Given \( \{x_t\} \), if \( V_z(t, z^*_t) \) exists it equals

\[
\Gamma(t) = \int_t^{T^*} e^{-\int_t^s r + \lambda + \mu(1-z^*_u)du} \mu(V(0, 1) - V(s, z^*_s))ds.
\]

Value of self-esteem over \( dt \)

- \( dz \) raises breakthrough by \( \mu dz dt \).
- Value of breakthrough is \( V(0, 1) - V(s, z^*_t) \).

Discounting the dividends

- Payoffs discounted at rate \( r \).
- \( dz \) disappears with prob. \( \mu z^*_t dt \), if breakthrough arrives.
- \( dz \) changes by \( g_z(a, z_t) = -(\lambda + \mu(1 - 2z^*_t)) \).
Derivation of Investment Incentives

▶ Give firm cash value of any breakthrough,

\[
V(t, z^*_t) = \int_t^{\tau^*} e^{-r(s-t)}(x_s - ca^*_s - k + \mu z^*_s(V(0, 1) - V(s, z^*_s)))ds.
\]

▶ Apply the envelope theorem,

\[
V_z(t, z^*_t) = \int_t^{\tau^*} e^{-r(s-t)} \frac{\partial z^*_s}{\partial z^*_t} \left( \mu(V(0, 1) - V(s, z^*_s)) - \mu z^*_s V_z(s, z^*_s) \right) ds.
\]

▶ The partial derivative equals,

\[
\partial z^*_s / \partial z^*_t = \exp \left( - \int_t^s (\lambda + \mu(1 - 2z^*_u))du \right).
\]

▶ Placing \( \mu z^*_s V_z(s, z^*_s) \) into the exponent,

\[
V_z(t, z^*_t) = \Gamma(t) := \int_t^{\tau^*} e^{-\int_t^s (r + \lambda + \mu(1-z^*_u))du} \mu(V(0, 1) - V(s, z^*_s))ds.
\]
Property 1: Shirk at the End

Theorem 1. Given \( \{x_t\} \), any optimal strategy \( \{a_t^*\}, \tau^* \), exhibits shirking \( a_t^* = 0 \) on \( [\tau^* - \epsilon, \tau^*] \).

Idea

- At \( t \to \tau^* \), so \( \Gamma(t) \to 0 \).
- Need technology shock and breakthrough before \( \tau^* \) for investment to pay off.
- Shirking accelerates the demise of the firm.
Property 2: Incentives are Single-Peaked

**Theorem 2.** If \( \{x_t\} \) decreases, investment incentives \( \Gamma(t) \) are single-peaked with \( \Gamma(0) > 0, \dot{\Gamma}(0) > 0 \) and \( \Gamma(\tau^*) = 0 \).

**Proof**

- Differentiating \( \Gamma(t) \) with \( \rho(t) := r + \lambda + \mu(1 - z_t^*) \),
  \[ \dot{\Gamma}(t) = \rho(t)\Gamma(t) - \mu(V(0, 1) - V(t, z_t^*)). \]
- Differentiating again,
  \[ \ddot{\Gamma}(t) = \rho(t)\dot{\Gamma}(t) + \dot{\rho}(t)\Gamma(t) + \mu\dot{z}_t^*\Gamma(t) + \mu V_t(t, z_t^*) \]
  \[ = \rho(t)\dot{\Gamma}(t) + \mu V_t(t, z_t^*). \]
- If \( \{x_t\} \) is decreasing \( V_t < 0 \), and \( \dot{\Gamma}(t) = 0 \) implies \( \ddot{\Gamma}(t) < 0 \).

**Countervailing forces:** As \( t \) rises,
- Dividends \( V(0, 1) - V(t, z_t^*) \) grow, and incentives increase.
- Get close to exit and incentives decrease.
**Property 3: Exit Condition**

**Theorem 3.** *If* \( \{x_t\} \) *is continuous, then* \( \tau^* \) *satisfies*

\[
V(\tau^*, z_{\tau^*}) = \left( x_{\tau^*} - k \right) + \mu z_{\tau^*} V(0, 1) = 0.
\]

*flow profit* \quad *option value*
Equilibrium
Definition

Equilibrium beliefs

- Reputation \( x_t = E^F[\theta^t|h^t = \emptyset, t \leq \tilde{\tau}] \) given by Bayes’ rule.
- Under point beliefs, \( \dot{x} = \lambda(\tilde{a} - x)dt - \mu x(1 - x)dt \).
- Can hold any beliefs after \( \tau(F) := \min\{t : F(\tilde{\tau} \leq t) = 1\} \).

Recursive equilibrium

- Given \( \{x_t\} \), any strategy \( (\{a_t\}, \tau) \in \text{supp}(F) \) is optimal.
- Reputation \( \{x_t\} \) derived from \( F \) via Bayes’ rule for \( t < \tau(F) \).
Existence

**Theorem 4.** *An equilibrium exists.*

**Idea**

- Strategy space compact in weak topology.
- Bayes rule, best response correspondences u.h.c.
- Apply Kakutani-Fan-Glicksberg Theorem.
Pure Strategy Equilibria

In a pure strategy equilibrium, $x_t = z_t^*$. 

- $\{x_t\}$ decreases and incentives are single-peaked (Theorem 2).

Changes in costs

- High costs: Full shirk equilibrium.
- Intermediate costs: Shirk-work-shirk equilibrium.
- Low costs: Work-shirk equilibrium.
Simulation Parameters

Restaurant accounting

- Revenues: \( x \) million.
- Capital cost: $500k.
- Investment cost: $125k.
- Interest rate: 20%.

Arrival rates

- Breakthroughs arrive once a year.
- Technology shocks arrive every 5 years.
Value Function and Firm Distribution

Figure: Capital cost $k = 0.5$, investment cost $c = 0.1$, interest rate $r = 0.2$, max. effort $\bar{a} = 0.99$, breakthroughs $\mu = 1$, technology shocks $\lambda = 0.1$. 
Investment Incentives

Figure: Capital cost $k = 0.5$, investment cost $c = 0.1$, interest rate $r = 0.2$, max. effort $\bar{a} = 0.99$, breakthroughs $\mu = 1$, technology shocks $\lambda = 0.1$. 
Typical Life-cycles

Figure: Capital cost $k = 0.5$, investment cost $c = 0.1$, interest rate $r = 0.2$, max. effort $\bar{a} = 0.99$, breakthroughs $\mu = 1$, technology shocks $\lambda = 0.1$. 
Mixed Strategy Equilibria

Exit

- Firm shirks near exit point (Theorem 1).
- Firms with less self-esteem exits gradually.
- Firm with most self-esteem exits suddenly.

Reputational dynamics

- $\{x_t\}$ decreases until firms start to exit.
- $\{x_t\}$ increases when firms gradually exit.
Illustration of Mixed Strategy Equilibrium

\[ z(t) \]

Work Region

Shirk Region

\[ z_t^- \]

\[ z_t^+ \]

\[ x_t \]

\[ 0 \quad \tau_0 \quad \tau \quad \tau(\overline{F}) \]
Competitive Equilibrium

Agent’s preferences

- Firm $i$ has expected output $x_{t,i}$
- Total output of experience good is $X_t = \int_i x_{t,i} \, di$.
- Consumers have utility $U(X_t) + N_t$.

Equilibrium

- Competitive equilibrium yields price $P_t = U'(X_t)$.
- Stationary equilibrium: $P_t$ independent of $t$.
- Firm $i$’s revenue is $x_{t,i}P$ and value is $V_i(t, z_t; P)$.

Entry

- Firm pays $\xi$ to enter and is high quality with probability $\bar{x}$.
- Given a pure equilibrium, let $z_i = \bar{x}$.
- Free entry determines price level: $V(\bar{t}, \bar{z}; P) = \xi$. 
Model Variation:
Observable Investment
Observable Investment

Investment $a_t$ is publicly observed

- Reputation and self-esteem coincide, $x_t = z_t$.

Optimal strategies

- Optimal investment

$$\hat{a}_t = \begin{cases} 
0 & \text{if } \lambda \hat{V}_z(\hat{z}_t) < c \\
1 & \text{if } \lambda \hat{V}_z(\hat{z}_t) > c 
\end{cases}$$

- Investment incentives

$$\hat{\Gamma}(t) = \int_t^\tau e^{-\int_t^s r + \lambda + \mu (1 - \hat{z}_u) du} \left[ 1 + \mu (\hat{V}(1) - \hat{V}(\hat{z}_s)) \right] ds.$$ 

- Optimal exit

$$\hat{V}(\hat{z}_t) = \hat{z}_\tau - k + \mu \hat{z}_\tau \hat{V}(1) = 0.$$
Characterizing Equilibrium

**Theorem 5.** If investment is observed, investment incentives $\hat{\Gamma}(t)$ are decreasing with $\hat{\Gamma}(0) > 0$ and $\hat{\Gamma}(\hat{\tau}) = 0$.

**Proof**

- Value $\hat{V}(\cdot)$ is strictly convex.
- Self-esteem $\hat{z}_t$ strictly decreases over time.
- Hence $\hat{V}_z(z_t)$ strictly decreases with $\hat{V}_z(z_{\hat{\tau}}) = 0$.

**Idea:** Investment is beneficial if

- There is a technology shock.
- There is a resulting breakthrough prior to exit time.
Value Function and Firm Distribution

Figure: Capital cost $k = 0.5$, investment cost $c = 0.1$, interest rate $r = 0.2$, max. effort $\bar{a} = 0.99$, breakthroughs $\mu = 1$, technology shocks $\lambda = 0.1$. 
Impact of Moral Hazard

**Theorem 6.** *If investment is observed, the firm works longer than in any baseline equilibrium.*

Idea

- When observed firm increases investment, belief also rises.
- Such favorable beliefs are good for the firm.
- Optimal investment choice higher for observed firm.

With observable investment,

- No shirk region at the top.
- Work until lower reputation.
- Value higher, so exit later.
Model Variation:
Privately Known Quality
Privately Known Quality

Firm knows $\theta_t$

- Investment $a_t$ still unknown, so there is moral hazard.

Recursive strategies

- Firm knows quality and time since breakthrough.
- Chooses investment $a_t$ and exit time $\tau$.
- Value function $V(t, \theta_t)$.

Optimal investment $a_t(\theta)$

- Independent of quality $a_t(\theta) = a_t$ and given by:

\[ a_t = \begin{cases} 
1 & \text{if } \lambda \Delta(t) > c \\
0 & \text{if } \lambda \Delta(t) < c 
\end{cases} \]

where $\Delta(t) := V(t, 1) - V(t, 0)$ is value of quality.
- Tech. shock has probability $\lambda dt$, yielding benefit $\Delta(t)$ of work.
Exit Choice

Assuming \( \{x_t\} \) continuously decreases

- Low quality firm exits gradually when \( t > \tau^L \).
- In equilibrium, high quality firm never exits.

Assuming firm works at end,

- Exit condition becomes

\[
V(t, 0) = (x_t - c - k) + \lambda V(t, 1) = 0.
\]

flow profit + option value
**Theorem 7.** If quality is privately observed, investment incentives $\Delta(t)$ are increasing with $\Delta(0) > 0$.

**Proof**

- The value of quality is present value of dividends:

$$
\Delta(t) = \int_t^\infty e^{-(r+\lambda)(s-t)} \mu[V(0,1) - V(s,1)] ds.
$$

- Investment incentives $\lambda \Delta(t)$ increase in $t$. 
Impact of Private Information

Known quality

- Work pays off if tech. shock (prob. $\lambda dt$).
- Fight to bitter end.
- Low firm gradually exits; high never does.

Unknown quality

- Work pays off if tech. shock & breakthrough ($\lambda dt \times \mu dt$).
- Coast into liquidation.
- Firm exits after $\tau$ periods without breakthrough.
Value Function and Firm Distribution

**Figure:** Capital cost $k = 0.5$, investment cost $c = 0.1$, interest rate $r = 0.2$, max. effort $\bar{a} = 0.99$, breakthroughs $\mu = 1$, technology shocks $\lambda = 0.1$. 
**Typical Life-cycles**

![Graphs showing typical life-cycles with various parameters.](image)

**Figure:** Capital cost $k = 0.5$, investment cost $c = 0.1$, interest rate $r = 0.2$, max. effort $\bar{a} = 0.99$, breakthroughs $\mu = 1$, technology shocks $\lambda = 0.1$. 
Conclusion

Model

- Firm dynamics in which main asset is firm’s reputation.
- Characterize investment and exit dynamics over life-cycle.

Equilibrium characterization

- Incentives depend on reputation and self-esteem.
- Shirk-work-shirk equilibrium.

Benchmarks

- Observed investment: Work-shirk equilibrium.
- Privately known quality: Shirk-work equilibrium.
APPENDIX
Imperfect Private Information

Private good news signals arrive at rate $\nu$

- Self-esteem jumps to 1 when private/public signal arrive.
- Else, drift is $\dot{z}_t = \lambda(a_t - z_t) - (\mu + \nu)z_t(1 - z_t)$.

Equilibrium

- Model is recursive since time of last public breakthrough.
- Investment incentives equal

$$\Gamma(t) = \int_t^{\tau^*} e^{-\int_t^s \rho(u) du} [\mu(V(0, 1) - V(s, z^*_s)) + \nu(V(s, 1) - V(s, z^*_s))] ds$$

where $\rho(u) = r + \lambda + (\mu + \nu)(1 - z^*_u)$.

- Shirk at the end, $t \to \tau^*$. 
Brownian Motion

Market observes signal $Y_t$

- $Y_t$ evolves according to $dY = \mu_B \theta_t dt + dW$.
- Investment incentives are

$$V_z(x_t, z_t) = \mathbb{E} \left[ \int_t^{\tau^*} e^{-\int_t^s \rho_u du + \int_t^s (1-2z_u)\mu_B} dW_u D(x_s, z_s) ds \right]$$

where $\rho_u = r + \lambda + \frac{1}{2} \mu_B^2 (1 - 2z_u)^2$

and $D(x, z) = \mu_B (x(1-x)V_x(x, z) + z(1-z)V_z(x, z))$.

Results similar to good news case

- Shirk at end, as $t \to \tau^*$.
- Shirk at start if $\bar{a} \approx 1$.
- Work in the middle if $c$ not too large.