Competitive Information Disclosure and Consumer Search

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Motivation

Buyers search for information

- Proliferation of products, and sellers selling same product.
- Consumers search across sellers to learn about products.
- Sellers manage buyers' learning by disclosing information.

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Model

- Sellers sell an identical set of products.
- Sellers choose disclosure policies.
- Buyers search sequentially and randomly.

Question

- Does competition force sellers to reveal all their information?

Results

Buyers' beliefs are public

- The monopoly disclosure policy is an equilibrium.
- With "dispersed products", monopoly is the only equilibrium.
- Idea: sellers can discriminate between new and old buyers.

Buyers' beliefs are private

- Full information is a limit equilibrium.
- With "suff. dispersed products", full info is only limit eqm.
- Idea: old buyers can mimic new buyers.

Implications

- Tracking software helps sellers implicitly collude.
- Letting customers delete cookies does not solve problem.

Literature

Information disclosure

- Kamenica and Gentzkow (2011)
- Rayo and Segal (2010)

Disclosure and competition

- Gentzkow and Kamenica (2012)
- Li and Norman (2014)
- Hoffmann, Inderst and Ottaviani (2014)

Search

- Diamond (1971)

Public Beliefs

Model

Basics

- One buyer; Infinite sellers.
- Finite set of states S.
- Buyers starts with prior $p \in \Delta S$.
- Sellers sell identical finite set of products $U \subset \mathbb{R}^S$. Let $0 \in U$.

Actions

- Buyer picks a seller at random.
- Receives a signal from seller, and forms posterior $q \in \Delta S$.
- Chooses (1) Buy product, (2) Exit, or (3) Search at cost c.

Disclosure policy

- Seller observes prior p.
- Chooses *disclosure policy* K s.t. $\int_{\Delta S} K(p, dq) = p$.

Equilibrium (K, Q)

Buyer's strategy

- Buyer's optimal choice $u^*(q) = \arg\max_{u \in U} q \cdot u$.
- Buyer's continuation value

$$V_K(q) := -c + \int_{\Delta S} \max\{r \cdot u^*(r), V_K(r)\} K(q, dr)$$

- Buyer purchases if posterior lies in *acceptance set*

$$Q := \{q \in \Delta S | q \cdot u^*(q) \geq V_K(q)\}$$

Seller's strategy

- Profits $\tilde{\pi}(u)$ from u, with $\pi(q) = \max_{u \in u^*(q)} \tilde{\pi}(u)$.
- Seller optimizes:

$$\int_Q \pi(q) K(p, dq) \geq \int_Q \pi(q) L(p, dq) \qquad \forall L$$

- Tie-break: If "no information" optimal, then choose this.

Example 1: Single Product

Buyer considers purchasing a 3D TV

- Two states $S = \{L, H\}$ with $q = \Pr(H)$.
- Utility of TV $u = (u(L), u(H)) = (-1, 1)$.
- Buyer purchases u_1 if $q \in [\frac{1}{2}, 1]$.
- Profits $\tilde{\pi}(u_1) = 1$.

Monopoly disclosure policy

- Perfect bad news policy $p \to \{0, \frac{1}{2}\}$. More formally,

$$K(p) = \begin{cases} (1-2p)\delta_{\{0\}} + 2p\delta_{\{\frac{1}{2}\}} & \text{if } p \in \left[0, \frac{1}{2}\right) \\ p & \text{if } p \in \left[\frac{1}{2}, 1\right] \end{cases}$$

Equilibrium

- Monopoly policy is an equilibrium.
- Monopoly policy is unique equilibrium.

Optimal Disclosure Policies

Firm's optimal policy

- Optimal profits coincide with convex hull of $\pi(q)\mathbf{1}_Q$.
- Absorbing beliefs $A_K = \{p \in \Delta S | K(p) = \delta_p\}$.

Lemma 1.
If K is optimal given Q then supp$(K) =$ cl(A_K).

Proof

- Suppose $p \to \{q_1, q_2\}$ and $q_1 \to \{q_{11}, q_{12}\}$.
- Then composition, $p \to \{q_{11}, q_{12}, q_2\}$ raises profits.

Implications

- If buyer continued searching, they would get no information.
- Hence $V_K(q) = -c + \int_{\Delta S} r \cdot u^*(q) K(q, dr)$.

Equilibrium is Monotone

- Fixing K, let V_K^c and Q_c be defined as above for each c.

Lemma 2.
If $(K, Q_{\bar{c}})$ is an equilibrium, then (K, Q_c) is an equilibrium $\forall c \leq \bar{c}$.

When the cost falls from $\bar{c}$ to c,

- Profit from K is constant, since supp$(K) \subset Q_c$.
- Scope for deviations smaller since $Q_c \subset Q_{\bar{c}}$.

Monopoly is an Equilibrium

- Apply the above result for $\bar{c} = \infty$.

Theorem 1.
The monopoly policy is an equilibrium.

If all sellers choose the monopoly policy,

- A buyer purchases at the first seller.
- Sellers don't defect since making monopoly profits.

Example 2: Horizontal Differentiation

Buyer considers purchasing either 3D or 4k TV

- Two states $S = \{L, H\}$ with $q = \Pr(H)$.
- Utilities: 3D TV $u_1 = (-1, \frac{1}{2})$ and 4k TV $u_2 = (\frac{1}{2}, -1)$.
- Buyer's purchases u_2 if $q \in [0, \frac{1}{3}]$ and u_1 if $q \in [\frac{2}{3}, 1]$.
- Profits: $\tilde{\pi}(u_1) = 1$ and $\tilde{\pi}(u_2) = \frac{1}{2}$.

Monopoly disclosure policy

- Perfect bad news policy $p \to \{0, \frac{2}{3}\}$. More formally,

$$K(p) = \begin{cases} (1 - \frac{3}{2}p)\delta_{\{0\}} + \frac{3}{2}p\delta_{\{\frac{2}{3}\}} & \text{if } p \in \left[0, \frac{2}{3}\right) \\ p & \text{if } p \in \left[\frac{2}{3}, 1\right] \end{cases}$$

Equilibrium

- Monopoly policy is unique equilibrium.

Example 3: Vertical Differentiation

Buyer considers purchasing good/cheap 3D TVs

- Two states $S = \{L, H\}$ with $q = \Pr(H)$.
- Utilities: Good TV $u_1 = (-1, 1)$ and cheap TV $u_2 = (-\frac{1}{3}, \frac{2}{3})$.
- Buyer's chooses u_2 if $q \in [\frac{1}{3}, \frac{2}{3})$ and u_1 if $q \in [\frac{2}{3}, 1]$
- Profits: $\tilde{\pi}(u_1) = \frac{3}{2}$ and $\tilde{\pi}(u_2) = 1$.

Monopoly policy, and an equilibrium

$$K(p) = \begin{cases} (1-3p)\delta_{\{0\}} + 3p\delta_{\{\frac{1}{3}\}} & \text{if } p \in \left[0, \frac{1}{3}\right) \\ (2-3p)\delta_{\{\frac{1}{3}\}} + (3p-1)\delta_{\{\frac{2}{3}\}} & \text{if } p \in \left[\frac{1}{3}, \frac{2}{3}\right) \\ p & \text{if } p \in \left[\frac{2}{3}, 1\right] \end{cases}$$

Another equilibrium

$$K(p) = \begin{cases} \frac{2-3p}{2}\delta_{\{0\}} + \frac{3p}{2}\delta_{\{\frac{2}{3}\}} & \text{if } p \in \left[0, \frac{2}{3}\right) \\ p & \text{if } p \in \left[\frac{2}{3}, 1\right] \end{cases}$$

Uniqueness of Monopoly Policy

- Products are *dispersed* if $q \cdot u \geq 0 \Rightarrow q \cdot u' \leq 0$, $\forall u, u' \in U$.

Theorem 2.
If products are dispersed and induce different profits then any equilibrium policy is a monopoly policy.

Idea

- Monopoly policy has lexicographic perfect bad news property.
- Prove result by induction.

Local Optimality

- K is *locally optimal* if there exists $\epsilon > 0$ s.t.

$$\int_{\Delta S} \pi(q) K(p, dq) \geq \int_{\Delta S} \pi(q) L(p, dq)$$

$\forall L$ with $\text{supp}(L) \subset B_\epsilon(\text{supp}(K))$.

Proposition 1.

Assume K is continuous.
(a) Any equilibrium policy is locally optimal.
(b) If K is locally optimal, $\exists c_\epsilon$ s.t. K is an eqm $\forall c \leq c_\epsilon$.

Idea

(a) $q \in \text{supp}(K)$ gets no info, so $q \in Q$.

If K continuous, $q \in B_\epsilon(\text{supp}(K))$ gets little info, so $q \in Q$.

(b) As $c \to 0$ so $Q_c \to \text{supp}(K)$.

Fixing ϵ, when $c \leq c_\epsilon$ then $Q_c \subset B_\epsilon(\text{supp}(K))$.

Non-Markovian Equilibria

So far considered Markovian equilibria

- Seller's policy depends on buyer's belief p.
- What if seller can also condition on when buyer visits?

Example 1 (cont.)

- Monopoly strategy is unique rationalizable strategy.
- No matter what seller $n+1$ does,

$$Q_n \supset \overline{Q}_n := \left\{q \in [0,1] : \max\{2q-1, 0\} \geq V_{\overline{K}}(q)\right\} = [0,c] \cup [1-c, 1]$$

- Seller n will use perfect bad news signal.
- Given most information is $p \to \{0, 1-c\}$, we have

$$Q_{n-1} \supset \overline{Q}_{n-1} = \left[0, \frac{(1-c)}{2(1-c)-1}c\right] \cup \left[(1-c)^2, 1\right]$$

- If $n \geq -\frac{\log(2)}{\log(1-c)}$, $Q_1 \supset [\frac{1}{2}, 1]$ and seller 1 chooses monopoly.

Private Beliefs

Model

Seller's strategy

- Seller chooses signal that is independent of other's signals.
- Seller's *signal policy* is dist. of posteriors $\mu(q)$ for buyer p.
- If start with prior r, Bayes' rule implies posterior is

$$[\phi_r(q)](s) := q(s)\frac{r(s)}{p(s)} \Big/ \sum_{s'} q(s')\frac{r(s')}{p(s')}$$

Equilibrium

- Buyer purchases if $q \in Q = \{q \in \Delta S | q \cdot u^*(q) \geq V(\mu, q)\}$.
- Seller chooses optimal policy: $\int_Q \pi(q)\mu(dq) \geq \int_Q \pi(q)\nu(dq)$.

Example 1 (cont.)

If $p \geq \frac{1}{2}$, no information is an equilibrium

- If other sellers give "no info", then best response is "no info".

More informative equilibria

- Firm uses perfect bad news signal $p \to \{0, b\}$, where $b \geq \frac{1}{2}$.
- At b, prefers to purchase now if

$$2b - 1 \geq \frac{b}{\phi_b(b)}(2\phi_b(b) - 1) - c$$

- This yields $b^2 - b[(1+p) - c(1-p)] + p \geq 0$.

As search costs decline, $c \to 0$

- If $p \geq \frac{1}{2}$, "no info" and "full info" are limit equilibria.
- If $p < \frac{1}{2}$, "full info" is unique limit equilibrium!

Equilibrium Existence

- U is *robust* if $\exists s(u)$ where u chosen at $\delta_{s(u)}$ $\forall u \in U$.

Proposition 2.

If U is robust then a symmetric equilibrium exists.

Idea

- Look for fixed point of $\mu \to V(\mu, q) \to Q(\mu) \twoheadrightarrow \varphi(\mu)$.
- Problem: If good stuck in middle $Q_c(\mu) \twoheadrightarrow \varphi(\mu)$ is not uhc.
- Robustness: Can construct nearby policy with little lost profit.
- Apply Kakutani-Fan-Glicksberg on space of signal policies.

Full Information is a Limit Equilibria

- Let $\bar{\mu}$ be the fully informative policy.

Theorem 3.
If preferences are strict at each vertex δ_s, then there is a sequence of equilibria s.t. $V(\mu, p) \to V(\bar{\mu}, p)$ as $c \to 0$.

Idea

- As $c \to 0$, a buyer can visit $1/\sqrt{c}$ sellers at cost $\sqrt{c} \to 0$.
- If other sellers provide full information, seller 1 must match.

Proof

- Consider policies μ with support in $\cup_s B_\epsilon(\delta_s)$, denoted M_ϵ.
- When $c \leq c_\epsilon$, $\varphi(\mu) \subset M_\epsilon$ for all $\mu \in M_\epsilon$.
- Apply above existence proof on M_ϵ and let $\epsilon \to 0$.

Possible Limit Equilibria

Information partition

- Partition states $Z = \{z_1, \ldots, z_{|Z|}\}$.
- Buyer learns which partition occurs.
- If $s, s' \in z$, learns nothing, $q(s')/q(s) = p(s')/p(s)$.

Boundary equilibrium

- Yields $|Z| - 1$ dimensional simplex Δ_Z with vertices p_z.
- Orthogonal component $\Delta_z^- := \{q \in \Delta S : q(s) = 0 \text{ for } s \notin z\}$.
- Z is a *boundary equilibrium* if seller doesn't reveal info at p_z.

Proposition 3.

(a) If preferences are strict at each p_z, Z is a boundary eqm iff p_z leads buyer to choose most profitable item in Δ_z^-.
(b) Any limit equilibrium is a boundary eqm.

Uniqueness of Full Information Policy

- Products are *sufficiently dispersed* if they are dispersed and the consumer doesn't purchase at any p_z with $|z| > 1$.

Theorem 4.
If products are sufficiently dispersed then any limit equilibrium is a full information policy.

Idea

- Sellers provide info on each dimension to induce purchase.
- As $c \to 0$ buyer learns everything.

Extensions

Choice of "Do Not Track"

Suppose buyers can become anonymous at cost k

- If buyer is tracked, sellers can see belief.
- If buyer is anonymous, looks same as new buyer.

Example 1 (cont.)

		Buyers $-i$	
		Anonymous	Tracked
Buyer i	Anonymous	$0.3 - k$	$0.15 - k$
	Tracked	0.3	0

Implications

- If $-i$ become anonymous $\Rightarrow$ positive externality on buyer i.
- There is no equilibrium where everyone is anonymous.
- If $k \geq 0.15$, there is equilibrium where everyone is tracked.

Intermediate Observability

Suppose seller can observe

- Which sellers buyer previously visited.
- Disclosure policies of these sellers.
- Then chooses independent disclosure policy.

Monopoly policy is a sequential equilibrium

- Seller 1 provides monopoly information.
- Sellers $n \geq 2$ believe monopoly posterior and provide no info.

Example 1: Monopoly policy is only sequential equilibrium.

- Each seller n faces set of form $[0, \alpha_n] \cup [\beta_n, 1]$.
- Each seller uses perfect bad new signal.
- Seller conditions on no bad news, so as if knows buyers belief.

Conclusion

Do sellers release all their information?

- Sequential search model.
- Seller chooses any disclosure policies.

Main results

- If beliefs public, monopoly policy is always equilibrium.
- If beliefs private, full information is limit equilibrium.
- Being anonymous exerts positive externality on other buyers.

Going forward

- Heterogeneous priors.
- Menus of contracts.
- Choice of prices and information.