

# Relational Contracts and the Value of Loyalty

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# Motivation

## Holdup problem is pervasive

- ▶ Developing economies (McMillan and Woodruff, 99)
- ▶ Developed countries (Macaulay, 67)

## Holdup explains forms of organisations

- ▶ Organisation of communities (Grief, 93)
- ▶ Make vs Buy decisions (Williamson, 85)

## How does Holdup affect supply relationships?

- ▶ Holdup problem mitigated by ongoing relationships.
- ▶ Maintaining relationships can reduce the scope of trade.

# Toyota vs. GM

## General Motors in 1980s

- ▶ Competitive bidding each year.
- ▶ Use cheapest supplier.
- ▶ Outsource 30% of production.
- ▶ Check quality of part before installing.

## Toyota in 1980s

- ▶ Automatically renew contracts for life of vehicle.
- ▶ Preferred supplier policy for new models.
- ▶ Outsource 70% of production.
- ▶ Trust suppliers to verify quality.

# Government Procurement (Kelman, 1990)

## Government Procurement in 1980s

- ▶ Full and open competition (e.g. competitive bidding).
- ▶ Could not use subjective information (e.g. prior performance)

## Public vs. Private

- ▶ Government uses lowest bidder more often.
- ▶ Private firms more loyal to suppliers.
- ▶ Private firms more satisfied with performance.

# Motivation

This paper will . . .

- ▶ Derive the optimal relational contract.
- ▶ Show relational contracts induce loyalty.
- ▶ Characterise distortions induced by ongoing relationships.

We will have predictions about

- ▶ Switching between suppliers.
- ▶ Time path of prices.
- ▶ When trade will take place at all.

# The Theory

One principal and  $N$  agents.

- ▶ Each period, principal invests in one agent.
- ▶ Investment costs vary across agents and over time.
- ▶ Agent can then hold up principal.

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- ▶ Investment costs vary across agents and over time.
- ▶ Agent can then hold up principal.

Agents can garner rents through threat of holdup.

- ▶ Rents same if trade once or one hundred times.
- ▶ Rents acts like fixed cost of new relationship.

Principal divides agents into 'insiders' and 'outsiders'.

- ▶ Trade with insiders efficiently.
- ▶ Trade is biased against outsiders.
- ▶ This is self-enforcing if parties are patient enough.

# Literature

- ▶ Calzolari and Spagnolo (2006).
- ▶ Relational contracts with random hiring: Shapiro and Stiglitz (1984) and Greif (1993, 2003).
- ▶ Relational contracts with contractible transfers: MacLeod and Malcomson (1989), Levin (2002, 2003).
- ▶ Community enforcement: Kandori (1992), Ghosh and Ray (1996), Sobel (2006).



# Outline

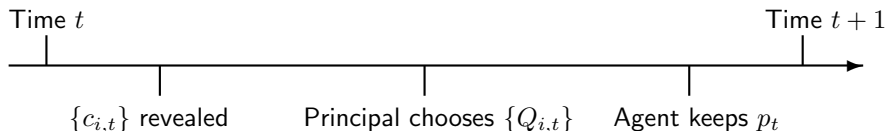
- 1 Introduction
- 2 Model
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- 4 Full Problem
- 5 Private Cost Information
- 6 On Transfers
- 7 Conclusion

## Model: Stage Game

One principal and  $N$  agents. Time  $t \in \{1, 2, \dots\}$ .

- ❶ Costs  $\{c_{i,t}\}$  publicly revealed.
- ❷ Principal chooses  $Q_{i,t} \in \{0, 1\}$  s.t.  $\sum_i Q_{i,t} \leq 1$ .  
Winning agent produces and sells product worth  $v$ .
- ❸ Agent keeps  $p_t \in [0, v]$ , and gives back  $v - p_t$  to principal.

Investment  $Q_{i,t}$  and prices  $p_t$  are noncontractible.



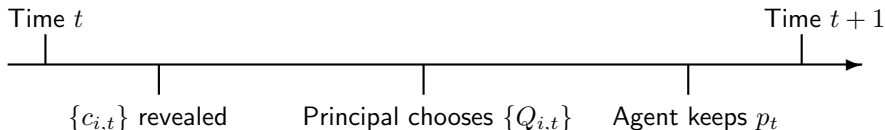
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Investment  $Q_{i,t}$  and prices  $p_t$  are noncontractible.

**Holdup Problem:** No investment in unique stage game equilibrium.



# Model: Repeated Game

Relationships bilateral.

- ▶ Agent  $i$  observes costs  $\{c_{i,t}\}$  and  $Q_{i,t}$ .

Relational contract  $\langle Q_{i,t}, p_t \rangle$  specifies

- ▶ Investments:  $Q_{i,t} : h_i^{t-1} \times [\underline{c}, \bar{c}]^N \rightarrow \{0, 1\}$ .
- ▶ Prices:  $p_t : h_i^{t-1} \times [\underline{c}, \bar{c}]^N \rightarrow [0, v]$ .

Equilibrium

- ▶ Contract is **agent-self-enforcing (ASE)** if agents' strategies form SPNE, taking principal's investment strategy as given.
- ▶ Contract is **self-enforcing (SE)** if both agents' and principal's strategies form SPNE.

# One-Sided Commitment

## Assumption

- ▶ Principal commits to (contingent) strategy,  $Q_{i,t}$ .
- ▶ Allows us to focus on agents' incentives.

Agent  $i$ 's utility at time  $t$  is

$$U_{i,t} := E_t \left[ \sum_{s \geq t} \delta^{t-s} p_s Q_{i,s} \right]$$

## Lemma 1.

*Contract  $\langle Q_{i,t}, p_t \rangle$  is agent-self-enforcing if and only if*

$$(U_{i,t} - v)Q_{i,t} \geq 0 \quad (\forall i)(\forall t) \quad (\text{DEA})$$

# Dynamic Enforcement Constraint

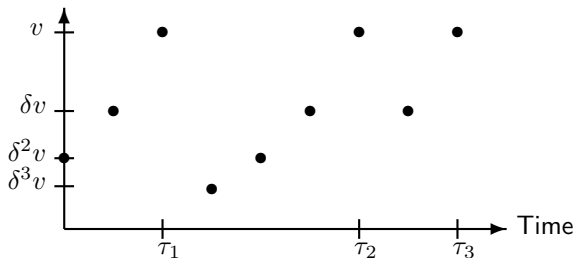
## Lemma 2.

*Contract  $\langle Q_{i,t}, p_t \rangle$  is agent-self-enforcing if and only if*

$$U_{i,t} \geq E_t[v\delta^{\tau_i(t)-t}] \quad (\forall i)(\forall t) \quad (\text{DEA}')$$

*where  $\tau_i(t) := \min\{s \geq t : Q_{i,s} = 1\}$  is time of next trade.*

**Proof.**



# Principal's Problem

The profit at time  $t$  from relationship  $i$  is

$$\Pi_{i,t} := E_t \left[ \sum_{s \geq t} \delta^{t-s} (v - c_{i,t} - p_t) Q_{i,t} \right]$$

Total profit is  $\Pi_t := \sum_i \Pi_{i,t}$ .

**Principal's problem** is to maximise initial profit

$$\begin{aligned} \Pi_0 &:= E_0 \left[ \sum_{s \geq 1} \sum_i \delta^{t-s} (v - c_{i,t} - p_t) Q_{i,t} \right] \\ \text{s.t.} \quad & (U_{i,t} - v) Q_{i,t} \geq 0 \quad (\forall i)(\forall t) \end{aligned} \quad (\text{DEA})$$

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$$\begin{aligned} \Pi_0 &:= E_0 \left[ \sum_{s \geq 1} \sum_i \delta^{t-s} (v - c_{i,t}) Q_{i,t} \right] - \sum_i U_{i,0} \\ \text{s.t.} \quad &U_{i,t} \geq E_t[v \delta^{\tau_i(t)-t}] \quad (\forall i)(\forall t) \quad (\text{DEA}') \end{aligned}$$

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# Optimal ASE Contract

The set of **insiders** at time  $t$  is  $\mathcal{I}_t := \{i : \tau_i(0) < t\}$

## Property 1.

*Trade with insiders is efficient. Suppose  $i \in \mathcal{I}_t$ . Then  $Q_{i,t} = 1$  if  $c_{i,t} < v$  and  $c_{i,t} < c_{j,t}$  ( $\forall j$ ).*

## The Idea

- ▶ First time agent trades, they gets rents  $v$ .
- ▶ This payment can be delayed and used to stop future holdup.
- ▶ Thus rents act like fixed cost of new relationship

# Optimal ASE Contract

## Property 2.

*Trade is biased against outsiders. Suppose  $i \notin \mathcal{I}_t$ . Then  $Q_{i,t} = 0$  if either:*

- ①  $(v - c_{i,t}) < v(1 - \delta)$ ; or
- ②  $(c_{j,t} - c_{i,t}) < v(1 - \delta)$  for  $j \in \mathcal{I}_t$ .

## The Idea

- ▶ Abstain if profit less than rental value of rents.
- ▶ Prefer insider if profit gain less than rental value of rents.
- ▶ May prefer relatively inefficient outsider (if costs not IID).

## Theory of endogenous switching costs

- ▶ Pay to switch to new agent, but not to revert back.

# Prices

## General prices

- ▶ Pick  $U_{i,t}$  such that (DEA') holds ( $\forall t$ ) and binds at  $t = 0$ .
- ▶ Prices can then be backed out of utility:

$$p_{i,t} = U_{i,t} - E_t[\delta^{\tau_i(t+1)} U_{i,\tau_i(t+1)}]$$

## Fastest prices

- ▶ These have property that (DEA') binds ( $\forall t$ ),

$$p_{i,t} = v E_t[1 - \delta^{\tau_i(t+1)}]$$

- ▶ Fastest prices maximise continuation profits,  $\Pi_{i,t}$ .
- ▶ Full problem: Investment rule implementable only if it can be implemented by fastest prices.

## Example: IID Costs

- ▶ Number of insiders,  $n_t$ , follows time-invariant markov chain.
- ▶ Stay inside if best insider cost  $c_{1:n}$  falls below cutoff,  $c_n^*$ .

Insiders, $n_t$	Cutoff, $c_n^*$	Prob( $n_{t+1} = n_t$ )	Value fn., $\Phi(n)$
0	0	0	83.6
1	0.358	0.358	85.3
2	0.398	0.637	87.0
3	0.454	0.837	88.6
4	0.549	0.959	90.2
5	0.834	0.999	91.7
6	1	1	92.9

Table:  $v = 2$ ,  $c_{i,t} \sim [0, 1]$ ,  $N = \infty$  and  $\delta = 0.98$ .

# Predictions

- ① More loyalty in countries with poorer legal systems.
  - Johnson et al (2002)
- ② More loyalty where goods are more specific.
  - Johnson et al (2002)
- ③ Firms who are less loyal receive lower quality.
  - Kelman (1990), GM vs. Toyota.
- ④ Trade harder as end game approaches.
  - Bankruptcy of GM and suppliers.

## Full Problem

**Principal's problem** is to maximise profits

$$\begin{aligned}\Pi_0 &= E_0 \left[ \sum_{s \geq 1} \sum_i \delta^{t-s} (v - c_{i,t} - p_t) Q_{i,t} \right] \\ \text{s.t.} \quad & \Pi_{i,t} Q_{i,t} \geq 0 \quad (\forall i)(\forall t) && \text{(DEP)} \\ & (U_{i,t} - v) Q_{i,t} \geq 0 \quad (\forall i)(\forall t) && \text{(DEA)}\end{aligned}$$

### Question

- Can we implement optimal ASE contract?



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# Time Inconsistency

## Example 1

- ▶ Suppose  $N = 1$ ,  $v = 1$  and  $\delta = 3/4$ .
- ▶ Costs:  $c_t = 1/2$  for  $t \leq 10$ , and  $c_t = 0.99$  for  $t > 10$ .

## What goes wrong:

- ▶ Optimal ASE contract has  $Q_{i,t} = 1$  ( $\forall t$ ).
- ▶ By backwards induction,  $Q_{i,t} = 0$  ( $\forall t$ ).

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## Optimal ASE contract is not time consistent.

- ▶ Rents of insiders are sunk, so agent used efficiently.
- ▶ But payment of rents is delayed to prevent future holdup.
- ▶ Principal may later regret promising to use agent efficiently.

## IID Costs

### Proposition 3.

*Suppose that costs are IID and  $\underline{c} > 0$ . Then  $\exists \hat{\delta}$ , independent of  $N$ , such that the optimal ASE contract satisfies (DEP) when  $\delta > \hat{\delta}$ .*

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- ▶ For fixed  $N$ , result is trivial.
  - $W_{i,t} \rightarrow \infty$  as  $\delta \rightarrow 1$ .
- ▶ Problem: If  $N = \infty$ , then  $\sup_t n_t \rightarrow \infty$  as  $\delta \rightarrow 1$ .
  - Marginal welfare,  $E[c_{1:n} - c_{1:n+1}]$ , falls quickly in  $n$ .
  - Average welfare,  $E[v - c_{1:n}]/n$ , falls more slowly in  $n$ .
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## Example 2

- ▶ Suppose  $c_{i,t} \sim U[0, 1]$  and  $v > 1$ .
- ▶ Then (DEP) satisfied when  $\delta \geq \hat{\delta} = (1 + (v - 1)^3)^{-1}$ .

## Private Cost Information

Suppose  $\{c_{i,t}\}$  are privately known by principal.

### Problem

- ▶ The optimal ASE contract is not incentive compatible.
- ▶ Principal lies about costs because of time inconsistency.

### Example 3

- ▶ Suppose  $N = 1$ ,  $v = 1$ ,  $c \sim U[0, 2]$ , and  $\delta = 9/10$
- ▶ Optimal ASE contract: Outsiders trade if  $c \leq 0.80$ ;  
Insiders trade if  $c \leq 1$ .
- ▶ This contract is self-enforcing and generates prices,  $p_t = 0.18$ .
- ▶ Principal will overstate cost if  $c \in [0.82, 1]$ .
- ▶ Similarly, she may lie to use outsider over insider.



# Maintenance Contracts

A **maintenance contract** has payments:

$$\begin{aligned} p_{i,t} &= (1 - \delta)v && \text{if } i \in \mathcal{I}_t \\ p_{i,t} &= 0 && \text{if } i \notin \mathcal{I}_t \end{aligned}$$

Investments  $Q_{i,t}$  chosen to maximise profits  $\Pi_0$  as in optimal ASE contract.

## More formally

1. Principal observes her costs.
2. Principal makes public cost reports, determining  $\langle Q_{i,t}, p_{i,t} \rangle$ .
3. Principal chooses in whom to invest.
4. Winning agent chooses whether to hold up principal.

# Maintenance Contracts

## Proposition 5.

*The maintenance contract is an optimal ASE contract, and is incentive compatible for principal. It is self-enforcing if*

$$W_{i,t} \geq v \quad \text{for all } i \in \mathcal{I}_t. \quad (\text{DEP}^{\text{MC}})$$

## Benefit of MC

- ▶ Incentive Compatibility

## Cost of MC

- ▶  $(\text{DEP}^{\text{MC}})$  is stricter than  $(\text{DEP})$ .
- ▶ However, under IID costs  $(\text{DEP}^{\text{MC}})$  holds if  $\delta > \hat{\delta}$ .

# Agents' Rents

Agents' obtain rents.

- ▶ Crucial to this paper.
- ▶ But principal may be able to fully extract.

## 1. Up-front payments.

- ▶ At time 0, agent pays principal all rents.

## 2. Contractible transfers.

- ▶ Set transfer equal to  $v$ .
- ▶ Agent “buys the firm”.

# Motivation 1: Wealth Constraints

## General Contract $\langle Q_{i,t}, \phi_{i,t}, \phi_{i,t}^0 \rangle$

- ▶  $\phi_{i,t}$  is voluntary payment from  $i$  to principal.
- ▶  $\phi_{i,t}^0$  is contractible payment from  $i$  to principal.

## Proposition 7.

*Suppose the agent has zero wealth. Then any self-enforcing contract  $\langle Q_{i,t}, \phi_{i,t}, \phi_{i,t}^0 \rangle$  delivers the same payoffs as a contract of the form  $\langle Q_{i,t}, p_t \rangle$ .*

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## Case Study: McDonalds (Kaufman and Lafontaine, 1994).

- ▶ In 1980s, franchisees made **ex ante** rents of \$400K.
- ▶ Franchise fee was only \$22.5K.

## Motivation 2: Cowboys

### Free Entry of Principals

- ▶ Suppose there are many ‘cowboy principals’ in the world.
- ▶ These cowboys have costs  $c_{i,t} = \infty$   $(\forall i)(\forall t)$ .

### General Contract $\langle Q_{i,t}, \phi_{i,t}, \phi_{i,t}^0 \rangle$

- ▶ Contract is **cowboy-proof** if cowboys makes negative profits.

### Proposition 8.

*Any self-enforcing cowboy-proof contract  $\langle Q_{i,t}, \phi_{i,t}, \phi_{i,t}^0 \rangle$  delivers the same payoffs as a contract of the form  $\langle Q_{i,t}, p_t \rangle$ .*

# Strategy for Outsourcing

Kern, Willocks and van Heck (2002), “The Winner’s Curse in IT Outsourcing”, California Management Review.

*“The goal must be win-win, where the supplier can make a return. In a one-sided venture, the supplier has to try to cover its costs in any way possible, which is likely to effect services, operations and relations adversely.”*

# Contracts without Rents

## Optimal contract exhibits loyalty

- ▶ Multi-sourcing reduces the frequency of trade.
- ▶ Hence defection more likely.

## Optimal contract

- ▶ Contract is stationary.
- ▶ Bias trade towards most recently used agent.



# Extensions

## Incentives to innovate

- ▶ How does contract affect entry of new agents?
- ▶ How does contract affect incentives to invest in R&D?
- ▶ How does potential entry affect optimal contract?

## Different quantity levels, $Q \in \{0, 1, \dots, L\}$

- ▶ Slow build up of trade.

## Renegotiation-proofness

- ▶ Equilibrium is  $\epsilon$ -renegotiation-proof if  $N \geq \hat{N}_\epsilon$ .

# Summary

Agents' ability to holdup principal gives them rents.

- ▶ These rents are independent of number of trades.
- ▶ Act like fixed cost of relationship.

Characterisation of optimal ASE contract.

- ▶ Principal divides agents into 'insiders' and 'outsiders'.
- ▶ Trade biased towards insiders.

ASE contract is robust.

- ▶ If parties patient, contract is self-enforcing.
- ▶ With maintenance payments, contract robust to private info.

# Full Problem with IID Costs and $N = 1$

Suppose  $N = 1$ . The optimal ASE contract obeys

$$\begin{aligned} Q_t &= \mathbf{1}_{c_t \leq c^*} && \text{if } i \notin \mathcal{I}_t \\ Q_t &= \mathbf{1}_{c_t \leq v} && \text{if } i \in \mathcal{I}_t \end{aligned}$$

If  $\delta > \hat{\delta}$ , then optimal ASE contract is implementable.

## Proposition 4.

*Suppose  $N = 1$ . Then the optimal SE contract obeys*

$$\begin{aligned} Q_t &= \mathbf{1}_{c_t \leq \kappa^*} && \text{if } i \notin \mathcal{I}_t \\ Q_t &= \mathbf{1}_{c_t \leq \kappa^{**}} && \text{if } i \in \mathcal{I}_t \end{aligned}$$

where  $\kappa^* \leq \kappa^{**}$ ,  $\kappa^* \leq c^*$  and  $\kappa^{**} \leq v$ .

## A Complementary Theory for Loyalty

Suppose agents are impatient.

- ▶ If multi-source then reduce frequency of trade.
- ▶ Hence defection more likely.

Model with transfers.

- ▶ Optimal contract stationary.
- ▶ For fixed  $N$ , efficient contract enforceable if  $\delta \geq \delta_N$
- ▶ For fixed  $\delta$ , efficient contract not enforceable if  $N \geq N_\delta$ .

Optimal contract

- ▶ When  $N = 1$ , then trade if  $c_t \in [0, c^*] \subset [0, v]$ .
- ▶ When  $N = 2$ , bias trade towards most recently used agent.
- ▶ What happens as  $N$  grows large?