American Economic Review 101 (December 2011): 3349–3367
http://www.aeaweb.org/articles.php?doi=10.1257/aer.101.7.3349

Relational Contracts and the Value of Loyalty†

By Simon Board*

Firms routinely rely on the goodwill of their trading partners. In developed economies, ongoing relationships complement formal contracts in mitigating holdup and facilitating transactions (Stewart Macaulay 1963). In developing economies, where contracts enjoy little legal protection, long-term relationships are even more critical for trade (John McMillan and Christopher Woodruff 1999).

The major cost of maintaining a relationship is that, when there are many potential suppliers, it can reduce the scope of trade. The purpose of this paper is to make this trade-off precise. We consider a firm (principal) that would like to trade with different suppliers (agents) over time under the threat of holdup. We derive the optimal relational contract, show that it induces loyalty, and characterize the resulting distortions. The model enables us to make predictions as to when a firm will switch suppliers, and how these switches depend on the length of the relationship, the number of suppliers, and the shocks to the economy.

The following examples illustrate the practical importance of loyalty in overcoming the holdup problem. In 1984, the US government started awarding \$200 billion worth of contracts through "full and open competition," either in the form of sealed bids or competitive negotiations. In order to reduce corruption, the evaluation panel was instructed to ignore subjective information, such as prior performance. Steven Kelman (1990) examines computer procurement, showing that the government was considerably less loyal than private firms (awarding 58 percent of contracts to the incumbent versus 78 percent), and that it was more likely to use the cheapest bidder (65 percent versus 41 percent). As a result, he finds that government contractors overpromised more than private contractors, leading managers to be 50 percent more dissatisfied and to be involved in more formal disputes. The government contracts rated particularly badly on "keeping promises" and "sticking to the contracted delivery schedule," with vendors holding up the government by withdrawing key personnel, providing poor advice and few creative ideas. He concludes that "Lacking the ability to recoup transaction specific investments through the assurance of repeat business, [government] vendors fail to make investments that require conscious effort and expenditure of resources" (Kelman 1990, p. 72).

* Department of Economics, 8283 Bunche Hall, UCLA, Los Angeles, CA 90095 (e-mail: sboard@econ.ucla.edu). I am grateful to the three referees, Li Hao, Christian Hellwig, Hugo Hopenhayn, Jon Levin, Igor Livshits, Bentley MacLeod, Niko Matouschek, Robert McMillan, Moritz Meyer-ter-Vehn, Bill Rogerson, Andy Skrzypacz, Bill Zame, and especially John McMillan. I also thank seminar audiences at CEMFI, CETC (Montreal), HBS, IIOC (Savannah), NASM (Duke), NBER, Northwestern, UBC, UC Berkeley, UCLA, and Western Ontario.

† To view additional materials, visit the article page at http://www.aeaweb.org/articles.php?doi=10.1257/aer.101.7.3349.

The importance of loyalty is also illustrated by the experience of the 1980s automobile industry (Paul Milgrom and John Roberts 1997). During this period, General Motors awarded one-year contracts for each component through competitive bidding. While GM exploited the static gains from trade, it had concerns about quality, necessitating the inspection of deliveries. In contrast, Toyota was more loyal, automatically renewing contracts for the life of the model and, for new models, favoring preferred bidders and suppliers with high performance ratings. As a result, Toyota overcame the holdup problem, trusting suppliers to meet the required quality standards, and installing parts without inspection, and delegating much of the design to its suppliers. In the words of Roberts (2004, p. 203), "compared to arm's length, short-run dealings, the stronger incentives that can be provided for mutually advantageous, value-creating behavior in an ongoing relationship [favors Toyota's approach]."

Finally, Simon Johnson, McMillan, and Woodruff (2002) analyze the incidence of loyalty using a survey of transitional Eastern European countries, where "relational contracting is the main mechanism governing contracting" (p. 260). In their survey, when asked whether they would change suppliers for a 10 percent price discount, over half the firms said they would pass up on the new deal, in whole or in part. Johnson et al. find that the degree of loyalty is significantly higher in countries with less effective court systems and for goods that are custom-built, i.e., where there is more scope for holdup. They conclude that "ongoing relationships can improve efficiency by supporting deals that the legal system is unable to enforce. But exclusion is the corollary of ongoing relationships."

Together, these examples illustrate two key features. First, when given the choice, firms are loyal to their suppliers, rather than simply maximizing the static gains from trade. Second, loyalty mitigates the holdup problem. The aim of this paper is to analyze the trade-off faced by such firms, and to derive the optimal relational contract.

Outline of the Paper.—We start with a basic holdup model. Each period, a principal invests in one of N agents.[1] The winning agent then has the opportunity to hold up the principal. We suppose the cost of investment differs across time and across agents, as technology and product requirements change, leading to variation in the principal's efficient trading partner. A relational contract then specifies how the investment by the principal and the price paid to the agent depend on current costs and the history of the game.

In Section II, we suppose that the principal commits to a specified investment strategy, allowing us to focus on the agents' incentives. When the principal invests in agent i, the agent has the option to hold up the principal. In order to prevent this opportunism, the principal must give the agent a sufficiently large rent, so that he resists the temptation. This rent can come both in terms of payment today and promised payments in future periods. Crucially, the principal can use these delayed payments in order to prevent future incidences of holdup. This means that the

[1] The principal is female, while the agents are male.

principal must pay the agent only one rent, independent of the number of times they trade. That is, the rent acts like a fixed cost of initiating a new relationship.

When designing the contract, the principal must trade off exploiting the gains from trade against the cost of starting new relationships. We show that the principal's profit-maximizing contract therefore divides agents into "insiders," with whom she has previously traded, and "outsiders," with whom she has never traded. The principal then uses insiders efficiently, while being biased against outsiders. The model thus provides a theory of endogenous switching costs, where there is a cost to use a new agent, but no cost to return to an old agent.

The insider-outsider contract has one large defect: it is not time consistent. From the principal's time-0 perspective, each agent must be paid a rent for the first trade, but not for subsequent trades. This has the consequence that insiders trade efficiently. The problem is that many of the agents' rents come from promised future payments, designed to prevent holdup during later trades. The principal thus has an incentive to avoid trade with insiders in later periods in order to avoid these promised future payments.

The time inconsistency has two implications. First, when the principal is impatient, she will be tempted to default. In Section III, we examine this problem, supposing that the relationship is enforced by bilateral punishment: that is, a defection by the principal against agent i is observed only by agent i. We show that when the principal is sufficiently patient and costs are i.i.d., the insider-outsider contract is self-enforcing. This is even the case if there are an infinite number of agents. Intuitively, each agent must be paid a rent, so the principal will only ever wish to trade with a finite number of partners. As the discount factor approaches one, the maximum number of insiders grows to infinity, but it grows sufficiently slowly that the profit per agent increases without bound.

The second implication of time inconsistency is that the principal has an incentive to exaggerate the cost of investing in insiders in order to avoid trading with them as frequently. In Section IV, we suppose that the principal's costs are private information and show that the principal can overcome this problem by using an "employment contract," where insiders are paid a fixed amount per period, independent of whether trade occurs or not. The advantage of the employment contract is that, since the payments to insiders are sunk, the principal has no incentive to misstate her true costs. The disadvantage is that the principal has more incentive to renege than when using the optimal prices from Section III. Nevertheless, under i.i.d. costs, the employment contract is self-enforcing when the principal is sufficiently patient. The model thereby provides a theory of the firm where both firm size and growth are endogenous.

This paper builds on a number of relational contracting papers. Bentley MacLeod and James Malcomson (1989) consider a model with one principal and one agent, while Jonathan Levin (2002) allows the principal to employ multiple workers, comparing multilateral and bilateral contracts. These papers differ from ours in that they allow for contractible transfers. As a result, the profit-maximizing contract is stationary and maximizes joint surplus.

Jonathan Thomas and Tim Worrall (1994) analyze a game between a principal and a single agent where the productivity of the relationship depends on the capital stock, which can be stolen by the agent at any time. In the optimal contract the principal gradually increases the capital stock, eventually converging to the efficient level, while the payments to the agent are backloaded to prevent holdup. The forces driving the current paper are similar, although we focus on multiple suppliers rather than the scale of investment.

Curtis Taylor and Steven Wiggins (1997) examine the trade-off between enforcing quality via inspections and via a relational contract in a stationary environment with identical suppliers. We allow for heterogenous suppliers and characterize the optimal relational contract.

As in the current paper, Giacome Calzolari and Giancarlo Spagnolo (2009) analyze a principal who procures from multiple agents under incomplete contracts. They consider an incomplete information model where the principal first selects $n < N$ agents and then, in subsequent periods, chooses the cheapest via an auction. In contrast, we study a complete information model and derive the optimal contract. When costs are i.i.d., this converges to the stationary contract used by Calzolari and Spagnolo.

David McAdams (2010) supposes agents are matched randomly into pairs and then play a partnership game with stochastic returns. Agents can choose to continue or quit but, as in the community enforcement literature, cannot sustain multiple relationships and cannot choose with whom they match after quitting.

Sylvain Chassang (2010) supposes that a principal contracts with an agent who may choose one of many actions with payoffs unknown to the principal. To overcome moral hazard, the principal punishes bad outcomes by suspending trade. This efficiency loss reduces the scope of experimentation, similar to the way the agents' rents reduce the scope of trade in the current paper.

Omitted proofs are in the Appendix.

I. The Environment

The economy consists of a principal and N agents. Time $t \in \{0, 1, 2, \dots\}$ is discrete and infinite. At time 0, the principal designs a contract to maximize her expected profits. Each period $t \in \{1, 2, \dots\}$ then consists of three stages (see Figure 1):

(i) The cost of investing in agent i, $c_{i,t} \in [\underline{c}, \overline{c}] \subset [0, \infty)$, is publicly revealed. These costs are distributed according to some stochastic process.

(ii) The principal chooses to invest in at most one of the agents. Denote the probability that the principal invests in i by $Q_{i,t} \in \{0, 1\}$, where $\sum_i Q_{i,t} \leq 1$.

(iii) The winning agent produces and sells a product with value v. He then chooses to keep $p_t \in [0, v]$ and pay the principal $v - p_t$.

We assume that prices p_t are observable but not contractible, either because courts are inefficient or because the transfer is unverifiable. The stage game of this model thus exhibits the holdup problem: the agent has all the ex post bargaining power, so expropriates the quasi-rents, v; anticipating being held up, the principal then abstains from investing.

This paper considers the infinitely repeated version of the holdup game, where all parties have discount rate $\delta \in (0, 1)$. We aim to model a decentralized market,

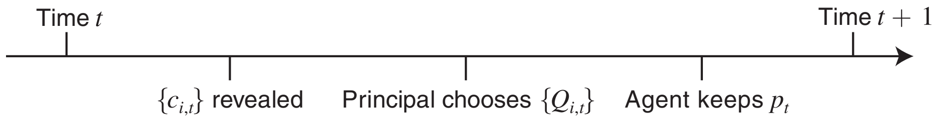

Figure 1. Time Line

so assume that contracts are maintained by bilateral punishment: that is, a deviation in the relationship between the principal and agent i cannot be observed by agents $j \neq i$. Formally, at time t, agent i observes $\mathbf{h}_{i,t} := \{c_{1,t}, \ldots, c_{N,t}, Q_{i,t}, p_t Q_{i,t}\}$. The agent's history at time t is thus $\mathbf{h}_i^t := (\mathbf{h}_{i,1}, \ldots, \mathbf{h}_{i,t})$, while the principal's history is $\mathbf{h}^t := (\mathbf{h}_1^t, \ldots, \mathbf{h}_N^t)$. A relational contract $\langle Q_{i,t}, p_t \rangle$ is defined to be a history-contingent plan of allocations and prices. The principal's investment strategy is a mapping $Q_{i,t}$: $\mathbf{h}_i^{t-1} \times [\underline{c}, \overline{c}]^N \to \{0, 1\}$, while the winning agent's pricing strategy is a mapping p_t: $\mathbf{h}_i^{t-1} \times [\underline{c}, \overline{c}]^N \times \{0, 1\} \to [0, v]$.

In Section II, we assume the principal commits to her investment strategy, allowing us to focus on the agents' incentives. A contract is *agent-self-enforcing* (ASE) if the agents' strategies form a subgame perfect equilibrium, taking the principal's strategy as given. In Section III, we analyze the problem where the principal cannot commit to her strategy. A contract is then *self-enforcing* if both the agents' and the principal's strategies form a subgame perfect equilibrium. Among the class of self-enforcing contracts, we then look for the contract that maximizes the principal's profit.

Welfare at time t from the principal's relationship with agent i is

$$W_{i,t} := E_t\left[\sum_{s=t}^{\infty} \delta^{s-t}(v - c_{i,s})Q_{i,s}\right],$$

where "E_t" is the expectation in period t, after $\{c_{i,t}\}$ have been revealed. Total welfare is $W_t := \sum_i W_{i,t}$. Agent i's utility at time t is

$$(1) \qquad U_{i,t} := E_t\left[\sum_{s=t}^{\infty} \delta^{s-t} p_s Q_{i,s}\right].$$

The principal's profit at time t from agent i is

$$(2) \qquad \Pi_{i,t} := E_t\left[\sum_{s=t}^{\infty} \delta^{s-t}(v - c_{i,s} - p_s)Q_{i,s}\right].$$

The principal's total profit is $\Pi_t := \sum_i \Pi_{i,t}$.

We have phrased the model in terms of a trust game, but it can be interpreted more broadly as a model of moral hazard or of Grossman-Hart-Moore incomplete contracts. For moral hazard, suppose that at stage 3 of the game, the principal pays the agent a wage v, and the agent chooses effort $e = v - p$. For incomplete contracts, suppose that at stage 3 of the game, the principal and agent decide whether or not to trade, creating value v, and that the agent has all the bargaining power.

The model assumes that agents can make payments to the principal only when they trade. Importantly, this means that the agent cannot make payments before the time of first trade (i.e., up-front bonds), allowing the principal to extract their rents. This assumption seems empirically reasonable. Patrick Kaufmann and Francine Lafontaine (1994) show that an average McDonald's franchisee makes ex post rents of \$400,000, but faces a franchise fee of only \$20,000. McMillan and Woodruff (1999, Table 1) find that only 35 percent of customer relationships in Vietnam involve any advance payment. Theoretically, this assumption can be motivated by assuming that agents have no wealth, or that contracts have to be robust against hit-and-run principals who take any up-front payment but have no intention of investing.[2]

II. One-Sided Commitment

In this section we suppose that the principal can commit to her investment strategy, $Q_{i,t}$. We then solve for the optimal ASE contract.

LEMMA 1: *A contract $\langle Q_{i,t}, p_t \rangle$ is ASE if and only if*

$$\text{(DEA)} \qquad (U_{i,t} - v) Q_{i,t} \geq 0 \qquad (\forall i)\,(\forall t).$$

PROOF:

Fix $\langle Q_{i,t}, p_t \rangle$ and assume that the agent's dynamic enforcement constraint (DEA) holds. Suppose the principal uses the grim trigger strategy: if i deviates at time t, then $Q_{i,s} = 0$ for $s > t$. Whenever the agent is used, $Q_{i,t} = 1$, the agent will not deviate since $U_{i,t} \geq v$. The contract is thus agent-self-enforcing. Conversely, if (DEA) does not hold, there exists a time t when $Q_{i,t} = 1$ and $U_{i,t} < v$. The agent will therefore defect and obtain v.

(DEA) is necessary and sufficient for the agent not to defect. Sufficiency follows from the use of the grim-trigger strategy. Necessity follows from the fact that the grim-trigger punishment attains the agent's min-max utility.

The *principal's problem* is to choose the contract $\langle Q_{i,t}, p_t \rangle$ to maximize time-0 profit,

$$\text{(3)} \qquad \Pi_0 = E_0 \left[\sum_{s=1}^{\infty} \sum_i \delta^s (v - c_{i,s} - p_s) Q_{i,s} \right],$$

subject to the agents' dynamic enforcement constraints (DEA). This problem can be simplified by noting that the principal's profit equals welfare minus the sum of agents' utilities. The principal's problem is then to maximize

$$\text{(4)} \qquad \Pi_0 = E_0 \left[\sum_{s=1}^{\infty} \sum_i \delta^s (v - c_{i,s}) Q_{i,s} \right] - \sum_i U_{i,0},$$

subject to (DEA).

[2] See Board (2009) for a more formal treatment. What if we allowed the principal to use up-front bonds? Suppose the contract is enforced bilaterally. When δ is high, the principal chooses the cheapest agent. When δ is low there is loyalty since the principal must be incentivized not to run away with the bonds. The critical δ depends on the environment: if $N = \infty$ and costs are i.i.d., then we are always in the second scenario.

In order to evaluate (4), we can place a lower bound on agents' utilities at each point in time. Let $\tau_i(t) := \min\{s \geq t: Q_{i,s} = 1\}$ be the time agent i next trades. Using (DEA) and $U_{i,t} = E_t[U_{i,\tau_i(t)}\delta^{\tau_i(t)-t}]$, a contract $\langle Q_{i,t}, p_t\rangle$ is ASE if and only if

$$U_{i,t} \geq E_t[v\,\delta^{\tau_i(t)-t}] \qquad (\forall i)\,(\forall t). \tag{DEA$'$}$$

The principal's problem is thus to choose $\langle Q_{i,t}, p_t\rangle$ to maximize profit (4) subject to (DEA$'$). Agents' utilities enter profit only via the time-0 term so, if it is feasible, profit is maximized by ensuring that (DEA$'$) binds at time-0,

$$U_{i,0} = E_0[v\,\delta^{\tau_i(0)}]. \tag{5}$$

In Section IIC we show that this is indeed feasible: there exist prices $p_t \in [0, v]$ such that (5) holds, and (DEA$'$) is satisfied at all other times. The principal's problem therefore reduces to choosing $\langle Q_{i,t}\rangle$ to maximize profit

$$\Pi_0 = E_0\left[\sum_{s=1}^{\infty}\sum_i \delta^s(v - c_{i,s})Q_{i,s}\right] - E_0\left[\sum_i v\,\delta^{\tau_i(0)}\right]. \tag{6}$$

We thus solve the problem in two steps. First, we choose the allocations $Q_{i,t}$ to maximize profits (6). Second, we use the agents' dynamic enforcement constraints to derive prices.

A. *The General Solution*

The principal's problem (6) is an optimal stopping problem and is hard to characterize fully. Fortunately, a number of economically relevant results are straightforward to derive. Denote the set of *insiders* at time t by $\mathcal{I}_t := \{i: \tau_i(0) < t\}$.

PROPOSITION 1: *An optimal ASE contract has the following properties:*

(*i*) *Trade with insiders is efficient. Suppose* $i \in \mathcal{I}_t$. *Then,* $Q_{i,t} = 1$ *if*

$$c_{i,t} < v \qquad \text{and} \qquad c_{i,t} < c_{j,t}\ (\forall j). \tag{7}$$

(*ii*) *Trade is biased against outsiders. Suppose* $i \notin \mathcal{I}_t$. *Then,* $Q_{i,t} = 0$ *if either*

$$(v - c_{i,t}) < (1 - \delta)v \qquad \text{or} \tag{8A}$$

$$(c_{j,t} - c_{i,t}) < (1 - \delta)v \qquad \text{for some } j \in \mathcal{I}_t. \tag{8B}$$

Part (i) says that if an insider has a lower cost than all other agents, then he will be chosen by the principal. In addition, the principal treats all insiders equally: conditional on trading with an insider, she always chooses the cheapest.[3] Intuitively, each agent obtains rents of v the first time he trades, by threatening to hold up the principal. Agents care about the lifetime value of their utility, so, by delaying payments to the agent, the principal can use the same sum of money to avoid holdup during later trades with the same agent. This means that the agent earns only one rent, no matter how many times he trades. This rent acts like a fixed cost of initiating a new relationship; once the fixed cost has been paid, the principal uses the agent efficiently.

Part (ii) says that trade is biased against outsiders. There are three types of biases. First, the principal may abstain rather than trade with an outsider, $i \notin \mathcal{I}_t$. If the principal refrains from investing in i, she loses the gains from trade, $v - c_{i,t}$. She postpones paying the agent's rent by at least one period, however, gaining the rental value of the rent, $(1 - \delta)v$. This yields equation (8A).

The second bias is that the principal may prefer an insider, j, over an outsider, i. If the principal trades with the insider rather than the outsider, she loses the cost difference, $c_{j,t} - c_{i,t}$. She delays the rent by at least one period, however, gaining $(1 - \delta)v$. This yields equation (8B). This result means that an agent who enters the market late will struggle to trade with the principal. Indeed, Example 1 below shows that, when costs are i.i.d., the insider advantage increases monotonically with the number of insiders, and eventually grows so large than no more outsiders are used.

The third bias is that, when trading with an outsider, the principal may not choose the agent with the lowest cost. For example, the principal is biased toward "larger" agents who are more efficient, on average. This is formalized by the following result.

PROPOSITION 2: *Assume agent i's costs $c_{i,t}$ are i.i.d. draws from an i-specific distribution, and the distribution of $c_{1,t}$ is smaller than $c_{2,t}$ in terms of first-order stochastic dominance. If agents 1 and 2 are outsiders and $c_{1,t} = c_{2,t}$ then the principal prefers to use agent 1 over agent 2.*

Intuitively, the principal cares about the cost in both the current period and in future periods, after the agent has become an insider. She thus prefers an agent who is more efficient on average, even if not the most efficient in the current period. Empirically, this suggests a bias toward larger, more established firms. For example, Kelman (1990, p. 8) finds the largest computer vendor, IBM, had more than twice the market share among private companies than with the government. The result also suggests a bias toward generalists rather than specialists. Indeed, it seems natural for a household to use a general contractor rather than a series of specialists (e.g., plumbers, painters, carpenters) in order to reduce their exposure to holdup.

Proposition 2 has the corollary that two agents may like to merge in order to make themselves larger and give themselves a competitive advantage. To illustrate, suppose costs are i.i.d. and that agents 1 and 2 can merge, causing them to act as a single entity and giving them joint cost of $\min\{c_1, c_2\}$. Such a merger reduces their rents if

[3] This follows from equation (13) in the Appendix.

TABLE 1—THE EVOLUTION OF TRADE

Insiders, n_t	Cutoff, Δ_n^*	$\Pr(n_{t+1} = n_t)$	Value function, Π_n
0	0	0	83.6
1	0.358	0.358	85.3
2	0.398	0.637	87.0
3	0.454	0.837	88.6
4	0.549	0.959	90.2
5	0.834	0.999	91.7
6	1	1	92.9

Notes: This example assumes the value of trade is $v = 2$, costs are $c_{i,t} \sim [0, 1]$, the number of suppliers is $N = \infty$, and the discount rate is $\delta = 0.98$. First-best profits are 100; under the insider-outsider contract they equal 83.6.

they trade, but increases the probability of trade. Consequently, if N is sufficiently large, such a merger will increase their joint utility.[4]

When choosing among outsiders, the principal also prefers more stable suppliers, who are expected to be in the market for a long time, over short-term players. Intuitively, when using a stable supplier, the principal can amortize the rents over a greater number of periods. This means that longevity can become self-fulfilling: firms that are expected to leave the market obtain no business and have no incentive to stay.

A few general points about Proposition 1 are worth emphasizing. First, it provides a foundation for switching costs. Unlike typical models of switching costs, however, there is a cost to using a new partner for the first time, but not to returning at a later date. Second, the optimal contract is not stationary, as typically assumed in efficiency wage models. Third, we made no assumptions about the stochastic process governing costs. With more structure, we can be more precise about the structure of the optimal contract, as shown by the following example.

EXAMPLE 1 (i.i.d. Costs): *Suppose costs are i.i.d. both over time and across agents, with support* $[\underline{c}, \overline{c}]$. *Let* $\omega(n) := E[\max\{v - c_{1:n}, 0\}]$ *be the expected welfare from n agents and* $c_{1:n} := \min\{c_1, \ldots, c_n\}$. *The maximum number of insiders,* n^*, *is the unique integer satisfying*

$$(9) \qquad \frac{\delta}{1 - \delta} \Delta\omega(n^*) \geq \max\{v - \overline{c}, 0\} + \underline{c} \geq \frac{\delta}{1 - \delta} \Delta\omega(n^* + 1),$$

where $\Delta\omega(n) := \omega(n) - \omega(n - 1)$. *The first inequality in equation (9) says that if there are* $n^* - 1$ *insiders, the principal prefers to use one last outsider and pay rent v, rather than sticking to the current set of insiders. Similarly, the second inequality says the principal prefers not to use agent* $n^* + 1$.

[4]PROOF: Before 1 and 2 merge, they have $U_{i,0} = O(1/N)$ as $N \to \infty$. Next, suppose agents 1 and 2 merge. Suppose the principal has no insiders. If she uses firm $i \notin \{1, 2\}$, her profit is at most $-\underline{c} + \Pi_1$, where Π_n is the profit from n insiders. If she uses the new merged firm, her profit is $-\min\{c_1, c_2\} + \Pi_2$. Observe that an extra insider is valuable in the states when it is unprofitable to bring in an outsider: that is, $\Pi_2 - \Pi_1 \geq E[c_1 - \min(c_1, c_2)]\Pr(c_1 - \underline{c} < v(1 - \delta)) =: k > 0$. Hence, the principal chooses the merged firm whenever $\min\{c_1, c_2\} - \underline{c} < k$. As a result, the agents' joint utility is bounded above zero as $N \to \infty$.

The number of insiders n_t serves as the state variable, where the transition depends on the difference between the costs of the cheapest insider and the cheapest outsider, Δ. Using n^* as the end point of the dynamic program, it is straightforward to solve for the optimal cutoffs, Δ_n^*. Table 1 provides a numerical illustration, in which there are an infinite number of agents willing to trade with the principal. Despite all this choice, the principal only ever uses six agents and the principal's adoption of new agents rapidly diminishes over time. More generally, Board (2009) shows that if either $\underline{c} > 0$ or $v > \overline{c}$ then Δ_n^* and $\Pr(n_{t+1} = n_t | n_t = n)$ increase in n.

B. *Empirical Implications*

The optimal contract in Proposition 1 predicts that the principal is loyal to her agents, treating all previous trading partners equally, while being biased against new agents. This contract resembles the supply associations in Japanese industry. For example, Toyota invites preferred suppliers (defined by size, dependency, and performance) to join its supply associations (e.g., Kyohokai, Seihokai, Eihokai). These suppliers account for a large fraction of Toyota's parts supply (typically over 80 percent) and are implicitly promised a share of future business. As in Proposition 1, new firms occasionally join, but old firms rarely leave. In 1992, Toyota's Tokai Kyohokai consisted of 141 members; over the previous 20 years, 29 firms joined but only 5 left (Banri Asanuma 1989; Mari Sako 1996).

The insider-outsider contract is seen elsewhere. In the UK legal industry, Olivier Chatain (2009, 2010) finds that, when choosing legal representation for a new area, clients have a large bias toward insiders, raising the trading probability from 8 percent to 35 percent. This insider bias is approximately independent of the number of services the law firm currently provides the client, as in Proposition 1. Similarly, Johnson, McMillan, and Woodruff (2002) find that, while Eastern European firms are biased against new suppliers, trust builds up quite quickly, within the first two months of the relationship.

These findings, while consistent with our endogenous fixed cost model, are also consistent with a model where there is an exogenous fixed cost for the first trade. Fortunately, the models yield a number of different predictions. First, in the holdup model, the legal environment matters. It therefore predicts that firms exhibit less loyalty in countries with better legal systems, and that any improvement in courts has the biggest impact when goods are custom built and when relationships are new, as found by Johnson, McMillan, and Woodruff (2002). Second, it predicts that the degree of loyalty increases when there is more asset specificity, as in Johnson, McMillan, and Woodruff (2002) or Asanuma (1989). Third, if the principal chooses not to be loyal, the holdup model predicts that the principal will receive lower quality services, as found by Kelman (1990) and Milgrom and Roberts (1997). Fourth, to mitigate the holdup problem, such disloyal firms will try to introduce more complete contracts, as in Kelman (1990) and Milgrom and Roberts (1997). Fifth, loyalty should decrease if the punishment payoff worsens. This means that membership of a trade association should reduce switching costs, as found by Johnson, McMillan, and Woodruff (2002). Sixth, the holdup model predicts that as the end game approaches (i.e., δ rises) the degree of loyalty rises (especially when there is no commitment).

This predicts that firms will be reluctant to trade with partners near bankruptcy, as seen recently in the car industry.[5]

C. *Prices*

The optimal investment function $Q_{i,t}$ is chosen to maximize profits (6). Prices, $p_t \in [0, v]$, can then be chosen in a number of ways such that (i) utilities satisfy the agents' dynamic enforcement constraints (DEA$'$) and, (ii) the principal gives no rents away (5). Given agents' utilities, prices are then determined by backward induction,

$$p_t = U_{i,t} - E_t\big[\delta^{\tau_i(t+1)} U_{i,\tau_i(t+1)}\big], \tag{10}$$

whenever $Q_{i,t} = 1$.

The *fastest prices* have the property that the dynamic enforcement constraints (DEA$'$) bind in all periods. These prices are so called because the principal makes payments to the agent as early as possible. Using (10), they are given by

$$p_t = v\,E_t\big[1 - \delta^{\tau_i(t+1)}\big], \tag{11}$$

whenever $Q_{i,t} = 1$.[6] By pushing payments forward, the fastest prices maximize the principal's continuation profit in each period. Consequently, when the principal cannot commit to the contract (Section III), an investment rule is implementable only if it can be implemented by the fastest prices. This is formalized by the following result.

PROPOSITION 3: *Fix an allocation rule $Q_{i,t}$. Denote the profit under the fastest prices by $\Pi'_{i,t}$ and the profit under any other price system by $\Pi''_{i,t}$. Then, $\Pi'_{i,t} \geq \Pi''_{i,t}\,(\forall i)\,(\forall t)$.*

PROOF:

Profit at time t is given by $\Pi_{i,t} = W_{i,t} - U_{i,t}$. Utilities must obey (DEA$'$), so $\Pi_{i,t}$ is maximized by setting $U_{i,t} = E_t\big[v\delta^{\tau_i(t)-t}\big]$, yielding $\Pi_{i,t} = \Pi'_{i,t}$.

III. No Commitment

We now suppose the principal cannot commit to her investment plan.[7] A contract $\langle Q_{i,t}, p_t\rangle$ is self-enforcing if and only if it satisfies the agents' dynamic enforcement constraint (DEA) and the principal's own dynamic enforcement constraint (DEP). We assume relationships are enforced by bilateral punishments, so a defection by

[5] See *Financial Times*, "US Carmakers Face Components Challenge" (March 8, 2006) and "Parts Suppliers Wary of Carmakers" (August 15, 2008).

[6] In contrast, the *slowest prices* have the property that the principal delays payment by as much as possible. This means that there exists a t_i^* such that, whenever $Q_{i,t} = 1$, $p_t = 0$ for $t < t_i^*$ and $p_t = v$ for $t > t_i^*$.

[7] This commitment problem is an important practical issue: Roberts (2004, p. 205) writes that "Toyota could, if it chooses, squeeze suppliers, demanding unreasonable concessions and expropriating the returns they expected when they made the investments to serve it. [. . .] Then the threat of mistreatment—not to mention the actual experience of it—might lead the suppliers to behave less cooperatively or to expend resources trying to protect themselves, either of which is value destroying."

the principal on agent i will be punished by the termination of relationship i. Thus, the constraint is

$$\text{(DEP)} \qquad \Pi_{i,t} Q_{i,t} \geq 0 \qquad (\forall i)(\forall t).$$

Appealing to Proposition 3, (DEA′) binds in each period, enabling us to rewrite (DEP) as

$$\text{(DEP}') \qquad (W_{i,t} - v) Q_{i,t} \geq 0 \qquad (\forall i)(\forall t).$$

Bilateral enforcement is realistic in a large decentralized market where agent i's information about agent j's relationship is poor. For example, McMillan and Woodruff (1999) report that in Vietnam only 19 percent of interviewees thought that a cheating customer would be blacklisted by other firms in the industry. Similarly, in the United States, firms often refuse to report negative information about former suppliers or employees for fear of legal action (Kelman 1990, p. 49).[8]

To motivate the discussion, Example 2 shows that the principal's dynamic enforcement constraint may strongly restrict the set of possible trading opportunities.

EXAMPLE 2 (Unraveling Trade): *Suppose* $N = 1$ *and* $c_{1,t} \in (0, v)$ *are deterministic and increasing such that* $\lim_{t\to\infty} c_{1,t} > \delta v$ *and* $\sum_{t=1}^{\infty} \delta^t (v - c_{1,t}) > v$. *In this case, the optimal ASE contract obeys* $Q_{1,t} = 1\ (\forall t)$. *Since gains from trade disappear over time, however, any contract eventually satisfies* $W_{1,t} < v$ *for* $t \geq t^*$. *Thus, trade is not feasible at time* t^* *and, by backward induction, the only solution to satisfy (DEP) is* $Q_{1,t} = 0\,(\forall t)$.

Example 2 shows that the insider-outsider contract is not time consistent. From the principal's time-0 perspective, she has to pay rents v to an agent in order to stop him defecting. After these rents have been paid, the principal uses the agent efficiently. The problem is that many of these rents come in terms of promised future utility, which are needed to stop future defections. These postponed payments mean that the principal may later regret promising to use the agent efficiently. If the principal defects in these later periods, she then raises the required price in earlier periods, exerting a negative externality on her former self.

The relational contract turns around the holdup problem. In a one-shot game, the principal is concerned that the agent will cheat after the specific investment. This problem is solved through a long-term relationship, by delaying the payment of the agent's rents. This relational contract then introduces a second holdup problem: the principal is tempted to defect and refuse to pay these promised rents.

[8] If trade is not anonymous, or agents can communicate with each other, then we can use multilateral enforcement, whereby a deviation against one agent leads to the termination of all relationships. For example, Toyota's supply associations share information so that "any mistreatment of a single supplier will be known to all" (Roberts 2004, p. 206). In this case, the principal faces the aggregate dynamic enforcement constraint $\Pi_t \geq 0\ (\forall t)$. While the unravelling problem in Example 2 persists, the optimal ASE contract is self-enforcing under a wider variety of circumstances.

A. *I.I.D. Costs*

With this new constraint, the *principal's problem* is to choose $\langle Q_{i,t} \rangle$ to maximize time-0 profit (3) subject to (DEP$'$). To examine further the principal's optimal contract let us suppose that costs are i.i.d. Proposition 4 shows that, when the principal is sufficiently patient, the insider-outsider contract is self-enforcing.

PROPOSITION 4: *Suppose that costs are i.i.d., and that either $\underline{c} > 0$ or $v > \overline{c}$. Then, there exists $\hat{\delta} < 1$, independent of N, such that the optimal ASE contract satisfies (DEP) when $\delta > \hat{\delta}$.*

EXAMPLE 3 (Uniform Cost): *Suppose $N = \infty$, $c_{i,t} \sim U[0, 1]$, and $v > 1$. Then,* Board (2009) *shows that (DEP) is satisfied if $\delta \geq \hat{\delta} := 1/(1 + (v - 1)^3)$. For example, if $v = 2$, then $\hat{\delta} = 1/2$, so the example in Table 1 is self-enforcing. As $v \to 1$, so $\hat{\delta} \to 1$.*

The principal's dynamic enforcement constraint requires that, when the principal trades with agent i, the future profits associated with i are positive. For fixed N, this trivially holds as the discount rate approaches one and profit per agent tends to infinity. Hence, there exists a $\hat{\delta}_N$ such that the optimal contract satisfies (DEP) when $\delta \geq \hat{\delta}_N$.

Proposition 4 makes a stronger statement: there exists a critical discount factor, $\hat{\delta}$, independent of the number of agents, such that the optimal contract satisfies (DEP) when $\delta \geq \hat{\delta}$. With an infinite number of agents, the problem is that the number of trading partners will increase without bound as $\delta \to 1$. Despite this, the average profit per agent tends to infinity as the parties become more patient. Intuitively, the principal's investment in new partners is limited by having to pay rent v for every new relationship. This means that the maximum number of insiders is determined by the marginal benefit of an extra agent, $E[c_{1:n-1} - c_{1:n}]$, as in equation (9). This marginal benefit is of order $o(n)$ and therefore decreases more rapidly than the average benefit of each relationship, $E[v - c_{1:n}]/n$. The average profit per agent thus increases in δ, and the principal will refrain from defecting when sufficiently patient.

For low δ, the insider-outsider contract does not satisfy the principal's dynamic enforcement constraint. As a result, she will treat insiders inefficiently, in order to increase her continuation profits. This leads to two biases in the way insiders are treated. First, insiders are not used as much as is efficient. If the principal abstains from trade when costs are high (and welfare low) then this eases (DEP$'$). Second, the principal is biased toward insiders she used more recently. Such a bias induces a second-order loss in total welfare but leads to a first-order increase in the welfare of the trading agent's relationship $W_{i,t}$, thereby relaxing (DEP$'$). In addition, there are the usual biases associated with outsiders, as discussed in Section IIA.

IV. Private Cost Information

The time inconsistency of the optimal contract also causes a problem when costs $c_{i,t} \in [\underline{c}, \overline{c}]$ are privately observed by the principal. To motivate the following discussion, Example 4 shows that the optimal contract may not be incentive compatible, even if it is self-enforcing.[9]

EXAMPLE 4: *Suppose* $N = 1$, $v = 1$, *costs are* $c_{1,t} \sim U[0, 2]$, *and* $\delta = 9/10$. *In the insider-outsider contract, the principal trades with an outsider if* $c_{1,t} \leq 0.80$, *and trades with an insider if* $c_{1,t} \leq 1$. *This contract is self-enforcing and generates prices* $p_t = 0.18$. *The contract is not incentive compatible, however. If* $c_{1,t} \in [0.82, 1]$, *the principal has an incentive to overstate her cost in order to avoid trade with an insider.*

Example 4 shows that the principal may exaggerate her cost to avoid trade with an insider. Similarly, she may misstate her costs in order to trade with an outsider over an insider. Intuitively, since many of an agent's rents take the form of delayed payments, the principal has an incentive to avoid these future trades by pretending that her costs are artificially high.

A. *Employment Contract*

We now show that if the principal can expand the space of contracts to allow payments from the principal to the agent even if no trade occurs, then she can implement the optimal ASE contract. Since the principal is assumed to have zero wealth, we continue to assume the agent cannot pay the principal before the time of first trade, $\tau_i(0)$.[10]

At time 0, the principal designs a contract $\langle Q_{i,t}, p_{i,t} \rangle$ to maximize her expected profits. Each period $t \in \{1, 2 \ldots\}$ then consists of four stages:

(i) The principal privately observes her costs $\{c_{i,t}\}$. These costs are distributed according to some stochastic process.

(ii) The principal makes public cost reports $\{\hat{c}_{i,t}\}$. These reports determine the allocation and payments specified by the contract.

(iii) The principal chooses to invest in at most one of the agents. Denote the probability the principal invests in i by $Q_{i,t} \in \{0, 1\}$, where $\sum_i Q_{i,t} \leq 1$.

(iv) If agent i wins, he produces and sells a product worth v. He then chooses to keep $p_{i,t} \in \mathbb{R}$, and pays the principal $v - p_{i,t}$. If agent i loses, he receives $p_{i,t} \in \mathbb{R}$ from the principal. All payments are voluntary; negative payments cannot be made before the time of first trade, $\tau_i(0)$.

Given that punishments are bilateral, at time t, agent i observes $\hat{\mathbf{h}}_{i,t} = \{\hat{c}_{1,t}, \ldots, \hat{c}_{N,t}, Q_{i,t}, p_{i,t}\}$. The agent's history at time t is thus $\hat{\mathbf{h}}_i^t := (\hat{\mathbf{h}}_{i,1}, \ldots, \hat{\mathbf{h}}_{i,t})$,

[9] This is a real problem for suppliers: for example, Atlantic, a supplier "attained the coveted Spear 1 supplier status at GM. That designation, GM claimed, would surely lead to more business with the manufacturer and its suppliers. But soon thereafter, GM reduced its orders with Atlantic without explanation" (Jeffrey Liker and Thomas Choi 2004).

[10] This expansion of payments is not useful in the complete information model (i.e., Proposition 3 still holds).

while the principal's history is $\mathbf{h}^t := (\mathbf{h}_1^t, \ldots, \mathbf{h}_N^t, \mathbf{c}_1^t, \ldots, \mathbf{c}_N^t)$, where $\mathbf{c}_i^t$ is the vector of i's past costs. The relational contract $\langle Q_{i,t}, p_{i,t}\rangle$ specifies an investment strategy $Q_{i,t}$: $\hat{\mathbf{h}}_i^{t-1} \times [\underline{c}, \overline{c}]^N \to \{0, 1\}$ and a pricing strategy $p_{i,t}$: $\hat{\mathbf{h}}_i^{t-1} \times [\underline{c}, \overline{c}]^N \times \{0, 1\} \to [0, v]$. The contract is *incentive compatible* if the principal cannot improve her payoffs by misreporting at stage 2.

Consider the following *employment contract*. Payments are given by

$$p_{i,t} = (1 - \delta)v \qquad \text{if } i \in \mathcal{I}_t, \tag{12}$$

$$p_{i,t} = 0 \qquad \text{if } i \notin \mathcal{I}_t.$$

The investment strategy $Q_{i,t}$ is then chosen to maximize time-0 profits (6).

PROPOSITION 5: *Suppose that costs are privately known by the principal. The employment contract* (*12*) *is an optimal ASE contract and is incentive compatible. Moreover*, *it is self-enforcing if*

$$W_{i,t} \geq v \qquad \textit{for all } i \in \mathcal{I}_t. \tag{DEP$^{\text{EC}}$}$$

The employment contract pays each insider $(1 - \delta)v$ per period, independent of whether trade occurs or not. That is, each "employee" submits to the authority of the principal, working when told, in exchange for a constant wage. The model thereby provides a theory of the firm, where both firm size and growth are determined endogenously.

The benefit of the employment contract is that the principal's payments to insiders are independent of her cost declarations. As a consequence, the principal's investment strategy is incentive compatible.

The cost of the employment contract is that the principal's dynamic enforcement constraint (DEP$^{\text{EC}}$) is stricter than under the fastest price contract (DEP). Under the employment contract, an insider receives utility $U_{i,t} = v$ whether or not he trades; under the fastest price contract, an insider receives $U_{i,t} = v$ when he trades and strictly less in other periods. When costs are i.i.d., the future profit from the relationship with agent i does not depend on whether they are trading this period. As a result, the optimal ASE contract is implementable under (DEP$^{\text{EC}}$) if and only if it is implementable under (DEP).[11] This is not the case, however, for other cost structures: if costs follow a Markov chain, then the optimal ASE contract may satisfy (DEP) but not (DEP$^{\text{EC}}$).[12]

[11] PROOF: Suppose $\mathcal{I}_t$ are the insiders under an optimal ASE contract and (DEP) holds. If $c_{i,t} = v \leq c_{j,t}$ for $i \in \mathcal{I}_t$ and $j \neq i$, the contract specifies $Q_{i,t} = 1$. Then, (DEP) implies that $v \leq W_{i,t} = \delta E_t[W_{i,t+1}]$ and, by the symmetry of the optimal ASE contract, $v \leq \delta E_t[W_{j,t+1}]$ for all $j \in \mathcal{I}_t$, implying (DEP$^{\text{EC}}$). For the converse, (DEP$^{\text{EC}}$) clearly implies (DEP).

[12] For example, suppose $N = 1$ and the firm starts with cost $c_t = 0$, which transitions into the absorbing state $c_t = \infty$ with positive probability each period. No trade is possible under (DEP$^{\text{EC}}$), but the optimal ASE contract will satisfy (DEP) if δ is sufficiently high.

Appendix

PROOF OF PROPOSITION 1:

(i) Fix t and suppose that $i \in \mathcal{I}_t$ obeys (7). By the principle of optimality, the principal wishes to maximize

$$\Pi_t = E_t\left[\sum_{s=t}^{\infty}\sum_i \delta^{s-t}(v - c_{i,s})Q_{i,s}\right] - E_t\left[\sum_{k\notin\mathcal{I}_t} \delta^{\tau_k(t)-t} v\right]. \tag{13}$$

Using (7), the principal prefers trading to both abstaining or trading with another agent, j.

(ii) Fix t and suppose $i \notin \mathcal{I}_t$ obeys (8A). Fix an allocation $\langle Q_{i,t}\rangle$ such that the principal trades with agent i in period t. Using (13), profit is given by

$$\begin{aligned}
\Pi_t(i) &= -c_{i,t} + E_t\left[\sum_{s=t+1}^{\infty}\sum_i \delta^{s-t}(v - c_{i,s})Q_{i,s}\right] - E_t\left[\sum_{k\notin\mathcal{I}_t\cup\{i\}} \delta^{\tau_k(t+1)-t} v\right] \\
&< -\delta v + E_t\left[\sum_{s=t+1}^{\infty}\sum_i \delta^{s-t}(v - c_{i,s})Q_{i,s}\right] - E_t\left[\sum_{k\notin\mathcal{I}_t\cup\{i\}} \delta^{\tau_k(t+1)-t} v\right] \\
&\leq 0 + E_t\left[\sum_{s=t+1}^{\infty}\sum_i \delta^{s-t}(v - c_{i,s})Q_{i,s}\right] - E_t\left[\sum_{k\notin\mathcal{I}_t} \delta^{\tau_k(t+1)-t} v\right] = \Pi_t(\emptyset),
\end{aligned}$$

where the first inequality comes from (8A), and $\Pi_t(\emptyset)$ is the profit obtained by following the original plan, but abstaining from trade at period t. The proof for (8B) is analogous: in this case, trade with agent i is dominated by trade with agent j.

PROOF OF PROPOSITION 2:

Suppose at time t the principal uses agent 2, and let $\langle Q_{i,t}, p_t\rangle$ be the optimal relational contract. We are now going to swap agents 1 and 2. First, define an underlying state $\omega_{i,s}$ which is i.i.d. and uniformly distributed on $[0, 1]$, and let the cost be $c_{i,s} = F_i^{-1}(\omega_{i,s})$, where F_i is the distribution function of i's costs.

Define the new contract, $\langle \tilde{Q}_{i,t}, p_t\rangle$, as follows. In period t, the principal uses agent 1. In period $s > t$, we swap the roles of agents 1 and 2, so that we treat agent 1 (resp. 2) in state $\{\omega_{1,s}, \omega_{2,s}, \omega_{3,s}, \ldots, \omega_{N,s}\}$ exactly as the old contract treated agent 2 (resp. 1) in state $\{\omega_{2,s}, \omega_{1,s}, \omega_{3,s}, \ldots, \omega_{N,s}\}$.

This new contract satisfies (DEA) since agent 2 is in exactly the same position as agent 1 (and vice versa). It also yields the same rents as the old contract, since the prices are the same. Insiders are used efficiently, so $\langle \tilde{Q}_{i,t}, p_t\rangle$ uses agent 1 more than $\langle Q_{i,t}, p_t\rangle$. Since $c_2 \geq_{\text{FSD}} c_1$, $F_2^{-1}(\omega) \geq F_1^{-1}(\omega)$ for all ω, and welfare is higher under the new contract. Hence, profits are also higher.

PROOF OF PROPOSITION 4:

We seek to show that, for δ sufficiently high, (DEP) is slack under the optimal contract. If $Q_{i,t} = 1$, the profit from relationship i is

$$\Pi_{i,t} = -c_{i,t} + \delta E_t[W_{i,t+1}] \geq -v + \frac{1}{n^*}\frac{\delta}{1-\delta}\,\omega(n^*), \tag{14}$$

where n^* is the maximum number of agents, and $\omega(n) := E[\max\{v - c_{1:n}, 0\}]$ is welfare when using n agents. Let us define $\omega(0) := 0$. Two facts are worth noting. First, $\omega(n)$ is increasing in n and converges to $v - \underline{c}$. Second, the marginal welfare of an extra agent, $\Delta\omega(n) := \omega(n) - \omega(n-1)$, decreases in n and converges to zero. The proof rests on two lemmas.

LEMMA 2: *For any infinite, increasing sequence of integers* $\{n_i\}$,

$$\sum_{i\geq 1}\frac{n_i - n_{i-1}}{n_i} = \infty.$$

PROOF:

Pick an infinite subsequence of the integers $\{m_j\}$ as follows. Let $m_1 := n_1$ and $m_j = \min\{n_i\colon n_i \geq 2m_{j-1}\}$. Then,

$$\sum_{i\geq 1}\frac{n_i - n_{i-1}}{n_i} = \sum_j \sum_{\{i: m_j \geq n_i > m_{j-1}\}} \frac{n_i - n_{i-1}}{n_i}$$

$$\geq \sum_j \sum_{\{i: m_j \geq n_i > m_{j-1}\}} \frac{n_i - n_{i-1}}{m_j} = \sum_j \frac{m_j - m_{j-1}}{m_j}.$$

By construction, $\frac{m_j - m_{j-1}}{m_j} \geq ½$. Hence, the sum is infinite.

LEMMA 3: $n\Delta\omega(n) \to 0$ *as* $n \to \infty$.

PROOF:

Since $\Delta\omega(n) > 0$ $(\forall n)$, $\liminf n\Delta\omega(n) \geq 0$. By contradiction, suppose that $\limsup n\Delta\omega(n) = k > 0$. Then, there exists a subsequence of integers $\{n_i\}$ such that

$$n_i\Delta\omega(n_i) \geq k - \epsilon > 0 \qquad (\forall i). \tag{15}$$

Abusing notation, let $n_i(n) = \min\{n_i\colon n_i \geq n\}$ be the next integer in the subsequence after an arbitrary integer n. We now obtain the following contradiction:

$$v - \underline{c} = \sum_{n\geq 1}\Delta\omega(n) \geq \sum_{n\geq 1}\Delta\omega(n_i(n)) = \sum_{i\geq 1}(n_i - n_{i-1})\Delta\omega(n_i)$$

$$\geq (k - \epsilon)\sum_{i\geq 1}\frac{n_i - n_{i-1}}{n_i} = \infty.$$

The first line follows from $\omega(0) = 0$ and $\lim \omega(n) = v - \underline{c}$. The second uses the fact that $\Delta\omega(n)$ is decreasing. The fourth line uses (15), while the fifth line follows from Lemma 2.

We can now complete the proof of Proposition 4. Using equation (9), we can place an upper bound on the number of insiders, n^*. Substituting (9) into (14),

$$\Pi_{i,t} \geq -v + \frac{\omega(n^*)}{n^*\Delta\omega(n^*)}\left[\max\{v - \bar{c}, 0\} + \underline{c}\right]. \tag{16}$$

Since either $\underline{c} > 0$ or $v > \bar{c}$, then $\max\{v - \bar{c}, 0\} + \underline{c} > 0$. As $\delta \to 1$, so $n^*(\delta)$ increases monotonically without bound. Thus, $\omega(n^*) \to v - \underline{c}$ and, by Lemma 3, $n^*[\Delta\omega(n^*)] \to 0$. Hence, $\Pi_{i,t} \to \infty$, as required.

PROOF OF PROPOSITION 5:

First, we show the contract is ASE. Under the employment contract, if $Q_{i,t} = 1$, agent i anticipates future rents v. Hence, (DEA) holds, and the agent has no incentive to defect.

Second, we show incentive compatibility. At time t, the principal's profit is

$$\Pi_t = E_t\left[\sum_{s=t}^{\infty}\sum_i \delta^{s-t}(v - c_{i,s})Q_{i,s}\right] - E_t\left[\sum_{i\notin\mathcal{I}_t} \delta^{\tau_i(t)-t}v\right] - \sum_{i\in\mathcal{I}_t} v. \tag{17}$$

The last term in (17) is sunk and can be ignored. Hence, the optimal time-0 investment plan, which maximizes (13), also maximizes (17). The principal thus cannot gain by lying about her costs, altering the investment plan.

Third, the contract is self-enforcing if the principal has no incentive to deviate. Fix $t \geq \tau_i(0)$. If $Q_{i,t} = 1$, the principal must wish to invest in the agent. If $Q_{i,t} = 0$, the principal must wish to pay the agent $(1 - \delta)v$. Since $U_{i,t} = v$ for all $t \geq \tau_i(0)$, we require $\Pi_{i,t} = W_{i,t} - v \geq 0$ for all $t \geq \tau_i(0)$, which can be rewritten as (DEP^{EC}).

REFERENCES

Asanuma, Banri. 1989. "Manufacturer-Supplier Relationships in Japan and the Concept of Relation-Specific Skill." *Journal of the Japanese and International Economies*, 3(1): 1–30.

Board, Simon. 2009. "Relational Contracts and the Value of Loyalty." Unpublished.

Calzolari, Giacomo, and Giancarlo Spagnolo. 2009. "Relational Contracts and Competitive Screening." Unpublished.

Chassang, Sylvain. 2010. "Building Routines: Learning, Cooperation, and the Dynamics of Incomplete Relational Contracts." *American Economic Review*, 100(1): 448–65.

Chatain, Olivier. 2009. "Where Do Client-Specific Scope Economies Come From? A Revealed Preference Analysis." Unpublished.

Chatain, Olivier. 2010. "Value Creation, Competition and Performance in Buyer-Supplier Relationships." *Strategic Management Journal*, 32(1): 76–102.

Johnson, Simon, John McMillan, and Christopher Woodruff. 2002. "Courts and Relational Contracts." *Journal of Law, Economics, and Organization*, 18(1): 221–77.

Kaufmann, Patrick, and Francine Lafontaine. 1994. "Costs of Control: The Source of Economic Rents for McDonald's Franchisees." *Journal of Law and Economics*, 37(2): 417–53.

Kelman, Steve. 1990. *Procurement and Public Management*. Washington, DC: American Enterprise Institute Press.

Levin, Jonathan. 2002. "Multilateral Contracting and the Employment Relationship." *Quarterly Journal of Economics*, 117(3): 1075–1103.

Liker, **Jeffrey**, **and Thomas Choi.** 2004. "Building Deep Supplier Relationships." *Harvard Business Review*, 82(12), 104–13.

Macaulay, **Stewart.** 1963. "Non-Contractual Relations in Business: A Preliminary Study." *American Sociological Review*, 28(1), 55–67.

MacLeod, **Bentley**, **and James Malcomson.** 1989. "Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment." *Econometrica*, 57(2): 447–80.

McAdams, **David.** 2010. "Performance and Turnover in a Stochastic Partnership." Unpublished.

McMillan, **John**, **and Christopher Woodruff.** 1999. "Dispute Prevention without Courts in Vietnam." *Journal of Law, Economics, and Organization*, 15(3): 637–58.

Milgrom, **Paul**, **and John Roberts.** 1997. "Johnson Controls, Inc.: Automotive Systems Group, The Georgetown, KY Plant." Stanford Graduate School of Business Case S-BE-9.

Roberts, **John.** 2004. *The Modern Firm: Organizational Design for Performance and Growth*. New York: Oxford University Press.

Sako, **Mari.** 1996. "Suppliers' Associations in the Japanese Automobile Industry: Collective Action for Technology Diffusion." *Cambridge Journal of Economics*, 20(6): 651–71.

Taylor, **Curtis**, **and Steven Wiggins.** 1997. "Competition or Compensation: Supplier Incentives under the American and Japanese Subcontracting Systems." *American Economic Review*, 87(4): 598–618.

Thomas, **Jonathan**, **and Tim Worrall.** 1994. "Foreign Direct Investment and the Risk of Expropriation." *Review of Economic Studies*, 61(1): 81–108.