Durable–Goods Monopoly with Varying Demand

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Motivation

- Back to school sales
  - New influx of demand → reduce prices in September.
  - But causes people to delay purchase in August.
  - How much should reduce price?
- Pricing with varying demand
  - What happens if new demand falls over time?
  - What happens if new demand is uncertain?
- Objective: Derive optimal pricing strategy for durable–goods monopolist facing fluctuating demand from new cohorts.

Durable Goods Monopoly

- No new entry of consumers (Stokey, 1979)
  - Consumers enter market in period 1.
  - Firm choose prices $\{p_1, \ldots, p_T\}$.
  - Agents choose when to buy.
  - Solution: charge static monopoly price forever.

- Identical entry each period (Conlisk et al, 1984)
  - Solution: charge static monopoly price forever.

Varying Demand

- What if new demand varies over time?
  - Theory of *dynamic* pricing.
  - Scope for intertemporal price discrimination.
- Technique
  - Method to solve dynamic mechanism design problems.
  - Simple marginal revenue interpretation.
- Fast rises and slow falls
  - Demand growing $\Longrightarrow$ price increases quickly.
  - Demand dying $\Longrightarrow$ price decreases slowly.
- Application: propagation of demand cycles.
  - Prices exceed the average–demand price.
  - The lowest price is at last period of the "slump".

The Model

- Time is discrete, $t \in \{1, \dots, T\}$, where allow $T = \infty$.
  - Consumers' and firm's information represented by filtered space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, Q)$.
  - Common time–t discount rate, $\delta_t \in (\epsilon, 1-\epsilon)$, is $\mathcal{F}_t$–adapted.
  - Total discount factor $\Delta_t := \prod_{i=1}^{t} \delta_s$.
- Consider consumer with value $\theta \in [\underline{\theta}, \overline{\theta}]$
  - If buy at time t and price p_t get $(\theta - p_t)\Delta_t$.
  - If do not purchase get zero.
- Each period consumers of measure $f_t(\theta)$ enter market
  - Distribution function $F_t(\theta)$, survival function $\overline{F}_t(\theta)$.
  - Total measure $F_t(\overline{\theta})$.
  - New demand, $f_t(\theta)$, is $\mathcal{F}_t$–adapted.

Payoffs

- Consumer's Problem
  - Consider consumer (θ, t) with value θ who enters at time t.
  - Given sequence of $\mathcal{F}_t$–adapted prices $\{p_1, \dots, p_T\}$.
  - Choose purchasing time $\tau(\theta, t)$ to maximise expected utility.

$$u_t(\theta) = \mathcal{E}[(\theta - p_\tau)\Delta_\tau]$$

- Firm's Problem
  - Assume marginal cost is zero.
  - Choose $\mathcal{F}_t$–adapted prices $\{p_t\}$ to maximise expected profit

$$\Pi = \mathcal{E}\left[\sum_{t=1}^{T} \int_{\underline{\theta}}^{\overline{\theta}} \Delta_{\tau^*(\theta,t)} p_{\tau^*(\theta,t)} \, dF_t\right]$$

  where $\tau^*(\theta, t)$ maximises the consumer's utility, $u_t(\theta)$.
- Notable assumptions: No resale. Firm commits to prices.

Consumer Surplus and Welfare

- Purchase time optimal so use envelope theorem,

$$u_t(\theta) = \mathcal{E}\left[\int_{\underline{\theta}}^{\theta} \Delta_{\tau^*(x,t)}\, dx + u(\underline{\theta}, t)\right]$$

using Milgrom–Segal (2002) since space of stopping times complex.

- Consumer surplus from generation t,

$$\int_{\underline{\theta}}^{\overline{\theta}} u_t(\theta)\, dF_t = \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \Delta_{\tau^*(\theta,t)} \overline{F}_t(\theta)\, d\theta\right]$$

- Welfare from generation t,

$$W_t = \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \theta \Delta_{\tau^*(\theta,t)}\, dF_t\right]$$

Costs are zero so the welfare is maximised by setting $p_t = 0$.

Firm's Problem

- Define *marginal revenue* with respect to price as

$$m_t(\theta) := \theta f_t(\theta) - \overline{F}_t(\theta)$$

- Expected profit is welfare minus consumer surplus,

$$\Pi = \mathcal{E}\left[\sum_{t=1}^{T} \int_{\underline{\theta}}^{\overline{\theta}} \Delta_{\tau^*(\theta,t)} m_t(\theta)\, d\theta\right]$$

- Profit is discounted sum of marginal revenues.
- Marginal revenue sticks to each agent (θ, t).
- The *firm's problem* is to chooses prices $\{p_1, \ldots, p_T\}$ to maximise Π subject to $\tau^*(\theta, t)$ maximising $u_t(\theta)$.

Consumer's Problem and Cutoffs

Lemma 1. *The earliest purchasing rule, $\tau^*(\theta, t)$, obeys:*
[existence] $\tau^*(\theta, t)$ exists.
[θ–monotonicity] $\tau^*(\theta, t)$ is decreasing in θ.
[non–discrimination] $\tau^*(\theta, t_L) \geq t_H \Longrightarrow \tau^*(\theta, t_L) = \tau^*(\theta, t_H)$, for $t_H \geq t_L$.

- Characterise $\tau^*(\theta, t)$ by $\mathcal{F}_t$–adapted cutoffs

$$\theta_t^* := \inf\{\theta : \tau^*(\theta, t) = t\}$$

- Back out prices from cutoffs:

$$(\theta_t^* - p_t^*)\Delta_t = \max_{\tau \geq t+1} \mathcal{E}\left[(\theta_t^* - p_\tau^*)\Delta_\tau\right]$$

General Solution

Definition. *Cumulative marginal revenue* equals $M_1(\theta) := m_1(\theta)$ and $M_t(\theta) := m_t(\theta) + \min\{M_{t-1}(\theta), 0\}$.

Assumption (MON). $M_t(\theta)$ is quasi–increasing ($\forall t$).

Theorem 1. *Under (MON) the optimal cutoffs are $\theta_t^* = M_t^{-1}(0)$.*

- Period $t = 1$
  - Sell to agent θ iff $m_1(\theta) \geq 0$
- Period $t = 2$
  - Form cumulative MR, $M_2(\theta) = m_2(\theta) + \min\{M_{t-1}(\theta), 0\}$
  - Sell to agent θ iff $M_2(\theta) \geq 0$
- Cutoffs are determined by past demand.
- Prices are determined by future cutoffs.

(1) Monotone Deterministic Demand

- Suppose demand deterministic.

Proposition 2a. *Suppose demand is increasing,* $m_{t+1}^{-1}(0) \geq m_t^{-1}(0)$*. Then* $\theta_t^* = m_t^{-1}(0)$ *and prices are* $p_t^* = m_t^{-1}(0)$*.*

Proposition 2b. *Suppose demand is decreasing,* $m_{t+1}^{-1}(0) \leq m_t^{-1}(0)$*. Then* $\theta_t^* = m_{\leq t}^{-1}(0)$ *and prices are*

$$p_t^* = \sum_{s=t}^{T} \mathcal{E}\left[\left(\frac{\Delta_s}{\Delta_t} - \frac{\Delta_{s+1}}{\Delta_t} \right) m_{\leq s}^{-1}(0) \,\middle|\, \mathcal{F}_t \right]$$

- Myopic price: $p_t^M := m_t^{-1}(0)$.
- Average–Demand price: $p_t^A := m_{\leq T}^{-1}(0)$

(2) Deterministic Cycles

- Suppose demand follows K repetitions of $\{f_1(\theta), \ldots, f_T(\theta)\}$

Proposition 4. *For* $k \geq 2$*, cycles are stationary.*

Proposition 5. *For* $k \geq 2$*, optimal prices always lie above the average–demand price,* $m_{\leq T}^{-1}(0)$*.*

- Price discrimination bad for *all* customers.

Proposition 6. *For* $k \geq 2$*, if cycles are simple the price is lowest at the end of the slump.*

(3) IID Demand

- Demand drawn from $\{m_i(\theta)\}$ with prob $\{q_i\}$.
  - Average marginal revenue $m^A(\theta) = \sum_i q_i m_i(\theta)$.
  - Average–demand price $p^A := [m^A]^{-1}(0)$.

Proposition 7. *The SLLN implies* $\lim_{t\to\infty} \theta_t^* \geq p^A$ *and* $\lim_{t\to\infty} p_t^* \geq p^A$ *a.s..*

- Stochastic equivalent of Proposition 5 (i.e. with deterministic cycles, prices exceed the average–demand price).

Summary

- Derived optimal pricing strategy for durable–goods monopolist facing varying demand.
- Award good to agents with positive cumulative MR.
- Prices rise quickly and fall slowly.
- Asymmetry pushes prices upwards.