Durable–Goods Monopoly with Varying Demand

Simon Board*

This Version: May 27, 2005
First Version: June 2003

Abstract

This paper solves for the profit maximising strategy of a durable–goods monopolist when incoming demand varies over time. Each period, additional consumers enter the market; these consumers can then choose whether and when to purchase. We first characterise the consumer's utility maximisation problem and, under a monotonicity condition, show the profit maximising allocation can be solved through a myopic algorithm, which has an intuitive marginal revenue interpretation. Consumers' ability to delay creates an asymmetry in the optimal price path, which exhibits fast increases and slow declines. This asymmetry pushes the price level above that charged by a firm facing the average level of demand. Applications of this framework include deterministic demand cycles, one–off shocks and IID demand draws. The optimal policy outperforms renting and can be implemented by a time consistent best–price provision.

1 Introduction

In her classic 1979 paper, Nancy Stokey asks whether a firm that faces a fixed set of consumers can use time to discriminate between them. By lowering its price over time, the firm can increase its sales to low valuation consumers. Such a strategy, however, will lead some high valuation consumers to postpone their purchases. Stokey concludes that the profit maximising strategy is to forgo the opportunity to discriminate, setting the price equal to the static monopoly price and holding it there forever. Sales only occur in the first period and no consumer ever delays.

*Department of Economics, University of Toronto. Address: 150 St. George St., Toronto, ON, M5S 3G7, Canada. http://www.economics.utoronto.ca/board. I am indebted to my advisor, Jeremy Bulow, for his guidance and enthusiasm. I have received many helpful comments from David Ahn, David Baron, Willie Fuchs, Ilan Kremer, John McMillan, Robert McMillan, Barry Nalebuff, Carolyn Pitchik, Ilya Segal, Andy Skrzypacz and Jeff Zwiebel. I am especially grateful to Richard Saouma for initiating my interest in sales. I also wish to thank the seminar audiences at Cambridge, Essex, LSE, Northwestern, NYU, Oxford, Penn, Rochester, UC San Diego, Stanford, UT Austin, Toronto and Yale. An earlier version of this paper went under the rather nebulous title of "Dynamic Monopoly". JEL: C73, D82, E32, L12. Keywords: durable goods, pricing, mechanism design, optimal stopping.

Relaxing the assumption that demand is given at time zero and never refreshed, Conlisk, Gerstner, and Sobel (1984) and Sobel (1991) allow a homogenous set of consumers to enter the market each period. They show that, once again, the profit maximising strategy is to set the price equal to the static monopoly price. On the equilibrium path, there are sales in each period, but no consumer ever delays.

The purpose of the current paper is to ask: what happens when incoming demand varies over time? We explore how time can be used to discriminate between different generations, how prices will vary over time, and how many consumers will delay on the equilibrium path.

Variation in demand arises naturally in many markets. For example, each September, thousands of students return to universities and colleges across Canada. Canadian Tire, a large retailer, responds to this influx of new consumers by holding a back–to–school sale, reducing prices on furniture, stationary and kitchen utensils. While the price reduction helps increase profits from price–sensitive students, it also induces consumers who would have bought in July and August to delay their purchases. When choosing their September price, Canadian Tire must therefore trade off these two effects. Anticipating the back–to–school sale, Canadian Tire must also consider how to alter prices in August in order to mitigate consumer delay. Furthermore, the firm must take into account the fact that any reduction in the August price will lead even more July consumers to postpone their purchases.

Demand for a product may change over its life cycle. The Rogers–Bass "innovation adoption curve" classifies the types of consumers who adopt a new product at different stages of the diffusion process. For example, innovators, the first group to adopt, are described as "venturesome and daring" (Bass (1969, p. 216)). This suggests that consumers who considered purchasing Sony's PS2 when it launched in 2000 have different demand characteristics from those consumers entering the market five years later. Demand may also be uncertain over time. When a record company releases a new CD, it may capture a mainstream audience (with elastic demand) or just appeal to the artist's core fans (with inelastic demand).

Allowing different generations to have different demand functions has the potential to significantly complicate the analysis of the problem. If Canadian Tire charges a very low price in September, then consumers in July and August will delay their consumption. From the profit maximisation problem, one can then derive optimal prices for July, August and September. Given these optimal prices, one must then verify that some consumers from July and August do indeed delay their purchases. Of course, we could equally well have initially assumed that the September price will only be low enough so that consumers born in August, but not those born in July, will delay. Since each generation has a different demand function, this will lead to a different set of optimal prices. One can then compare these cases and pick the one that maximises profits. One can also see that, in general, this approach is very unwieldy, exemplifying the problem of working in prices. In addition, this illustrates that when demand is changing over time, the most challenging part of the problem is to identify which generations purchase in which periods.

Rather than working in prices, this paper will solve the problem by working in quantity space. We first analyse each consumer's purchase decision as an optimal stopping problem. Using this formulation, the consumers' purchasing rule is characterised by a sequence of cutoffs: at any time t, a consumer purchases if their valuation lies above the time–t cutoff. We then use mechanism design to eliminate prices and describe the firm's profit as a function of these cutoffs. This transformation allows us to reduce the firm's problem to choosing the sequence of cutoffs to maximise profit.

Under a monotonicity condition on marginal revenue, the profit–maximising cutoffs are characterised by a myopic algorithm, where the allocation at time t depends only upon the consumers who have entered the market up to time t. Consumers are forward–looking and prices depend upon the sequence of future demand; the optimal allocations, however, only depend upon past demand. The myopic nature of this algorithm enables us to analyse any stochastic sequence of demand functions: monotone demand paths, demand cycles, permanent or transitory demand shocks, IID demand draws, and so on. The optimal prices can then be derived from the consumers' utility maximisation problem.

The optimal myopic algorithm has an intuitive interpretation: The incoming demand curve in any period t can be associated with a marginal revenue curve, where marginal revenue is with respect to price, not quantity. In the first round, the firm sells the good to consumers with positive marginal revenue (net of costs). In each period thereafter, the firm adds the marginal revenue of the old consumers who have yet to buy to the marginal revenue of the new agents, forming a cumulative marginal revenue function. The firm then sells the good to agents with positive cumulative marginal revenue.

Consumers' ability to delay induces an asymmetry in the optimal price path. When demand grows stronger over time, in that valuations tend to rise, the firm will want to increase their price. Agents then have no incentive to delay and the firm can discriminate between the different generations, charging the monopoly price against the incoming generation—the *myopic price*.

When demand weakens over time, in that valuations tend to fall, the firm will want to decrease their price over time. Charging the myopic price, however, will now lead to falling prices, causing some consumers to delay their purchases. Anticipating this delay, the firm slows the rate at which prices fall. The price path is flattened so much that prices always stay above the price chosen by a firm who pools all generations together and prices against the average level of demand—the *average–demand price*.

The contrast between increases and decreases in demand is stark. If there is a permanent and unanticipated increase in demand, the price quickly jumps up to the new higher monopoly price. However, if demand falls, the price will jump down a little and slowly fall towards the lower monopoly price over time.

This asymmetry between demand increases and decreases crucially affects the firm's optimal pricing policy. When demand follows stationary cycles, it leads to sharp price increases and gentle declines. This is shown in *Figure 1*, where the lower panel describes which generations purchase in which periods.

During the first quarter of the cycle, as demand rises from its average level to the peak of the boom, the price rises quickly. For the other three–quarters of the cycle, as demand falls and then returns to its average position, the price slowly falls. These price cycles are stationary, showing no decline in amplitude no matter how long the market has existed. The price is minimised not in the period of lowest demand, but in the last period of the slump, just before demand returns to its average position. The asymmetry between increases and decreases in demand also raises the price level: price always exceeds the average–demand price. This means introducing variation in demand leads to an increase in all prices. In other words, when the firm has more information about demand cycles, *all* consumers are made worse off and social welfare is reduced.

The basic model makes two assumptions of note. First, there is no resale. There are many goods for which this is the right assumption: one–time experiences, (e.g. a trip to Disneyland), intermediate products (e.g. aluminium), regulated markets (e.g. plutonium), potential lemons (e.g. computers), goods with high transactions costs (e.g. fridges) or those with emotional attachment (e.g. diamonds). However such an assumption is not innocuous. With perfect resale, price variations decrease in amplitude as the market gets older; without resale, the market fluctuations never abate. With resale, the price responds symmetrically to changes in demand; without resale, price movements are highly asymmetric. Introducing resale has no effect on allocations if and only if new demand falls over time; otherwise the presence of resale lowers the monopolist's profits. Since renting is identical to commitment pricing with resale, renting attains maximal profits if and only if demand is declining.

Second, it is assumed that the monopolist can perfectly commit to a sequence of prices. With homogenous demand and no commitment, there are many equilibria ranging from the those which are very bad for the firm to others which are close to the full–commitment outcome, as examined by Conlisk, Gerstner, and Sobel (1984) and Sobel (1991). This paper should thus be viewed as establishing the best possible outcome for the firm. This seems particularly reasonable if the firm is concerned about its reputation across several durable–goods markets. In addition, there are contractual solutions to the commitment problem. We extend the result of Butz (1990) by showing that a best–price provision can implement the optimal scheme without pre–commitment. This time–consistency result, however, depends upon the absence of resale. With resale, we show that a best–price provision may not be time–consistent.

The precursors of this paper are the models of Stokey (1979), Conlisk, Gerstner, and Sobel

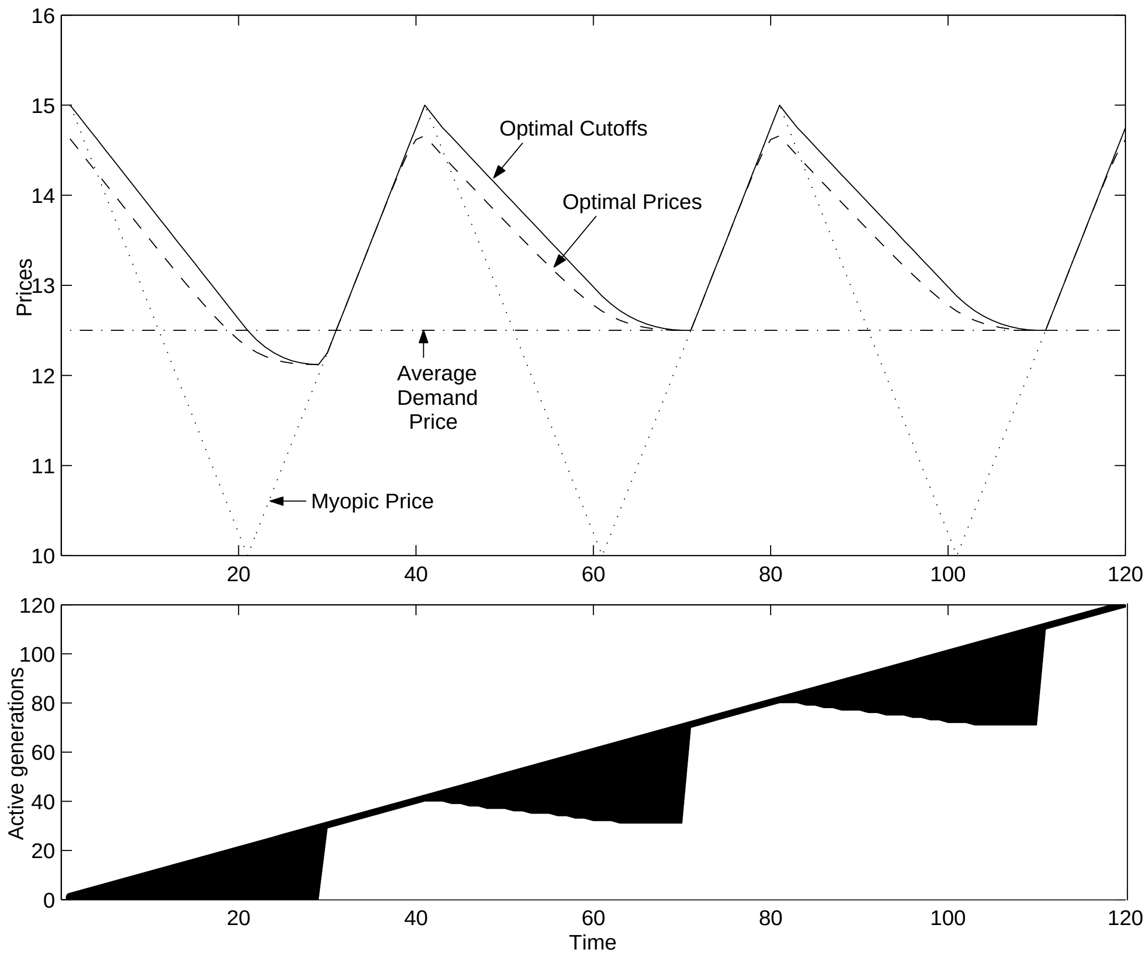

Figure 1: Price Cycles

(1984) and Sobel (1991), as examined in Section 3.2. Other authors introduce dynamics into durable goods models in different ways. Conlisk (1984), Laffont and Tirole (1996), Biehl (2001) and Board (2004) have stochastic valuations, cost variations have been analysed by Stokey (1979) and Levhari and Pindyck (1981), while Dudine, Hendel, and Lizzeri (2005) consider storable goods. When consumers and the firm have different discount rates, the optimal price may fall over time as examined by Sobel and Takahashi (1983), Landsberger and Meilijson (1985) and Wang (2001). There is also a line of work where the good depreciates over time and consumers are allowed to scrap the product, such as Rust (1985, 1986), Waldman (1996) and Hendel and Lizzeri (1999).

The current paper solves for the optimal price sequence of a durable–goods monopolist, but one can also look at more general mechanisms. For example, Segal (2003) shows that, without the entry of new generations, the optimal mechanism in a durable goods model can be implemented by a price mechanism if marginal costs are constant or there are large numbers of consumers. Both of these assumptions are satisfied in our model.

The paper is organised as follows: Section 2 describes the model. Section 3 derives the firm's optimal control problem which is solved in Section 4. Section 5 discusses applications including monotone demand paths, one–off shocks, demand cycles and IID demand. Section 6 examines the effect of information structures and discount rates. Section 7 analyses resale and renting, while Section 8 concludes. Omitted proofs are contained in the Appendix.

2 Model

Time is discrete, $t \in \{1, \ldots, T\}$, where we allow $T = \infty$. Demand and the discount rate are allowed to be uncertain, depending on the state of the world $\omega \in \Omega$. The information possessed by consumers and the firm is described by a filtered space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, Q)$, where $\mathcal{F}$ are the measurable sets, $\mathcal{F}_t$ is the information partition at time t, which grows finer over time, and Q is the probability measure. The common discount rate, $\delta_t \in [0, \overline{\delta}] \subset [0, 1)$, is $\mathcal{F}_t$–adapted, i.e. $\{\omega : \delta_t \leq x\} \in \mathcal{F}_t$ for all $x \in (0, 1)$. This means that at time t, consumers and the firm know δ_t. Let the discounting up to time t be $\Delta_t = \prod_{s=1}^{t} \delta_s$, where $\Delta_t := 0$ for $t \geq T + 1$.

A consumer with valuation $\theta \in [\underline{\theta}, \overline{\theta}]$ who purchases at time $t \in \{1, \ldots, T, \infty\}$ and price p_t obtains utility

$$(\theta - p_t)\Delta_t$$

A consumer always has the option not to purchase, in which case they have zero utility and are said to buy at time $t = \infty$.

Each period, consumers of measure $f_t(\theta)$ enter the market. Let $F_t(\theta)$ be the absolutely continuous distribution function, where $F_t(\overline{\theta})$ is the total number of agents (and not necessarily equal to one), and denote the survival function by $\overline{F}_t(\theta) := F_t(\overline{\theta}) - F_t(\theta)$. The time–$t$ demand function, $f_t(\theta)$, is $\mathcal{F}_t$–adapted, so while consumers and the firm may not know future demand, they do know current demand. This assumption is trivially satisfied if demand is deterministic.

Consider a consumer of type (θ, t) with valuation θ who enters in period t. Given a sequence of $\mathcal{F}_t$–adapted prices, $\{p_t\}$, they have the problem of choosing a purchasing time $\tau(\theta, t) : \Omega \to \{t, \ldots, T, \infty\}$ to maximise expected utility,

$$u_t(\theta) = \sup_{\tau} \mathcal{E}[(\theta - p_\tau)\Delta_\tau] \tag{2.1}$$

where "$\mathcal{E}$" is the expectation over Ω. The purchasing time is a random variable taking values in $\{t, \ldots, T, \infty\}$, where the decision to buy at time s depends on information available at time s, i.e. $\{\omega : \tau(\theta, t) \le s\} \in \mathcal{F}_s$ $(\forall s)$. Let $\tau^*(\theta, t)$ be the earliest solution to this problem, which exists by Lemma 1 in Section 3.3.

The firm's problem is to choose $\mathcal{F}_t$–adapted prices $\{p_t\}$ to maximise profit. Assume marginal cost is constant and normalise it to zero. This yields expected profit

$$\Pi = \mathcal{E}\left[\sum_{t=1}^{T} \int_{\underline{\theta}}^{\overline{\theta}} \Delta_{\tau^*(\theta,t)} p_{\tau^*(\theta,t)} \, dF_t\right] \tag{2.2}$$

where $\tau^*(\theta, t)$ maximises the consumer's utility (2.1).

3 Solution Technique

3.1 Firm's Problem

Consumers choose their purchase times optimally, so we can apply the envelope theorem to the utility maximisation problem (2.1). The space of stopping times is complicated so we will use the generalised envelope theorem of Milgrom and Segal (2002).[1] This yields utility,

$$u_t(\theta) = \mathcal{E}\left[\int_{\underline{\theta}}^{\theta} \Delta_{\tau^*(x,t)} \, dx + u_t(\underline{\theta})\right] \tag{3.1}$$

Since the seller will always choose prices $p_t \ge \underline{\theta}$ $(\forall t)$, it will be the case that $u_t(\underline{\theta}) = 0$ $(\forall t)$. Integrating by parts, *consumer surplus* from generation t is

$$\int_{\underline{\theta}}^{\overline{\theta}} u_t(\theta) \, dF_t = \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \Delta_{\tau^*(\theta,t)} \overline{F}_t(\theta) \, d\theta\right] \tag{3.2}$$

[1] This envelope theorem requires that the set of optimal purchasing times is nonempty, which is true by Lemma 1 in Section 3.3.

Expected *welfare* from generation t is defined by

$$W_t := \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \Delta_{\tau^*(\theta,t)}\theta \, dF_t\right] \tag{3.3}$$

with total welfare $W = \sum_t W_t$. Since costs are zero, the welfare maximising pricing scheme is to set all prices equal to zero. Expected profit equals welfare (3.3) minus consumer surplus (3.2),

$$\begin{aligned}\Pi &= \mathcal{E}\left[\sum_{t=1}^{T}\int_{\underline{\theta}}^{\overline{\theta}} \left[\Delta_{\tau^*(\theta,t)}\theta - u_t(\theta)\right] \, dF_t\right] \\ &= \mathcal{E}\left[\sum_{t=1}^{T}\int_{\underline{\theta}}^{\overline{\theta}} \Delta_{\tau^*(\theta,t)} m_t(\theta) \, d\theta\right] \end{aligned} \tag{3.4}$$

where $m_t(\theta) := \theta f_t(\theta) - \overline{F}_t(\theta)$ is *marginal revenue* with respect to price.[2]

Profit is thus the discounted sum of marginal revenues. Notice how the marginal revenue gained from agent (θ, t) is the same no matter when they choose to buy. That is, an agent's marginal revenue is determined when they are born, and sticks to them forever.

The *firm's problem* is to choose prices $\{p_t\}$ to maximise profit (3.4) subject to consumers choosing their purchasing time $\tau^*(\theta, t)$ to maximise utility (2.1).

3.2 Special Cases

We now consider three special cases that will serve as useful benchmarks for what follows. Example 1 comes from Stokey (1979), where there is a single demand curve of consumers and entry never occurs. Example 2 supposes the monopolist can set a different price schedule for each generation. Example 3 is the homogenous entry model of Conlisk, Gerstner, and Sobel (1984).

Example 1 (Single Generation). If $f_t(\theta) = 0$ for $t \geq 2$, then profit (3.4) reduces to

$$\Pi = \int_{\underline{\theta}}^{\overline{\theta}} \Delta_{\tau^*(\theta,t)} m_1(\theta) \, d\theta \tag{3.5}$$

[2] It is more common to use marginal revenue with respect to quantity, $MR_t(\theta) := m_t(\theta)/f_t(\theta)$. However, since we will be adding demand across generations, summing demand curves horizontally, it is easier to work with marginal revenue with respect to price. In any case, both $m_t(\theta)$ and $MR_t(\theta)$ have the same roots, and are thus interchangeable in pricing formulae.

To maximise (3.5) the firm would like to set purchasing times as follows:

$$\begin{aligned}\tau^*(\theta,1) &= 1 \qquad \text{if} \quad m_1(\theta)\geq 0\\ \tau^*(\theta,1) &= \infty \qquad \text{if} \quad m_1(\theta)<0\end{aligned}$$

That is, the firm would like positive marginal revenue consumers to purchase immediately, and the rest to never buy. If $m_1(\theta)$ is increasing this optimal policy can be implemented by setting

$$\begin{aligned}p_1^* &= m_1^{-1}(0)\\ p_t^* &\geq m_1^{-1}(0) \quad \text{if} \quad t\geq 2\end{aligned}$$

Since the price is increasing, consumers buy in period 1 or never buy at all. A consumer then buys in period 1 if and only if $\theta \geq m_1^{-1}(0)$, which is the same as $m_1(\theta)\geq 0$. △

Example 2 (Discrimination between Generations). Next, suppose the firm could tell the different cohorts apart and set a price p_t^s for generation s in time t. The firm would then implement Stokey's solution for each cohort. That is,

$$\begin{aligned}p_t^s &= m_s^{-1}(0) \quad \text{if} \quad t=s\\ p_t^s &\geq m_s^{-1}(0) \quad \text{if} \quad t\geq s+1\end{aligned}$$

Hence if the myopic monopoly price grows over time, $m_t^{-1}(0)\geq m_{t-1}^{-1}(0)$, the seller can simply charge the myopic price, $p_t^*=m_t^{-1}(0)$, and need not discriminate. △

Example 3 (Homogenous Demand). Finally, suppose demand is identical in each period, $f_t(\theta)=f_0(\theta)$ $(\forall t)$. The firm can implement the discriminatory optimum from Example 2 by setting $p_t^*=m_0^{-1}(0)$ $(\forall t)$. △

3.3 The Cutoff Approach

As suggested by Examples 1–3, rather than solving for prices directly, it is easier to solve for the optimal purchasing rule and back out prices. This approach works since prices $\{p_t\}$ only enter into profits (3.4) via the purchasing rule $\tau^*(\theta,t)$—a standard feature of quasi–linear mechanism design problems. This is analogous to solving a standard monopoly model in quantities and using the demand curve to derive prices.

Lemma 1. *The earliest purchasing rule, $\tau^*(\theta,t)$, has the following properties:*
[existence] $\tau^*(\theta,t)$ exists.
[θ–monotonicity] $\tau^*(\theta,t)$ is decreasing in θ.

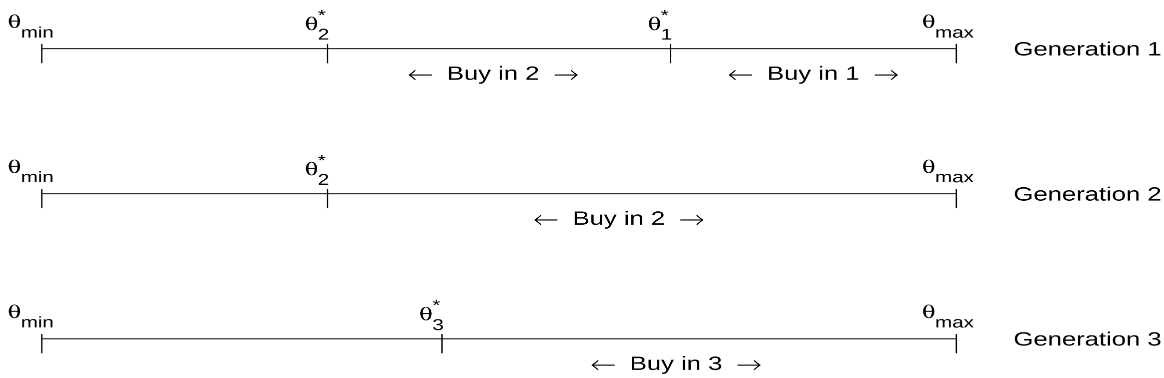

Figure 2: Cutoffs

[*non–discrimination*] *If* $\tau^*(\theta, t_L) \geq t_H$ *then* $\tau^*(\theta, t_L) = \tau^*(\theta, t_H)$, *for* $t_H \geq t_L$.
[*right–continuity*] $\{\theta : \tau^*(\theta, t_L) \leq t_H\}$ *is closed, for* $t_H \geq t_L$.

Proof. [existence], [θ–monotonicity], [non–discrimination] follow from Lemma 4 in Appendix A.1, which describes properties of the set of optimal stopping rules. These properties also apply to the least element by Topkis (1998, Theorem 2.4.3). [right–continuity] follows from the continuity of $u_t(\theta)$ in θ. $\square$

Lemma 1 implies the optimal stopping rule $\tau^*(\theta, t)$ can be characterised by a sequence of cutoffs. The *cutoff* θ_t^* is the lowest value agent from generation t that purchases in period t,

$$\theta_t^* := \min\{\theta : \tau^*(\theta, t) = t\}.$$

That is, consumers in the market in period t will buy if their valuation exceeds θ_t^*. If demand is uncertain, these cutoffs are $\mathcal{F}_t$–adapted random variables. For generation $t \leq t'$ the *updated cutoff*, $\theta^*(t'; t)$, is the lowest value agent from generation t who buys by time t',

$$\theta^*(t'; t) := \min_{t \geq s \geq t'} \theta_s^* \tag{3.6}$$

If $t' < t$ then set $\theta^*(t'; t) = \infty$. Consumer θ from generation t will then buy in period t' if

$$\theta \in [\theta^*(t'; t), \theta^*(t' - 1; t))$$

A simple three–period example is shown in *Figure 2*, where we suppose $\theta_1^* > \theta_3^* > \theta_2^*$.

The *firm's problem* is then to choose cutoffs $\{\theta_t^*\}$ to maximise profit (3.4).

We have shown that we can move from prices to cutoffs. The reverse is also true: prices can be backed out from any sequence of cutoffs. First suppose T is finite. In the last period, an agent with value θ_T^* is indifferent between buying and not, so the firm sets $p_T^* = \theta_T^*$. In earlier periods an agent with value θ_t^* should be indifferent between buying in period t and waiting. Hence prices are determined by the following algorithm:[3]

$$\begin{aligned} \Delta_t(\theta_t^* - p_t^*) &= \max_{\tau \geq t+1} \mathcal{E}\left[(\theta_t^* - p_\tau^*)\Delta_\tau | \mathcal{F}_t\right] \\ &= \mathcal{E}\left[(\theta_t^* - p^*_{\tau(\theta_t^*, t+1)})\Delta_{\tau(\theta_t^*, t+1)} | \mathcal{F}_t\right] \end{aligned} \tag{3.7}$$

where $\tau(\theta_t^*, t+1) = \min\{\tau \geq t+1 : \theta_t^* \geq \theta_\tau^*\}$. When T is infinite, there is no last period, but equation (3.7) remains valid. One can then calculate prices by truncating the problem, calculating prices for a finite T and letting $T \to \infty$, as shown by Chow, Robbins, and Siegmund (1971, Theorems 4.1 and 4.3).

4 Optimal Pricing

4.1 Ordering Demand Functions

It will be useful to consider a method to rank demand curves. Period H is said to have *higher demand* than period L if $m_H^{-1}(0) \geq m_L^{-1}(0)$, so the optimal static monopoly price is higher under $F_H(\theta)$ than $F_L(\theta)$. There may, however, be more people under the "low" demand, i.e. $F_H(\overline{\theta}) \leq F_L(\overline{\theta})$, as is the case in the "back–to–school" example in the Introduction.

A sufficient condition for period H to have higher demand than period L is that $F_H(\theta)$ is larger than $F_L(\theta)$ in hazard order, $\overline{F}_H(\theta)/f_H(\theta) \geq \overline{F}_L(\theta)/f_L(\theta)$. Suppose θ_L is distributed according to $F_L(\theta)$, which is log–concave. Then $F_H(\theta)$ is larger than $F_L(\theta)$ in hazard order, and consequently $m_H^{-1}(0) \geq m_L^{-1}(0)$, in the following examples:

(a) Upwards Shift. $\theta_H := \theta_L + \epsilon$ for some constant $\epsilon > 0$.

(b) Upwards Pivot. $\theta_H := \alpha\theta_L$ for $\alpha > 1$.

(c) Outwards Shift. $f_H(\theta) := \alpha f_L(\theta/\alpha)$ for $\alpha > 1$.

Let the marginal revenue from a set of generations $A \subset \{1, \dots, T\}$ be denoted $m_A(\theta) := \sum_{s \in A} m_s(\theta)$. Similarly, let $m_{\leq t}(\theta) := \sum_{k \leq t} m_k(\theta)$ be total marginal revenue of consumers who have entered the market by time t.

[3] If the cutoffs lie in $(\underline{\theta}, \overline{\theta})$ these prices are unique. An equivalent way to obtain prices is to equate (2.1) and (3.1) under the optimal purchasing time $\tau^*(\theta, t)$.

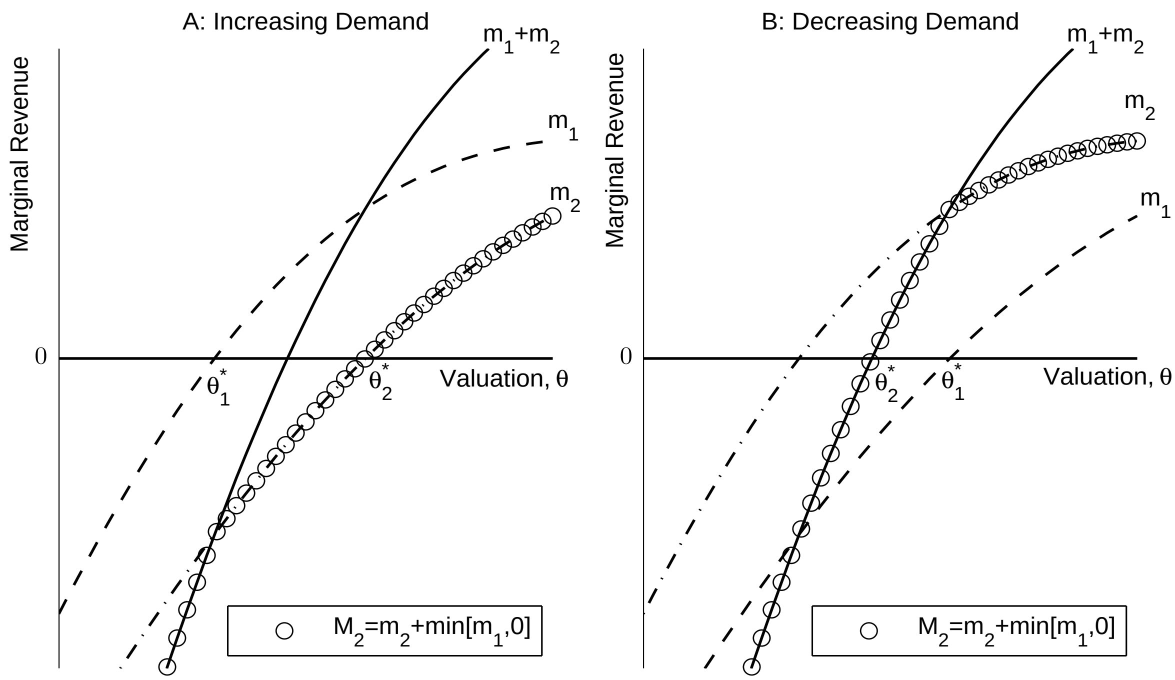

Figure 3: Two–Period Example

4.2 Deterministic Two–Period Model

To gain some intuition behind the solution, consider two–period model, where demand is deterministic and the discount rate δ is constant. In this case, profit (3.4) reduces to

$$\Pi = \int_{\theta_1^*}^{\overline{\theta}} m_1(\theta)\, d\theta + \delta \int_{\theta_2^*}^{\overline{\theta}} \Big[m_2(\theta) + 1_{\{\theta<\theta_1^*\}} m_1(\theta) \Big]\, d\theta$$

Assume that the marginal revenue functions, $m_t(\theta)$, are increasing. One can now derive the optimal policy via calculus; however, the approach is not particularly illuminating. In contrast, the following argument is easy to generalise.

First, consider the increasing demand case, $m_2^{-1}(0) \geq m_1^{-1}(0)$, as shown in *Figure 3A*.[4] Example 2 demonstrates that the optimal rule is the myopic policy $\theta_t^* = m_t^{-1}(0)$. To verify this, consider fixing the second period cutoff and choosing θ_1^*. If the firm sells to type $(\theta, 1)$ in period 1, they will obtain profit of $m_1(\theta)$. If the firm does not sell to type $(\theta, 1)$ in period 1, they may end up selling to the agent in period 2, yielding profit $\delta m_1(\theta)$, or they may never sell to the agent, yielding profit 0. If $m_1(\theta) \geq 0$ then $m_1(\theta) \geq \max\{\delta m_1(\theta), 0\}$, so the firm is always better off selling now, independent of future cutoffs. Conversely, if $m_1(\theta) < 0$ then

[4] *Figure 3* is created using the Weibull distribution.

$m_1(\theta) < \min\{\delta m_1(\theta), 0\}$ and the firm is always better off not selling. So the profit–maximising rule is simple: sell to an agent of type $(\theta, 1)$ in period 1 if and only if $m_1(\theta) \geq 0$.

In period 2, selling to agents with valuation $\theta \in \left[m_1^{-1}(0), \overline{\theta}\right]$ yields a marginal revenue of $m_2(\theta)$. For valuations $\theta \in \left[\underline{\theta}, m_1^{-1}(0)\right)$, the firm also sells to first generation agents and marginal revenue is $m_1(\theta) + m_2(\theta)$. Hence the firm faces the cumulative marginal revenue, $M_2(\theta) := m_2(\theta) + \min\{m_1(\theta), 0\}$. Since demand is increasing the firm can sell to all generation 2 agents with positive marginal revenue, $\theta \geq m_2^{-1}(0)$, without having to sell to any more generation 1 agents. One can imagine the cutoff θ_2^* being slowly reduced, including more and more agents. The firm then stops at $m_2^{-1}(0)$, since going further will include negative marginal revenue agents from generation 2 and will eventually include negative marginal revenue agents from generation 1. This yields the cutoffs $\theta_t^* = m_t^{-1}(0)$ for $t = 1, 2$.

Second, consider the decreasing demand case, $m_2^{-1}(0) \leq m_1^{-1}(0)$, as shown in *Figure 3B*. As in the increasing demand case, the firm should sell to an agent in period 1 if and only if $m_1(\theta) \geq 0$. However, the second period is different. If the firm were to sell to all the generation 2 agents with positive marginal revenue, $\theta \geq m_2^{-1}(0)$, they would include some generation 1 agents with negative marginal revenue who did not buy in the first round. Thus the firm increases the cutoff until the total marginal revenue from both generations, $m_{\leq 2}(\theta)$, equals zero. This yields the cutoffs $\theta_t^* = m_{\leq t}^{-1}(0)$ for $t = 1, 2$.

When demand is increasing, only generation 2 buys in period 2. Hence the firm only cares about the marginal revenue from the second generation when choosing its cutoff. In comparison, when demand is decreasing, both generations are active in period 2. Hence the firm cares about the total marginal revenue from both generations when choosing its cutoff.

4.3 General Solution

Definition 1. *Cumulative marginal revenue equals* $M_t(\theta) := m_t(\theta) + \min\{M_{t-1}(\theta), 0\}$, *where* $M_1(\theta) := m_1(\theta)$.

Assumption (A1). $M_t(\theta)$ is quasi–increasing $(\forall t)$.[5]

The assumption that $m_t(\theta)$ is quasi–increasing is often used in mechanism design and holds for most common distributions (e.g. normal, lognormal, exponential). If $m_t(\theta)$ is quasi–increasing and demand is increasing, in that $m_t^{-1}(0) \geq M_{t-1}^{-1}(0)$, then $M_t(\theta)$ is automatically quasi–increasing. If $m_t(\theta)$ is quasi–increasing and demand is decreasing, then $M_t(\theta)$ is quasi–increasing if the demand reduction is not too great.[6]

[5] A function $M(\theta)$ is (strictly) quasi–increasing if $M(\theta_L) \geq 0 \Longrightarrow M(\theta_H) \geq (>) 0$ for $\theta_H > \theta_L$. Define the root of a quasi–increasing function by $M^{-1}(0) := \sup\{\theta : M_t(\theta) < 0\}$. If $M(\theta) < 0$ $(\forall\theta)$ then $M^{-1}(0) := \overline{\theta}$. If $M(\theta) \geq 0$ $(\forall\theta)$ then $M^{-1}(0) := \underline{\theta}$.

[6] What does "too great" mean? First, consider the linear demand example, where $f_t(\theta) = 1$ on $[0, 2b_t]$. Marginal revenue, $m_t(\theta) = 2(\theta - b_t)$, is increasing on $[0, 2b_t]$. If $\max_t b_t \leq 2 \min_t b_t$, each $m_t(\theta)$ is strictly increasing

Theorem 1. *Under A1, the profit–maximising cutoffs are given by* $\theta_t^* = M_t^{-1}(0)$.

Proof. Let Π_t be expected profit from generation t, and $\Pi_{\geq t}$ be the profit from generations $\{t, \ldots, T\}$. Denote the positive and negative components by $M_t^+(\theta) := \max\{M_t(\theta), 0\}$ and $M_t^-(\theta) := \min\{M_t(\theta), 0\}$. The proof will proceed by induction, starting with period $t = 1$.

Fix $\{\tau(\theta, t')\}$ for $t' \geq 2$, and consider the optimal choice of $\tau(\theta, 1)$. Notice that the non–discrimination condition in Lemma 1 implies $\tau(\theta, 1) \in \{1, \tau(\theta, 2)\}$. Splitting up the profit equation (3.4), $\Pi = \Pi_1 + \Pi_{\geq 2}$,

$$\begin{aligned} \Pi &= \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \mathcal{E}\left[\Delta_{\tau(\theta,1)} M_1^+(\theta) + \Delta_{\tau(\theta,1)} M_1^-(\theta) \ \mid \mathcal{F}_1\right] d\theta\right] + \Pi_{\geq 2} \\ &\leq \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \mathcal{E}\left[\Delta_1 M_1^+(\theta) + \Delta_{\tau(\theta,2)} M_1^-(\theta) \ \mid \mathcal{F}_1\right] d\theta\right] + \Pi_{\geq 2} \end{aligned}$$

The second line solves for the optimal choice of $\tau(\theta, 1)$. Since $M_1(\theta)$ is measurable with respect to $\mathcal{F}_1$, the optimal choice is $\tau^*(\theta, 1) = 1$ if $M_1(\theta) \geq 0$ and $\tau^*(\theta, 1) = \tau(\theta, 2)$ if $M_1(\theta) < 0$. This is independent of the choice of $\tau(\theta, 2)$. If $M_1(\theta)$ is quasi–increasing, this purchasing rule can be implemented by setting $\theta_1^* = M_1^{-1}(0)$.

Continuing by induction, consider period t. Suppose $\theta_s^* = M_s^{-1}(0)$ for $s < t$. Fix $\{\tau(\theta, t')\}$ for $t' \geq t$, and consider the optimal choice of $\tau(\theta, t)$. The non–discrimination condition in Lemma 1 implies $\tau(\theta, t) \in \{t, \tau(\theta, t+1)\}$. Splitting the profit equation, $\Pi = \Pi_{\leq t-1} + \Pi_t + \Pi_{\geq t+1}$,

$$\begin{aligned} \Pi &= \mathcal{E}\left[\sum_{s=1}^{t-1} \int_{\underline{\theta}}^{\overline{\theta}} \Delta_s M_s^+(\theta)\, d\theta\right] + \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \mathcal{E}\left[\Delta_{\tau(\theta,t)} M_t^+(\theta) + \Delta_{\tau(\theta,t)} M_t^-(\theta) \ \mid \mathcal{F}_t\right] d\theta\right] + \Pi_{\geq t+1} \\ &\leq \mathcal{E}\left[\sum_{s=1}^{t-1} \int_{\underline{\theta}}^{\overline{\theta}} \Delta_s M_s^+(\theta)\, d\theta\right] + \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \mathcal{E}\left[\Delta_t M_t^+(\theta) + \Delta_{\tau(\theta,t+1)} M_t^-(\theta) \ \mid \mathcal{F}_t\right] d\theta\right] + \Pi_{\geq t+1} \end{aligned}$$

The optimal choice of stopping rule is $\tau^*(\theta, t) = t$ if $M_t(\theta) \geq 0$ and $\tau^*(\theta, t) = \tau(\theta, t+1)$ if $M_t(\theta) < 0$. If $M_t(\theta)$ is quasi–increasing, this stopping rule can be implemented by setting $\theta_t^* = M_t^{-1}(0)$. $\square$

In the first period, the monopolist can either sell to agent θ and gain $\Delta_1 m_1(\theta)$, or postpone selling to this agent and gain $\Delta_t m_1(\theta)$ if they eventually sell in period t. Since $\Delta_t < \Delta_1$ the

---

and continuous on $[0, \max_t b_t]$, implying $M_t(\theta)$ is quasi–increasing. Second, consider log–linear demand, where $f_t(\theta) = \exp(-t\lambda_t)/\lambda_t$ on $[0, \infty)$. Marginal revenue, $m_t(\theta) = \exp(-\theta\lambda_t)(\theta/\lambda_t - 1)$, is increasing on $[0, 2\lambda_t]$. If $\max_t \lambda_t \leq 2 \min_t \lambda_t$, each $m_t(\theta)$ is strictly increasing and continuous on $[0, \max_t \lambda_t]$, again implying $M_t(\theta)$ is quasi–increasing. Moreover, computational analysis suggests that with two generations and log–linear demand $M_t(\theta)$ is always quasi–increasing, suggesting the assumption is quite robust. These two examples were picked for there algebraic simplicity, but seem to be representative of many other distributions (e.g. normal, lognormal).

monopolist should sell to the agent if and only if $m_1(\theta) \geq 0$. In period 2, and every subsequent period, the firm sums the marginal revenue of the new consumers and that of old agents who have yet to buy. This cumulative marginal revenue is given by $M_t(\theta) = m_t(\theta) + M_1^-(\theta)$, whereupon the monopolist again sells to agents with valuation θ if and only if their marginal revenue is positive, $M_t(\theta) \geq 0$.

This algorithm is completely myopic: It says the optimal cutoff point at period t only depends upon the consumers who have entered by time t. That is, the optimal cutoff at time t is independent of future demand and the discount rate.

4.4 Active Generations

In the Introduction we said that the key part of the problem is to work out out which generations buy in a given period. At time t_H, we will say a generation t_L is active if some members of generation t_L purchase in period t_H. Theorem 2 characterises the set of active generations and uses this to provide an alternative derivation of the optimal cutoffs.

Definition 2. *The upper active set is* $\overline{A}(t_H) := \{t_L \leq t_H : \theta_t^* \leq \theta^*(t_H - 1; t_L)\}$.

Lemma 2. *The upper active set has the following properties:*
(a) $\overline{A}(t) = \{a, \dots, t\}$ *for some* $a \leq t$.
(b) $\overline{A}(t) \supset \overline{A}(t-1)$ *or* $\overline{A}(t) = \{t\}$.

Proof. (a) $t \in \overline{A}(t)$ since $M_t^{-1}(0) \in [\underline{\theta}, \overline{\theta}] < \infty$. $\overline{A}(t)$ is connected since $\theta^*(t_H - 1; t_L)$ is increasing in t_L. (b) If $t' \in \overline{A}(t-1)$ then $\theta^*(t-1; t') = \theta^*(t-1; t-1)$. If $\{t-1\} \in \overline{A}(t)$ then $t' \in \overline{A}(t)$. □

Lemma 2(a) says that generation $s+1$ is active when s is active, and that the current generation is always active. Lemma 2(b) says that once two generations are pooled, they are never separated. Define $\mathcal{A}(t) := \{\{a, \dots, t\} : a \leq t\}$ as the collection of possible active sets at time t.

Theorem 2. *Suppose A1 holds. Then* $M_t(\theta)$ *is the lower envelope of* $\{m_A(\theta) : A \in \mathcal{A}(t)\}$ *and the optimal cutoffs are given by*

$$\theta_t^* = \max_{A \in \mathcal{A}(t)} m_A^{-1}(0) \tag{4.1}$$

When $A = \overline{A}(t)$*, this maximum is obtained. Moreover, if* $M_t(\theta)$ *is strictly quasi–increasing and continuous then* $\overline{A}(t)$ *is the maximal set in* $\mathcal{A}(t)$ *such that* $m_A^{-1}(0) = \theta_t^*$.

Proof. Fix t and pick an arbitrary $A \in \mathcal{A}(t)$. That is, $A = \{a, \dots, t\}$ for some $a \leq t$. By construction,

$$\sum_{s=a}^{t} m_s(\theta) = M_t(\theta) + \sum_{s=a}^{t-1} M_s^+(\theta) - M_{a-1}^-(\theta) \tag{4.2}$$

Hence $M_t(\theta) \leq m_A(\theta)$. Cumulative marginal revenue can also be written as

$$M_t(\theta) = \sum_{s \leq t} m_s(\theta) \mathbf{1}_{\theta < \theta^*(t-1;s)} \tag{4.3}$$

so for any θ, $\exists A \in \mathcal{A}(t)$ such that $M_t(\theta) = m_A(\theta)$. That is, $M_t(\theta) = \min\{m_A(\theta) : A \in \mathcal{A}(t)\}$.

Since $M_t(\theta) \leq m_A(\theta)$, if $M_t(\theta) \geq 0$ then $m_A(\theta) \geq 0$, $\forall A \in \mathcal{A}(t)$. That is, $M_t^{-1}(0) \geq \max_{A \in \mathcal{A}(t)} m_A^{-1}(0) \geq m_{\overline{A}(t)}^{-1}(0)$. To obtain the reverse inequality, (4.3) implies that for small $\epsilon > 0$, $M_t(\theta_t^* - \epsilon) = m_{\overline{A}(t)}(\theta_t^* - \epsilon)$. Hence $M_t^{-1}(0) \leq m_{\overline{A}(t)}^{-1}(0)$. Putting this together, $M_t^{-1}(0) = m_{\overline{A}(t)}^{-1}(0)$.

Fix t and define $A^*(t)$ to be the maximal set such that $m_{A^*(t)}^{-1}(0) = M_t^{-1}(0)$. Since $m_{\overline{A}(t)}^{-1}(0) = M_t^{-1}(0)$ it must be that $\overline{A}(t) \subset A^*(t)$. In order to obtain a contradiction, suppose $A^*(t) = \overline{A}(t) \cup B$ for some nonempty set B, where $b = \max\{t : t \in B\}$. Since $b \notin \overline{A}(t)$, $\theta_b^* < \theta_t^*$. $M_b(\theta)$ lies below $m_B(\theta)$ and is strictly quasi–increasing so there is a small $\epsilon > 0$ such that $m_B(\theta_t^* - \epsilon) \geq M_b(\theta_t^* - \epsilon) > \epsilon$. Moreover, $M_t(\theta)$ is continuous so ϵ can be chosen sufficiently small such that $m_{\overline{A}(t)}(\theta_t^* - \epsilon) = M_t(\theta_t^* - \epsilon) \in (-\epsilon, 0)$. Hence $m_{A^*(t)}(\theta_t^* - \epsilon) = m_{\overline{A}(t)}(\theta_t^* - \epsilon) + m_B(\theta_t^* - \epsilon) > 0$, and $m_{\overline{A}(t)}^{-1}(0) > m_{A^*(t)}^{-1}(0)$, contradicting the assumption that $m_{A^*(t)}^{-1}(0) = M_t^{-1}(0)$. □

Theorem 2 can be interpreted in two steps. Firstly, the optimal cutoffs are determined by the marginal revenue of the active generations. This is as one would expect from the first order condition of the firm's profit maximising problem, taking the set of active generations as exogenous. Secondly, the set of active generations is chosen to maximise the cutoff. Intuitively one can think of quantity at time t slowly being released. The first units will go to members of the current generation with the highest valuations. Generation t continues to receive all the units until the value drops below $\theta^*(t-1; t-1)$, the highest unserved valuation from the previous generation. The next units then get split between the generations t and $t-1$. This continues until the valuation being served falls below $\theta^*(t-1; t-2)$, at which point generation $t-2$ also starts to receive the new units. This process continues until the firm stops issuing units, at which point the active generations are those that have received some positive quantity.

The following example shows the optimal policy can be deceptively simple.

Example 4 (Linear Demand). Suppose $f_t(\theta) = 1$ on $[0, 2b_t]$ where $b_t \in [10, 20]$.[7] Under Theorem 1 the monopolist should sell quantity b_t each period. This is the same as the quantity sold by a firm who could completely discriminate between different cohorts. This equivalence is analogous to the property that average quantity sold remains unaffected by third–degree price discrimination when demand is linear (Tirole (1988, p.139)). See Appendix A.2. △

[7] The bounds on b_t ensure $M_t(\theta)$ is quasi–increasing. See footnote 6.

4.5 Ironing

Theorem 1 assumes the cumulative marginal revenue $M_t(\theta)$ is quasi–increasing (A1). If this fails in a one–period model, one can calculate the ironed marginal revenue $\overline{M}_1(\theta)$ (e.g. Myerson (1981)). The firm then sells to an agent if and only if $\overline{M}_1(\theta) \geq 0$. Unfortunately, in the multi–period, the myopic policy of ironing each $M_t(\theta)$ individually may not work, as the following example shows.

Example 5. Suppose $T = 2$, δ constant, $[\underline{\theta}, \overline{\theta}] = [0, 1]$ with $m_1(\theta) = 1 - 4 \cdot \mathbf{1}_{[1/2,1]}$ and $m_2(\theta) = -10 + 20 \cdot \mathbf{1}_{[1/2,1]}$. This yields $\overline{M}_1(\theta) = -1$ so a myopic policy suggests not awarding the good to any agent. In period 2, $M_2(\theta) = -9 + 16 \cdot \mathbf{1}_{[1/2,1]}$ which is quasi–increasing with $M_2^{-1}(0) = 1/2$ yielding revenue $7\delta/2$. However if the firm sells to consumers $[0, 1]$ in period 1 and $[1/2, 1]$ in period 2 then revenue is $-1 + 10\delta/2$ which is preferable if $\delta \geq 2/3$. $\triangle$

The problem in Example 5 is that demand decreases over time.[8] In period 1, the existence of negative marginal revenue agents may stop the monopolist selling to all positive marginal revenue consumers. However, if these negative consumers end up buying anyway, the firm should take this into account in its ironing calculation. Since the firm now has to be forward–looking, the simple myopic policy in Theorem 1 no longer holds.

4.6 Best–Price Provisions and Time Consistent Pricing

Applying the revenue equivalence theorem, Proposition 1 shows that the optimal allocation (Theorem 1) can be implemented by a best–price provision. Moreover, the best–price provision is time–consistent so the firm need not commit to a sequence of prices at time 0, so long as they can promise to honour the best price agreement.[9]

A *best–price provision* works as follows: In each period the firm announces a price p_t^{BP}. If a consumer buys in period t and the price then falls, they are then given a rebate equal to the difference in the prices. In each subsequent period s, they are given a rebate equal to $\min\{p_t^{\mathrm{BP}}, \ldots, p_s^{\mathrm{BP}}\} - \min\{p_t^{\mathrm{BP}}, \ldots, p_{s-1}^{\mathrm{BP}}\}$. In discounted terms, the consumer purchasing in period t pays

$$\sum_{s=t}^{T} (\Delta_s - \Delta_{s+1}) \min\{p_t^{\mathrm{BP}}, \ldots, p_s^{\mathrm{BP}}\}$$

Proposition 1. *Suppose A1 holds. Then the firm's optimal policy under a best–price provision is to set $p_t^{BP} = M_t^{-1}(0)$, inducing the same allocation and profits as Theorem 1. This policy is time consistent.*

[8] If demand increases, in that $\overline{m}_2^{-1}(0) \geq \overline{m}_1^{-1}(0)$, then each period can be treated separately and the myopic ironing policy is optimal.

[9] Butz (1990) reaches a similar conclusion in a model with declining demand and resale. When demand is allowed to increase, however, the introduction of resale means that the best–price provision may not be time consistent, as shown in Section 7.2.

Proof. Under a best–price provision an agent of type (θ, t) will purchase as soon as the price falls below θ, so the set of implementable allocations is characterised by Lemma 1. Profits are given by equation (3.4) so the optimal cutoffs are given by Theorem 1 and implemented by setting $p_t^{\text{BP}} = \theta_t^*$.

To prove time consistency, let the time consistent cutoffs be denoted by θ_t^{BP}. At each point in time t, the firm chooses p_t^{BP} to maximise profits from period $s \geq t$,

$$\Pi_{\geq t}^{\text{BP}} = \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \sum_{s=t}^{T} \sum_{r=1}^{s} \Delta_s \mathbf{1}_{\tau(\theta,r)=s} m_r(\theta)\, d\theta\right]$$

Since $\theta_t^{\text{BP}} = p_t^{\text{BP}}$, we can also think of the firm choosing θ_t^{BP} directly. However, notice that the difference

$$\Pi - \Pi_{\geq t}^{\text{BP}} = \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \sum_{s=1}^{t-1} \sum_{r=1}^{s} \Delta_s \mathbf{1}_{\tau(\theta,r)=s} m_r(\theta)\, d\theta\right]$$

is independent of θ_t^{BP}, so the choice of θ_t^{BP} also maximises total profits, Π. □

5 Applications

This Section will apply the Theorems 1 and 2 to different paths of demand functions. We will examine how cutoffs change over time, how long agents wait before purchasing and the time path of prices.

Assumption (A2). $m_t(\theta)$ is strictly increasing and continuous in θ and $m_t^{-1}(0) \in (\underline{\theta}, \overline{\theta})$ $(\forall t)$.

This Section (as well as Section 7) uses Assumption A2 rather than A1. This stricter monotonicity assumption simplifies proofs and helps provide a cleaner characterisation of demand cycles (Section 5.3). If $m_t(\theta)$ is quasi–increasing and demand variation is not too great then A2 will hold for all relevant valuations. For linear and log–linear demand, this means demand cannot double between the lowest and highest periods (see footnote 6). Versions of many of these results also extend to A1.[10]

Lemma 3. *Suppose A2 holds. Then*
(a) $m_1^{-1}(0) > m_2^{-1}(0)$ *implies* $m_{\{1,2\}}^{-1}(0) \in (m_2^{-1}(0), m_1^{-1}(0))$
(b) $m_1^{-1}(0) \geq m_2^{-1}(0)$ *implies* $m_{\{1,2\}}^{-1}(0) \in [m_2^{-1}(0), m_1^{-1}(0)]$

Proof. Omitted. □

[10] To illustrate, A1 is sufficient for the "if" part of the monotone demand characterisation (Proposition 2). Similarly, when demand cycles, cutoffs are stationary (Proposition 4) and, using Theorem 2, cutoffs and prices exceed the average–demand price (Proposition 5).

5.1 Monotone Deterministic Demand: Fast Rises and Slow Falls

Definition 3. *Demand is increasing if* $m_{t+1}^{-1}(0) \geq m_t^{-1}(0)$*. Demand is weakly decreasing if* $m_{\leq t}^{-1}(0) \geq m_{t+1}^{-1}(0)$*.*

Increasing demand means that the myopic monopoly price against the incoming generation rises over time. Weakly decreasing demand means the myopic monopoly price against the incoming generation is lower than the monopoly price against the sum of the previous demands.

Proposition 2 characterises the optimal cutoffs and prices when demand is growing or falling. These results are simple extensions of the two–period solution in Section 4.2.

Proposition 2. *Suppose demand is deterministic and A2 holds. Optimal cutoffs are given by* $\theta_t^* = m_t^{-1}(0)$ $(\forall t)$ *if and only if demand is increasing. This can be implemented by prices*

$$p_t^* = m_t^{-1}(0)$$

Optimal cutoffs are given by $\theta_t^* = m_{\leq t}^{-1}(0)$ *if and only if demand is weakly decreasing. This can be implemented by prices*

$$p_t^* = \sum_{s=t}^{T} \mathcal{E}\left[\left(\frac{\Delta_s}{\Delta_t} - \frac{\Delta_{s+1}}{\Delta_t}\right) m_{\leq s}^{-1}(0) \,\middle|\, \mathcal{F}_t\right] \tag{5.1}$$

where $\Delta_{T+1} := 0$*.*

Proof. [If]. Increasing demand case: For $t = 1$, $\theta_1^* = m_1^{-1}(0)$. Suppose $\theta_s^* = m_s^{-1}(0)$ for $s < t$ and consider period t. $M_t(\theta) = m_t(\theta)$ on $[\theta_{t-1}^*, \overline{\theta}] = [m_{t-1}^{-1}(0), \overline{\theta}]$, using the induction hypothesis. Demand is increasing so $M_t^{-1}(0) = m_t^{-1}(0)$.

Decreasing demand case: For $t = 1$, $\theta_1^* = m_{\leq 1}^{-1}(0)$. Suppose $\theta_s^* = m_{\leq s}^{-1}(0)$ for $s < t$ and consider period t. $M_t(\theta) = m_{\leq t}(\theta)$ on $[\underline{\theta}, \min\{\theta_1^*, \ldots, \theta_{t-1}^*\}] = [\underline{\theta}, m_{\leq t-1}^{-1}(0)]$, using the induction hypothesis. Demand is weakly decreasing so Lemma 3 implies $m_{\leq t}^{-1}(0) \leq m_{\leq t-1}^{-1}(0)$ and hence $M_t^{-1}(0) = m_{\leq t}^{-1}(0)$.

[Only If]. Increasing demand case: Applying the contrapositive, suppose $m_t^{-1}(0) < m_{t-1}^{-1}(0)$. Theorem 2 means $\theta_t^* \geq m_{\{t-1,t\}}^{-1}(0) > m_t^{-1}(0)$, using Lemma 3.

Decreasing demand case: Applying the contrapositive, suppose $m_t^{-1}(0) > m_{\leq t-1}^{-1}(0)$. Theorem 2 means $\theta_t^* \geq m_t^{-1}(0) > m_{\leq t}^{-1}(0)$, using Lemma 3.

Prices can then be derived from equation (3.7). With increasing demand, this is immediate. With weakly decreasing demand, prices obey the AR(1) equation $(\theta_t^* - p_t^*) = \mathcal{E}[(\theta_t^* - p_{t+1}^*)\delta_{t+1} | \mathcal{F}_t]$ $\square$

When demand is increasing over time, the firm can charge the optimal myopic price, $p_t^* = m_t^{-1}(0)$. Since the price is increasing, no consumers will delay their purchases, and the problem can be broken into T disjoint sub–problems (see Example 2 in Section 3.2). In contrast, if the firm charges the myopic price when demand is decreasing then there will be much delay. The firm takes this into account and chooses the cutoff points so that at time t, agent θ buys if the "past average" marginal revenue, $m_{\leq t}(\theta)$, is positive. Prices are then given by a geometric sum of future "past average" monopoly prices.

Two price paths, mentioned in the introduction, will serve as useful benchmarks.

Definition 4. *The myopic price is* $p_t^M := m_t^{-1}(0)$. *The average–demand price is* $p^A := \lim_{t\to T} m_{\leq t}^{-1}(0)$, *assuming the limit exists.*

The myopic price is the price charged by a monopolist who only takes the current generation of consumers into account, ignoring the previous ones. By Example 2, this equals the optimal price when the monopolist can discriminate between generations. It is also the optimal price if consumers are banned from delaying consumption. The average–demand price is charged by a monopolist who faces average demand $\frac{1}{T}\sum_{t=1}^{T} F_t(\theta)$ each period. This is also the price charged by an uninformed firm which knows total demand over the T periods, but does not know the demand each period.[11]

Figure 4A compares different price paths under increasing demand.[12] As can be seen, $p_t^* = p_t^M$ is increasing. Since agents never delay their purchases under increasing demand, the optimal price path is independent of the discount factor.

This can be contrasted to decreasing demand, as shown in *Figure 4B*. Here p_t^* is decreasing, starting off below the myopic price and ending above it. The optimal price p_t^* converges to the average–demand price from above as $t \to T$. That is,

$$\lim_{t\to T}[p_t^* - p^A] = \lim_{t\to T}\sum_{s=t}^{T}\mathcal{E}\left[\left(\frac{\Delta_s}{\Delta_t} - \frac{\Delta_{s+1}}{\Delta_t}\right)(m_{\leq s}^{-1}(0) - m_{\leq t}^{-1}(0))\,\middle|\,\mathcal{F}_t\right] = 0$$

since every convergent sequence is cauchy. The discount factor is also relevant when demand decreases: price falls towards the average–demand price as agents become more patient. (For more on discounting, see Section 6.2.)

5.2 One–Off Shocks in Demand

The model can be used to rigorously analyse permanent and transitory shocks to demand. This subsection describes the demand path and sketches the consequences of a permanent shock.

[11] The uncertainty interpretation assumes the firm cannot update their strategy over time, perhaps because because sales are unobservable in the short term.

[12] *Figures 1, 4* and *5* are generated using linear demand curves. In particular, values have measure 1 on $[0, b_t]$ where $b_t \in [20, 30]$. The discount rate is $\delta = 0.9$ in *Figures 4–5* and $\delta = 0.75$ in *Figure 1*. Example 4 in Section 5.3 further analyses the linear demand specification.

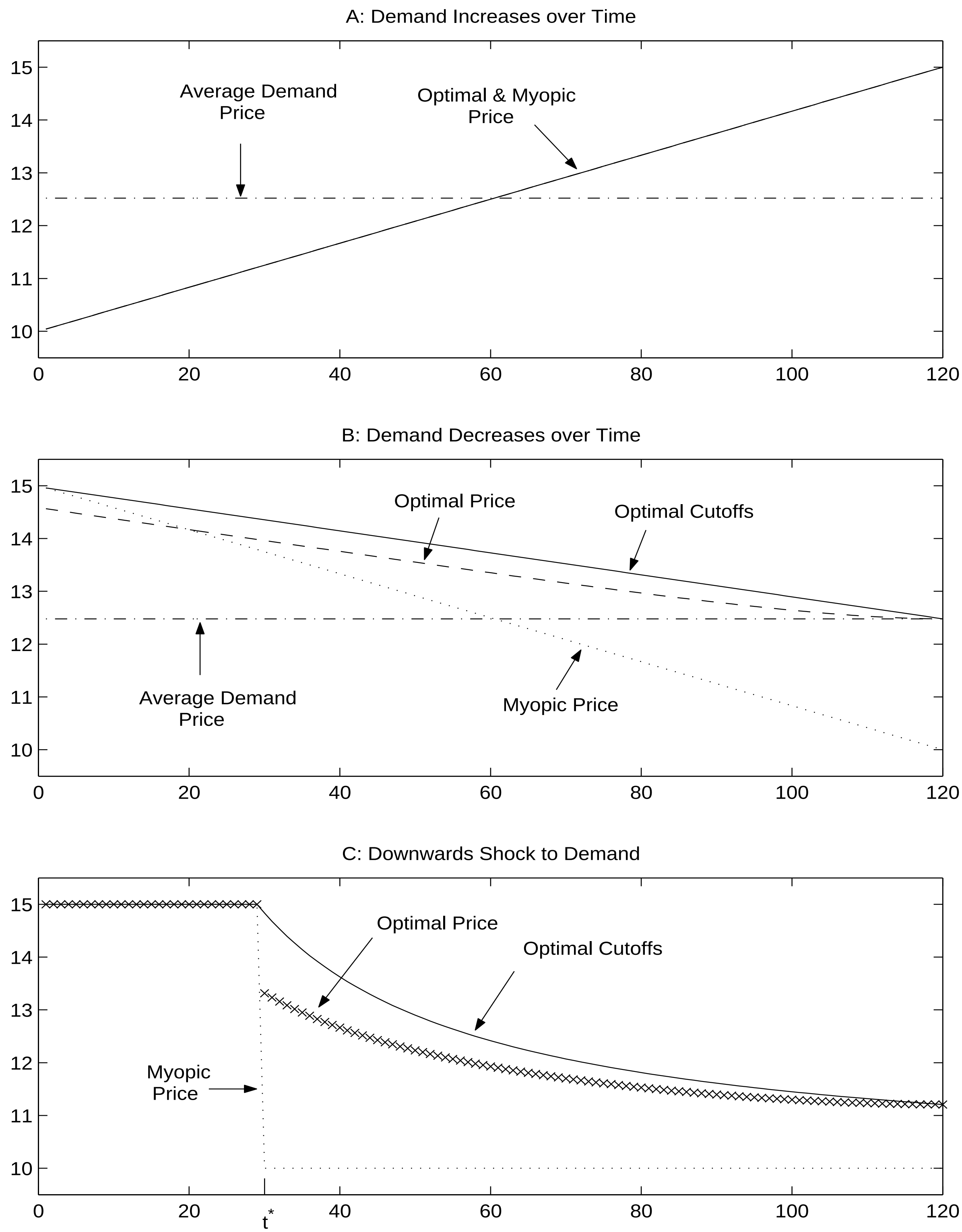

Figure 4: Monotone Demand Paths

Demand is constant for the first $t'-1$ periods, with $m_t(\theta) = m_\alpha(\theta)$. There are two states of the world. In state ω_α, demand stays at $m_t(\theta) = m_\alpha(\theta)$ for $t \in \{t', \dots, T\}$. In state ω_β, demand shifts to $m_t(\theta) = m_\beta(\theta)$ for $t \in \{t', \dots, T\}$. To complete the description of the world, suppose the state of the world is realised in period $t'' \leq t'$ and let the ex–ante probability of state ω_α be α.

First, suppose demand jumps upwards, $m_\beta^{-1}(0) \geq m_\alpha^{-1}(0)$. Prices start at $m_\alpha^{-1}(0)$ until time $t'-1$ and then, if demand increases, jump upwards to $m_\beta^{-1}(0)$.

Second, suppose demand jumps downwards, $m_\beta^{-1}(0) \leq m_\alpha^{-1}(0)$. Prices start at $m_\alpha^{-1}(0)$ and slowly decrease until time t''. In state ω_α the price then jumps upwards to $m_\alpha^{-1}(0)$. In state ω_β the price then jumps down a little and slowly converges to $m_\beta^{-1}(0)$. The case of $\alpha \approx 1$ and $t'' = t'$ is shown in *Figure 4C*.

5.3 Deterministic Demand Cycles

This subsection examines the optimal price path when demand follows deterministic cycles. The sequence of demand functions is described by K repetitions of $\{F_1, \dots, F_T\}$, where $T < \infty$ but we allow $K = \infty$. Denote the period t of cycle k by t_k. An example of this was seen in *Figure 1* in the Introduction, which also illustrates the set of active agents, $\underline{A}(t)$ (which is defined below). One can see the pattern of sharp price increases and slow declines; these intuitively follow from Proposition 2. When new demand is growing, the price rises quickly along with the myopic price and there is no delay. When new demand is falling, agents delay their purchases and the price falls much more slowly. The picture also illustrates other regularities that occur after the first cycle:

1. Cutoffs and prices follow a stationary pattern.

2. Cutoffs and prices always lie above the average–demand price.

3. The lowest cutoff and price occur in the last period of the slump.

Propositions 4–6 correspond to these results. First, it will useful to define the lower active set.

Definition 5. *The lower active set is* $\underline{A}(t_H) := \{t_L \leq t_H : \theta_t^* < \theta^*(t_H - 1; t_L)\}$.

The lower active set will often look very similar to the upper active set, but is particularly useful in the analysis of demand cycles. Lemma 2 applies to the lower active set, as does a version of Theorem 2.

Proposition 3. *Under A2,* $m_{\underline{A}(t)}^{-1}(0) = \theta_t^*$. *Moreover,* $\underline{A}(t)$ *is the minimal set in* $\mathcal{A}(t)$ *such that* $m_A^{-1}(0) = \theta_t^*$.

Proof. Using (4.3) and A2, $M_t(\theta_t^*) = m_{\underline{A}(t)}(\theta_t^*) = 0$. By A2, this uniquely defines $\theta_t^* = m_{\underline{A}(t)}^{-1}(0)$.

Fix t and let $A^*(t)$ be the minimal set such that $m_{A^*(t)}^{-1}(0) = \theta_t^*$. In order to attain a contradiction, suppose $\underline{A} = A^* \cup B$ for some nonempty set B, where $b = \max\{t : t \in B\}$. Since $b \in \underline{A}(t)$, we have $\theta_b^* > \theta_t^*$ and $M_b^-(\theta_t^*) < 0$. Using equation (4.2), $0 = M_t(\theta_t^*) \leq m_{A^*(t)}(\theta_t^*) + M_b^-(\theta_t^*) < 0$, yielding a contradiction. □

Proposition 4. *Suppose demand follows deterministic cycles and A2 holds. Then* $|\underline{A}(t)| \leq T$*. Hence if* $k \geq 2$*, the cycles are stationary,* $\theta_{t_k}^* = \theta_{t_2}^*$.

Proof. Suppose $|\underline{A}(t)| > T$. Define the set A such that $\underline{A}(t) = A \cup B$ where B consists of the union of $\{1, \ldots, T\}$ sets and $|A| \leq T$. Then $m_{\underline{A}(t)}^{-1}(0) \leq \max\{m_A^{-1}(0), m_{\{1,\ldots,T\}}^{-1}(0)\}$ by Lemma 3, contradicting the fact that $\underline{A}(t)$ is the smallest set to achieve the maximum in Theorem 2. □

Proposition 4 means the cutoffs will be the same for each cycle $k \geq 2$. This substantially simplifies analysis: when $k \geq 2$, we can use modular arithmetic to write the collection of possible active sets at time t, $\mathcal{A}(t)$, as

$$\mathcal{A}_K(t) = \{\{a, \ldots, t\} : a \in \{1, \ldots, T\}\} \quad (\text{mod–}T)$$

The cutoff and price are minimised at time

$$\underline{t} := \min\{\text{argmin}_{t \in \{1,\ldots,T\}}\{\theta_{t_k}^* : k \geq 2\}\}$$

A consumer who does not buy in period $\underline{t}$ will never buy. Hence the market effectively resets at period $\underline{t}$, enabling us to throw out all previous generations. This means that the starting position of the cycle only matters for the first $\underline{t}$ periods, at which point the stationary cycles start.

Prices are determined by equation (3.7). For cycles $k \in \{2, \ldots, K-1\}$, the prices are stationary with boundary condition $p_{\underline{t}}^* = \theta_{\underline{t}}^* = m_{\{1,\ldots,T\}}^{-1}(0)$, using Proposition 5(a). For the final cycle, where $k = K$ and $t > \underline{t}$, the relevant boundary condition is $p_T^* = \theta_T^*$. Since there is not as much scope for delay, prices in the last cycle may be higher than in previous cycles.

Proposition 5. *Suppose demand follows cycles with* $k \geq 2$ *and A2 holds. Then*
(a) The lowest optimal cutoff equals the average–demand price. Hence agents buy later under optimal pricing.
(b) The lowest optimal price equals the average–demand price.
(c) Under the optimal price path, in comparison to average–demand pricing, profits are higher, utility is lower for every type (θ, t) *and welfare is lower for every generation.*

Proof. (a) If $k \geq 2$ then $\{1, \ldots, T\} \in \mathcal{A}(t)$ and Theorem 2 implies $\theta^*_t \geq m^{-1}_{\{1,\ldots,T\}}(0)$. Equation (4.2) implies

$$m_{\{1,\ldots,T\}}(\theta^*_{\underline{t}}) = M_{\underline{t}_k}(\theta^*_{\underline{t}}) + \sum_{s=\underline{t}_{k-1}+1}^{\underline{t}_k - 1} M^+_s(\theta^*_{\underline{t}}) - M^-_{\underline{t}_{k-1}}(\theta^*_{\underline{t}}) = 0$$

using (1) the definition $\theta^*_{\underline{t}}$, (2) the fact that $\theta^*_s \geq \theta^*_{\underline{t}_k}$ for $s \in \{\underline{t}_{k-1} + 1, \ldots, \underline{t}_k - 1\}$ and (3) $\theta^*_{\underline{t}_k} \geq \theta^*_{\underline{t}_{k-1}}$. Thus $\theta^*_{\underline{t}} = m^{-1}_{\{1,\ldots,T\}}(0) = p^A_t$.

(b) $p^*_{\underline{t}} = \theta^*_{\underline{t}} = p^A_t$.

(c) Profit is lower under the average–demand price regime by revealed preference. The utility of any customer (3.1) and welfare of any generation (3.3) is higher under the average–demand price since the cutoffs are always higher. □

In each period, the cutoff is higher than if the monopolist faced average demand, the price is higher, and welfare and consumer surplus are lower. That is, all consumers are made worse off by the ability of the monopolist to discriminate between generations. Intuitively, during a high demand period, the price will be high, and only the high demand generations will be active. However, during a low demand period, both high and low generations will be active. Thus the high demand cohorts exert a negative externality on the low cohorts, raising the price to an average level.

With quasi–linear utility, the indirect utility function is convex in prices, suggesting price variation benefits consumers because of the option value. However, Proposition 5 shows that when prices are endogenous, price variation may hurt all consumers. The result can also be contrasted with the standard view that the welfare effect of third degree price discrimination is indeterminate (e.g. Tirole (1988, p. 137)). Inter– and intra–temporal price discrimination can have very different properties.

Define a cycle as *simple* if $m^{-1}_t(0) - m^{-1}_{t-1}(0)$ is nonzero and has at most two changes of sign. That is, each cycle has one "boom" and one "slump".

Proposition 6. *Suppose demand follows cycles with $k \geq 2$ and A2 holds. Then $\underline{t}$ obeys*

$$m^{-1}_{\underline{t}}(0) \leq m^{-1}_{\{1,\ldots,T\}}(0) \leq m^{-1}_{\underline{t}+1}(0)$$

When the cycle is simple, this uniquely defines $\underline{t}$.

Proof. First, Proposition 5(a) and Theorem 2 imply $m^{-1}_{\{1,\ldots,T\}}(0) = \theta^*_{\underline{t}} \geq m^{-1}_{\underline{t}}(0)$. Second, by the definition of $\underline{t}$, $\underline{A}(\underline{t}+1) = \{\underline{t}+1\}$. Hence $m^{-1}_{\underline{t}+1}(0) \geq m^{-1}_{\{1,\ldots,T\}}(0)$. When the cycle is simple, this uniquely defines $\underline{t}$ and implies $\underline{A}(\underline{t}) = \{1, \ldots, T\}$. □

In a simple cycle, $\underline{t}$ is uniquely defined as the last period of the slump, just before new demand returns to its long run average. In literary terms: The darkest hour is just before dawn.

5.4 IID Demand

The model allows demand to be uncertain enabling us to study the effect of incoming cohorts that are independently and identically distributed (IID).[13] Each period, demand is drawn from $\{m_x(\theta)\}_x$ with probability measure $\mu(x)$ for $x \in [0,1]$, where higher indices imply higher demand: $m_{x_H}^{-1}(0) \geq m_{x_L}^{-1}(0)$, for $x_H \geq x_L$.

In this setting, we can define the *average–demand price* by $p^A := [\int m_x(\theta)d\mu(x)]^{-1}(0)$. As in the deterministic setting, this is the optimal price for a monopolist who is ignorant about demand. Example 6 explicitly derives the optimal price schedule in a two–period model.

Example 6 (T=2 with IID Demand). Denote the demand in the first and second period by $m_1(\theta) = m_{x_1}(\theta)$ and $m_2(\theta) = m_{x_2}(\theta)$ respectively and suppose the discount rate is constant. Theorem 2 implies $\theta_1^* = m_{x_1}^{-1}(0)$ and $\theta_2^* = \max\{m_{\{x_1,x_2\}}^{-1}(0), m_{x_2}^{-1}(0)\}$. The second period price is $p_2^* = \theta_2^*$, while the initial price is given by

$$p_1^* = \left(1 - \delta \int_0^{x_1} d\mu(x)\right) m_{x_1}^{-1}(0) + \delta \int_0^{x_1} m_{\{x_1,x\}}^{-1}(0)\, d\mu(x) \tag{5.2}$$

If demand of the first generation is low, then demand is likely to increase and p_1^* is determined by the first generation's demand, $m_{x_1}^{-1}(0)$. If demand of the first generation is high, then demand is likely to decrease and p_1^* is determined by both generations' demand, $m_{\{x_1,x_2\}}^{-1}(0)$. The extreme cases when $x_1 \in \{0,1\}$ are directly analogous to the monotone demand results in Proposition 2. The general principle is that p_1^* is affected by a state of the world only if the first generation is active in that state. △

When there are more periods, it becomes harder to solve the option problem required to back out prices. However, one can make comparisons as the number of periods grows large. This next result is the stochastic analogue to Proposition 5.

Proposition 7. *Suppose demand is IID, $\{m_x(\theta)\}_x$ are uniformly bounded and A2 holds. Then* $\lim_{t\to\infty} \theta_t^* \geq p^A$ *and* $\lim_{t\to\infty} p_t^* \geq p^A$ *a.s..*

Proof. Define $g_t(\theta) = \frac{1}{t}\sum_{s=1}^t m_s(\theta)$ and $g(\theta) = \int m_x(\theta)d\mu(x)$. The strong law of large numbers implies that $g_t(\theta) \overset{as}{\to} g(\theta)$ pointwise $\forall \theta \in [\underline{\theta}, \overline{\theta}]$. The collection $\{m_x(\theta)\}_x$ is uniformly bounded and increasing so the Glivenko–Cantelli Theorem (Davidson (1994, Theorem 21.5)) states that $\sup_\theta |g_t(\theta) - g(\theta)| \overset{as}{\to} 0$. Since $g(\theta)$ is strictly increasing $m_{\{1,\ldots,t\}}^{-1}(0) = g_t^{-1}(0) \overset{as}{\to} g^{-1}(0) = p^A$. Finally, $\{1,\ldots,t\} \in \mathcal{A}(t)$ so Theorem 2 implies $\lim_{t\to\infty} \theta_t^* \geq p^A$ a.s.. □

[13] Other sequences of demand functions are have interesting properties. One nice example arises where demand curves are linear with the intercept following a random walk. In this model, the asymmetric treatment of increases and reductions in demand means the ex–ante expectation of θ_t^* is increasing in t.

In the long run, the price should be at least as high as the average demand price. Moreover, it will often be strictly higher (e.g. if $m_t^{-1}(0) > p^A$). This means that prices increase when there is more demand variation, or when the firm has more information about the incoming demand.

6 Comparative Statics

This Section explores the properties of the optimal policy given in Theorem 1. Consequently we only require the weaker monotonicity condition, A1.

6.1 Information Structure

Theorem 1 can be used to address the effect of varying uncertainty about future demand.

Proposition 8. *Consider two information structures $\mathcal{F}_t$ and $\mathcal{F}'_t$ such that $m_t(\theta)$ is $\mathcal{F}_t \cap \mathcal{F}'_t$-adapted and suppose A1 holds. Then utilities and profit are the same under both information structures.*

Proof. Utility and profit are determined by $\tau(\theta, t)$, as shown in equations (3.1) and (3.4). By Theorem 1, this is independent of the information structure if $m_t(\theta)$ is measurable with respect to $\mathcal{F}_t$. $\square$

Proposition 8 shows that under the optimal mechanism, payoffs are independent of information, if consumers and the firm know current demand. Prices, however, will be different under different information structures. Proposition 8 very much depends upon the simple structure of the optimal solution and may fail without A1.

To consider an application of Proposition 8, recall the examples of unexpected shocks in Section 5.2. In this example, demand may receive a permanent demand shock at time t', which agents find out about at time $t'' \leq t'$. Proposition 8 then says that agents should be indifferent over the announcement time, t''.

So far we have made two assumptions about agents' information. First, the firm and consumers possess the same information. If firms and consumers possess different information, and both know current demand, the monopolist would still like to implement the cutoffs given by Theorem 1. If the firm knows more than consumers, the prices are determined by consumers' information sets and payoffs are the same as under symmetric information. However, if consumers know more than firms, then there may be no prices to implement the optimal cutoffs.

Second, we suppose agents know demand $f_t(\theta)$ at time t. Without this assumption, there are two problems. First, the firm may engage in experimentation. Second, the firm may delay purchases until they have better information about demand.

6.2 Prices and Discount Rates

Proposition 9. *Suppose A1 holds. Then increasing δ_t a.e. reduces price p_s^* a.e., for $s < t$. As $\delta_t \stackrel{as}{\to} 1$ $(\forall t)$ so $(p_t^* - \min_{s \geq t} \theta_t^*) \stackrel{as}{\to} 0$. As $\delta_t \stackrel{as}{\to} 0$ $(\forall t)$ so $(p_t^* - \theta_t^*) \stackrel{as}{\to} 0$.*

Proof. (a) Pick t and consider increasing δ_t a.e.. By backwards induction, price p_s^* is independent of δ_t for $s \geq t$. Continuing by induction, pick $s < t$ and suppose future prices are (weakly) decreasing in δ_t. An increase in δ_t thus increases the utility of an agent with value θ_s^* if they choose to delay, given by the right hand side of equation (3.7). Thus the price p_s^* decreases.

(b) Suppose $\delta_t \stackrel{as}{\to} 1$ $(\forall t)$. Equation (3.7) implies that that agent waits for the lowest price, $p_t^* \stackrel{as}{\to} \min\{\theta_t^*, \min_{s>t}\{p_s^*\}\}$. With the boundary condition $p_T^* = \theta_T^*$, this implies $p_t^* \stackrel{as}{\to} \min_{s \geq t} \theta_t^*$.

(c) Suppose $\delta_t \stackrel{as}{\to} 0$ $(\forall t)$. Then $\Delta_{\tau(\theta_t^*, t+1)}/\Delta_t \stackrel{as}{\to} 0$ and the right hand side of equation (3.7) converges to zero. □

When agents are impatient ($\delta_t \approx 0$), the firm simply sets the price equal to the current optimal cutoff, $p_t^* = \theta_t^*$. As agents become more patient, consumers find it less costly to delay, reducing the prices required to implement any sequence of cutoffs. In the limit, when agents are completely patient, they wait for the lowest price and the price is determined by the lowest cutoff in all future periods, $p_t^* = \min_{s \geq t} \theta_t^*$.

Since discount rates are allowed to be uncertain, it is also easy to analyse the impact of changes in interest rates. For example, an unexpected increase in future interest rates will increase today's price. Again, this depends upon the myopic nature of the optimal policy in Theorem 1.

7 Resale and Renting

So far it has been assumed that there is no resale. This is reasonable if the good is a one–time experience or is associated with high transactions costs. This section analyses the opposite case: perfect resale.

Each period t, the consumer obtains discounted rental utility $(\Delta_t - \Delta_{t+1})\theta$, where $\Delta_{T+1} = 0$. As before, demand $f_t(\theta)$ enters the market each period, where $f_t(\theta)$ and Δ_t are $\mathcal{F}_t$–adapted. Throughout this section assume that A2 holds. Consider two alternative policies:

1. The firm rents the good at price R_t each period.

2. The firm sells the good, committing to a price schedule $\{p_t^R\}$, where perfect resale is possible and the seller is allowed to buy goods back.

Bulow (1982) and Butz (1990) show these policies induce the same profits and utilities.

Theorem 3. *Suppose A2 holds and the monopolist either rents the good or they sell the good and allow resale. Then the profit–maximising cutoffs are given by* $\theta_t^R = m_{\le t}^{-1}(0)$*.*

Proof. Both policies induce allocations of the form $\{\theta : \theta \ge \theta_t^R\}$ for some $\theta_t^R \in [\underline{\theta}, \overline{\theta}]$. When renting (policy 1), each period can be treated separately, so at time t the good should be allocated to $\{\theta : m_{\le t}(\theta) \ge 0\}$. Under A2, this is implemented by setting $\theta_t^R = m_{\le t}^{-1}(0)$. When the firm commits to a sequence of sale prices and allows resale (policy 2) the set of implementable allocations is also of the form $\{\theta : \theta \ge \theta_t^R\}$. Hence the revenue equivalence theorem implies that the optimal policy and profit are identical to those when renting. $\square$

When the monotonicity assumption A2 fails, the firm should simply iron $m_{\le t}(\theta)$ $(\forall t)$.

The rental price under the optimal strategy is $R_t = (1 - \delta_{t+1}) m_{\le t}^{-1}(0)$. The optimal price path with resale, p_t^R, can be derived from the AR(1) system,

$$(\theta_t^R - p_t^R) = \mathcal{E}[(\theta_t^R - p_{t+1}^R)\delta_{t+1} | \mathcal{F}_t]$$

The resale price is thus the geometric sum of future rental values, as given by equation (5.1). Hence, if limits exist, the resale price converges to the average–demand price, $p^A := \lim_{t \to T} m_{\le t}^{-1}(0)$. Intuitively, after enough time, the resale market grows very large and the firm loses the ability to discriminate.

7.1 The Effect of Resale

Figure 5 can be used to assess the effect of resale. With resale, troughs and peaks are treated symmetrically, and the cycles decrease in amplitude as the size of the resale market engulfs new production. In the limit, the resale price converges to the average–demand price. Without resale, price cycles are highly asymmetric and are stationary. The reason for this difference is that with resale, a low valuation consumer may buy if they anticipate the price to rise, and so all previous generations remain active. In contrast, without resale, only the high demand generations remain active.

Proposition 10. *Suppose A2 holds. Then cutoffs and profits are lower with resale than without resale. Moreover, resale has no effect on allocations if and only if demand is weakly decreasing.*

Proof. (a) Since $\{1, \ldots, t\} \in \mathcal{A}(t)$, $\theta_t^R \le \theta_t^*$ by Theorem 2.

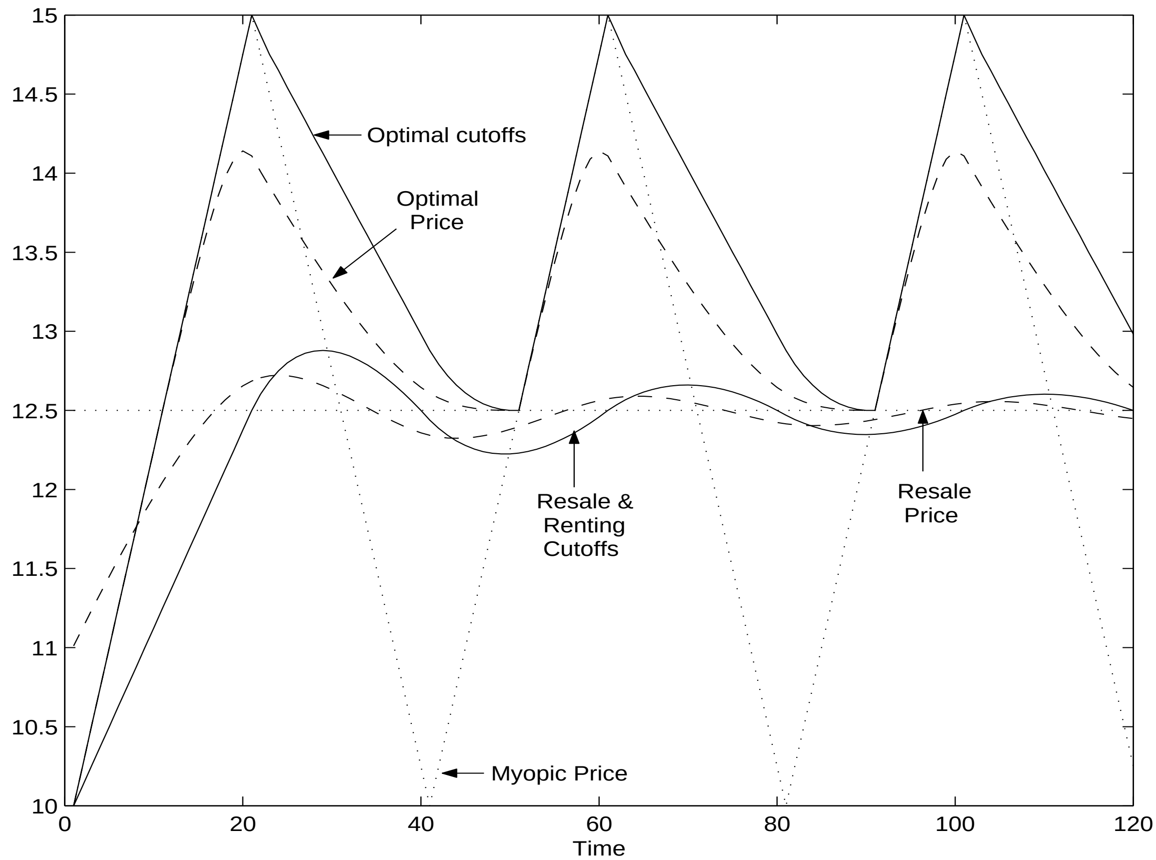

Figure 5: Resale

(b) Using Theorem 1, profits without resale are

$$\Pi = \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \sum_{s=1}^{T} \Delta_s M_s^+(\theta) d\theta\right] = \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \sum_{t=1}^{T} (\Delta_t - \Delta_{t+1}) \sum_{s=1}^{t} M_s^+(\theta)\, d\theta\right]$$

changing the order of summation and noting $\Delta_s = \sum_{t=s}^{T}(\Delta_t - \Delta_{t+1})$. Profits with resale are

$$\begin{aligned}
\Pi^R &= \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \sum_{t=1}^{T} (\Delta_t - \Delta_{t+1}) m_{\leq t}^+(\theta)\, d\theta\right] \\
&= \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \sum_{t=1}^{T} (\Delta_t - \Delta_{t+1}) \left[\max\left\{0, M_t(\theta) + \sum_{s=1}^{t-1} M_s^+(\theta)\right\}\right] d\theta\right] \\
&\leq \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \sum_{t=1}^{T} (\Delta_t - \Delta_{t+1}) \left[\max\{0, M_t(\theta)\} + \sum_{s=1}^{t-1} \max\{0, M_s^+(\theta)\}\right] d\theta\right] \\
&= \mathcal{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \sum_{t=1}^{T} (\Delta_t - \Delta_{t+1}) \left[\sum_{s=1}^{t} M_s^+(\theta)\right] d\theta\right] = \Pi
\end{aligned}$$

where the second line uses equation (4.2) and the third uses Jensen's inequality.

(c) Proposition 2 shows $\theta_t^* = m_{\leq t}^{-1}(0)$ if and only if $m_{\leq t-1}^{-1}(0) \geq m_t^{-1}(0)$. Thus when demand is not declining, $M_t(\theta) < 0$ and $m_{\leq t}(\theta) \geq 0$ for $\theta \in [m_{\leq t}^{-1}(0), M_t^{-1}(0))$ and the inequality in (b) is strict. □

With resale, Coase (1972) and Bulow (1982) argue that renting achieves the same profits as selling, as shown in Theorem 3. Without resale, Proposition 10 implies that selling strictly outperforms renting, unless demand is weakly decreasing with probability one.

The following examples examine the welfare effect of resale. Example 7 shows that with linear demand, resale improves welfare. However, Example 8 shows that the welfare effect may be ambiguous.

Example 7 (Linear Demand). Suppose $f_t(\theta) = 1$ on $[0, 2b_t]$, where $b_t \in [10, 20]$. With resale the optimal quantity sold in period t is b_t, which is the same as without resale (Example 4). Resale increases allocative efficiency and hence welfare. See Appendix A.2. △

Example 8. Suppose $T = 2$ and δ is constant. (a) Suppose $\theta_1 = 1$ with mass 1 and $\theta_2 \sim U[0, 4]$. With resale, cutoffs are $(\theta_1^R, \theta_2^R) = (1, 1)$ yielding welfare $(8 + 15\delta)/8$. Without resale, cutoffs are $(\theta_1^*, \theta_2^*) = (1, 2)$ and welfare is $(8 + 12\delta)/8$, lower than with resale. (b) Suppose $\theta_1 \sim U[0, 2]$ and $\theta_2 = 2$ with mass 1. With resale, cutoffs are $(\theta_1^R, \theta_2^R) = (1, 2)$ yielding welfare $(6 + 10\delta)/8$. Without resale, cutoffs are $(\theta_1^*, \theta_2^*) = (1, 2)$ and welfare is $(6 + 16\delta)/8$, higher than with resale. △

With linear demand curves, resale increases welfare, reduces profits and therefore increases consumer surplus. Yet it is not the case that all consumers are better off. To see this, consider a two–period model with increasing demand. The first (low–demand) cohort prefers no resale since they then avoid being pooled with the second (high–demand) cohort. For the same reason the second generation prefers resale.[14] However, if demand follows cycles, as in Section 5.3, all consumers will prefer resale, if they are born late enough. This follows because with resale, prices converge to the average–demand price; while without resale, prices always exceed the average–demand price (Proposition 5).

7.2 Best–Price Provisions and Resale

Section 4.6 shows that without resale, a best–price provision implements the optimal selling strategy in a time consistent manner. Butz (1990) argues that a best–price provision is also time consistent when resale is allowed. Butz, however, assumes that prices fall over time. In contrast, Example 9 shows that when prices can increase a best–price scheme is *not* time consistent.

Example 9. Suppose $T = 2$, demand is deterministic, δ is constant and A2 holds. Consider a firm that uses a best–price provision and chooses prices sequentially. If demand is decreasing, $m_1^{-1}(0) \geq m_2^{-1}(0)$, the subgame perfect outcome is $\theta_1^* = m_1^{-1}(0)$ and $\theta_2^* = m_{\{1,2\}}^{-1}(0)$. If demand is not decreasing, $m_1^{-1}(0) < m_2^{-1}(0)$, the subgame perfect outcome is $\theta_1^* \geq m_1^{-1}(0)$ and $\theta_2^* < m_{\{1,2\}}^{-1}(0)$. Hence the commitment solution is time consistent under a best–price policy if and only if demand is decreasing. See Appendix A.3 for a proof. $\triangle$

When demand is increasing, the commitment price p_t^R increases over time. A little extra production in the second period then lowers the price below p_2^R, but does not lead to rebates. Hence the firm does not internalise the effect of this extra production on its first period self.

The firm can, however, achieve the maximal profit from Theorem 3 in a time consistent manner through renting or a price–updating policy. *Price–updating* works as follows: In period t, the firm chooses a price p_t^{PU} and agents choose whether to purchase or not. In each subsequent period s, an agent who owns the good is asked to pay $p_{s+1}^{\mathrm{PU}} - p_s^{\mathrm{PU}}$, so decreasing the price leads to rebates, while increasing the price leads to surcharges.

Proposition 11. *Suppose there is resale and A2 holds. The firm's optimal policy under a price–updating scheme is to set $p_t^{PU} = m_{\leq t}^{-1}(0)$ inducing the same allocation and profits as Theorem 3. This policy is time consistent.*

[14] Naive intuition might suggest that the first (low–demand) generation are better off under resale since they have the option to sell to the second (high–demand) generation. However this is incorrect: the firm knows the first generation will resell and raises prices in the first period. Hence the agents with relatively low valuations, who buy in period 1 and resell in period 2, exert a negative externality on the high valuation agents from the first generation, who buy and never resell.

Proof. By purchasing the good in period t and reselling it at time $t+1$, an agent must pay $(1-\delta_{t+1})p_t^{\mathrm{PU}}$, taking into account the price updating and capital gain. The scheme is thus identical to renting with a rental price $(1-\delta_{t+1})p_t^{\mathrm{PU}}$ and is thus time consistent. $\square$

8 Conclusion

This paper characterises the monopolist's optimal pricing strategy with varying demand. When new demand grows stronger, the price rises quickly, unhindered by the presence of old consumers. In contrast, when new demand becomes weaker, the price falls slowly as consumers delay their purchases. This asymmetry between rises and falls leads to an increase in the price level which harms consumers and reduces welfare below that induced by a monopolist who charges the optimal monopoly price against the average level of demand.

There are a number of papers that analyse how markups change over the business cycle (e.g. Rotemberg and Woodford (1999)). Our paper, in contrast, has said little about how prices and quantities relate. Example 4 in Section 5.3 shows that with linear demand curves, the optimal quantity equals the myopic quantity and therefore markups are pro-cyclical. On the other hand, markups are counter-cyclical in the "back–to–school" example in the Introduction. In general, we have seen that the movement of the markup is determined by marginal revenues and can be partially separated from the movement of quantities. To make further predictions, one needs to restrict the possible sequence of demand curves, either through theoretical or empirical means.

A Omitted Material

A.1 Properties of the Consumer's Maximisation Problem

Lemmas 4 establishes some properties of the agent's utility maximisation problem (2.1).

Denote the set of maximisers by $\hat{\tau}(\theta, t)$. Comparing two stopping rules, let $\tau_H \geq \tau_L$ if $\tau_H(\omega) \geq \tau_L(\omega)$ (a.e.–$\omega \in \Omega$). Comparing two sets of stopping rules, $\hat{\tau}_H \geq \hat{\tau}_L$ in strict set order if $\tau' \in \hat{\tau}_H$ and $\tau'' \in \hat{\tau}_L$ imply that $\tau' \vee \tau'' \in \hat{\tau}_H$ and $\tau' \wedge \tau'' \in \hat{\tau}_L$.

Lemma 4. *The consumer's optimal purchase decision has the following properties:*
(a) $\hat{\tau}(\theta, t)$ is a nonempty sublattice and contains a greatest and least element.
(b) Every selection from $\hat{\tau}(\theta, t)$ is decreasing in θ.
(c) If $t_H \geq t_L$ then $\hat{\tau}(\theta, t_H) = \hat{\tau}(\theta, t_L) \cap \{t_H, \ldots, T\}$, in states where the latter is nonempty. Hence $\hat{\tau}(\theta, t)$ is increasing in t in strict set order.

Proof. (a) Nonemptiness of $\hat{\tau}(\theta, t)$ follows from Klass (1973, Theorem 1). The set of optimal rules is characterised by Klass (1973, Theorem 6) and contains a least and greatest element. The least element can be found by using the rule: stop when current utility is weakly greater than the continuation utility (Chow, Robbins, and Siegmund (1971, Theorem 4.2)). Since the set of purchasing times is a lattice and $u_t(\theta)$ is modular in τ (i.e. both super– and submodular), the set of maximisers is a sublattice by Topkis (1998, Theorem 2.7.1).

(b) $u_t(\theta)$ has strictly decreasing differences in (θ, τ), since $\delta_t < 1$, and is modular in τ. Hence every selection is decreasing by Topkis (1998, Theorem 2.8.4).

(c) If in states $A \in \mathcal{F}$, $\tau \in \hat{\tau}(\theta, t_L) \cap \{t_H, \ldots, T\}$ maximises the utility of (θ, t_L) it must also maximise the utility of (θ, t_H) who has a smaller choice set. □

Let $\tau^*(\theta, t)$ be the least element from $\hat{\tau}(\theta, t)$. If $\tau \in \hat{\tau}(\theta, t)$ then $\tau = \tau^*(\theta, t)$ (a.e.–θ) for any state ω, by Lemma 4(b). Consequently we can assume the consumer chooses purchasing rule $\tau^*(\theta, t)$ without affecting the firm's profits. Several properties of $\tau^*(\theta, t)$ are described in Lemma 1.

A.2 Linear Demand: Examples 4 and 7

The model has particularly clean predictions when demand is linear: $f_t(\theta) = 1$ on $[0, 2b_t]$, where $b_t \in [10, 20]$. This section demonstrates that:
(a) Under the optimal policy (Theorem 1) the firm sells b_t in period t.
(b) Under complete discrimination the firm sells b_t in period t. Moreover, welfare and consumer surplus are lower than under the optimal policy.
(c) Under resale/renting the firm sells b_t in period t. Moreover, welfare and consumer surplus

are higher than under the optimal policy.

Allocations, quantities and payoffs under optimal policy. Marginal revenue is $m_t(\theta) = 2(\theta - b_t)$, so Theorem 2 implies the optimal cutoff is

$$\theta_t^* = \frac{1}{|\overline{A}(t)|} \sum_{s \in \overline{A}(t)} b_s =: b_{\overline{A}(t)}$$

where $|\overline{A}(t)|$ is the number of elements in the upper active set.

To derive quantity sold let us use the following construction. Using Lemma 2 pick a subset of periods $\{t_1, \ldots, t_n\} \subset \{1, \ldots, t-1\}$ such that $\cap_i \overline{A}(t_i) = \emptyset$ and $\cup_i \overline{A}(t_i) = \{1, \ldots, t-1\}$. Then for generation $s \in \overline{A}(t_i)$, $\theta^*(t-1; s) = b_{\overline{A}(t_i)}$. By Lemma 2, we then have $\overline{A}(t) = \cap_{i=k}^n \overline{A}(t_i) \cup \{t\}$ for some $k \in \{1, \ldots, n\}$. In period t sales are made to agents born after period t_{k-1} and the quantity sold is

$$\begin{aligned}
Q_t &= \sum_{t_{k-1} < s \leq t-1} [\theta^*(t-1; s) - \theta^*(t; s)] + [2b_t - \theta^*(t; t)] \\
&= \sum_{i=k}^{n} \sum_{s \in \overline{A}(t_i)} [b_{\overline{A}(t_i)} - b_{\overline{A}(t)}] + [2b_t - b_{\overline{A}(t)}] \\
&= \left[\sum_{i=k}^{n} \sum_{s \in \overline{A}(t_i)} b_{\overline{A}(t_i)} + b_t \right] - \left[\sum_{s \in \overline{A}(t)} b_{\overline{A}(t)} \right] + b_t \\
&= \left[\sum_{s \in \overline{A}(t)} b_s \right] - \left[\sum_{s \in \overline{A}(t)} b_s \right] + b_t = b_t
\end{aligned}$$

Welfare from sales in period t equals,

$$\Delta_t \sum_{s \leq t} \int_{\theta^*(t;s)}^{\theta^*(t-1;s)} \theta \, d\theta = \frac{\Delta_t}{2} \left[\sum_{i=k}^{n} |\overline{A}(t_i)| b_{\overline{A}(t_i)}^2 + 4b_t^2 - |\overline{A}(t)| b_{\overline{A}(t)}^2 \right]$$

Allocations, quantities and payoffs under complete discrimination. Suppose the firm can completely discriminate between generations, as in Example 2. In period t an agent buys if they are from generation t and $\theta \geq b_t$. Hence quantity b_t is sold in period t. This is the same as under the optimal policy (see above).

Under complete discrimination welfare from sales in period t equals,

$$\Delta_t \int_{b_t}^{2b_t} \theta \, d\theta = \frac{3}{2} \Delta_t b_t^2$$

This is less than welfare without complete discrimination since Jensen's inequality implies

$$\sum_{i=k}^{n} \frac{|\overline{A}(t_i)|}{|\overline{A}(t)|} b^2_{\overline{A}(t_i)} + \frac{1}{|\overline{A}(t)|} b_t^2 \geq b^2_{\overline{A}(t)}$$

Hence the welfare generated at each point in time is larger without discrimination, and welfare across all periods must also be larger, independent of the discount rate. Intuitively, the same quantity is allocated as in the optimal policy, but less efficiently. Profit is larger and welfare lower under discrimination, so consumer surplus is also lower.

Allocations, quantities and payoffs under resale/renting. Suppose the good can be resold. Theorem 3 implies that the time t cutoff is given by

$$\theta_t^R = \frac{1}{t} \sum_{s \leq t} b_s$$

The quantity sold up to period t is $\sum_{s \leq t}(2b_t - \theta_t^R) = \sum_{s \leq t} b_s$. Hence quantity b_t is sold in period t. This is the same as without resale (see above).

Welfare with resale in period t is

$$(\Delta_t - \Delta_{t+1}) \sum_{s \leq t} \int_{\theta_t^R}^{2b_t} \theta \, d\theta = (\Delta_t - \Delta_{t+1}) \sum_{s \leq t} \left[2b_t^2 - [\theta_t^R]^2\right]$$

In rental terms, welfare without resale in period t is

$$(\Delta_t - \Delta_{t+1}) \sum_{s \leq t} \int_{\theta^*(t;s)}^{2b_t} \theta \, d\theta = (\Delta_t - \Delta_{t+1}) \sum_{s \leq t} \left[2b_t^2 - [\theta^*(t;s)]^2\right]$$

Welfare is then greater with resale since

$$\sum_{s \leq t} [\theta^*(t;s)]^2 \geq \frac{1}{t} \left[\sum_{s \leq t} \theta^*(t;s)\right]^2 = \frac{1}{t} \left[\sum_{s \leq t} b_s\right]^2 = \frac{1}{t} \left[t\theta_t^R\right]^2 = \sum_{s \leq t} [\theta_t^R]^2$$

where the first equality uses the fact that the firm sells quantity b_t each period under Theorem 1. Intuitively, welfare is higher with resale since the same quantity is allocated more efficiently. With resale, profits are lower and welfare higher, so consumer surplus is also higher.

A.3 Best–Price Policy with Resale: Example 9

Suppose $T = 2$, δ is constant and A2 holds. For convenience let us drop the "BP" superscripts.

Fix the period 1 price and cutoff (p_1, θ_1^*). In period 2 the firm chooses p_2 inducing cutoff

$\theta_2^* = p_2$. Price p_2 is chosen to maximise

$$\Pi_2 = p_2[\overline{F}_{\{1,2\}}(p_2) - \overline{F}_1(\theta_1^*)] - \overline{F}_1(\theta_1^*)\max\{0, p_1 - p_2\}$$

The optimal choice of p_2 is then given by

$$\begin{aligned} p_2^* &= m_{\{1,2\}}^{-1}(0) \quad \text{if} \quad p_1 \geq m_{\{1,2\}}^{-1}(0) \\ &< m_{\{1,2\}}^{-1}(0) \quad \text{if} \quad p_1 < m_{\{1,2\}}^{-1}(0) \end{aligned} \tag{A.1}$$

Thus $p_2^* \leq \max\{p_1, m_{\{1,2\}}^{-1}(0)\}$.

In period 1 consumers' purchasing decision depends upon the current price p_1 and their expectation of the second period price p_2^e. The utility of agent θ_1^* who buys in period 1 and sells in period 2 is

$$(1-\delta)\theta_1^* - p_1 + \delta p_2^e + \delta \max\{0, p_1 - p_2^e\} \tag{A.2}$$

Since agent θ_1^* must be indifferent between buying and not, setting (A.2) equal to zero yields the first period price

$$p_1 = (1-\delta)\theta_1^* + \delta \max\{\theta_1^*, p_2^e\} \tag{A.3}$$

Fix θ_1^* and consider two cases. First suppose $\theta_1^* \geq m_{\{1,2\}}^{-1}(0)$. Equation (A.3) implies $p_1 \geq \theta_1^* \geq m_{\{1,2\}}^{-1}(0)$. Equation (A.1) implies $\theta_2^* = p_2^* = m_{\{1,2\}}^{-1}(0)$.

Second suppose $\theta_1^* < m_{\{1,2\}}^{-1}(0)$. Since $p_2^* \leq m_{\{1,2\}}^{-1}(0)$, equation (A.3) implies $p_1 < m_{\{1,2\}}^{-1}(0)$. Equation (A.1) implies $\theta_2^* = p_2^* < m_{\{1,2\}}^{-1}(0)$.

Now let us examine the optimal period 1 cutoffs. First, suppose demand is decreasing. The firm can choose $\theta_1^* = m_1^{-1}(0) \geq m_{\{1,2\}}^{-1}(0)$ which induces $\theta_2^* = m_{\{1,2\}}^{-1}(0)$ in period 2. This implements the commitment optimum and is therefore optimal in period 1.

Next, suppose demand is not decreasing. If the firm chooses $\theta_1^* = m_1^{-1}(0) < m_{\{1,2\}}^{-1}(0)$ the optimal period 2 choice is $p_2^* < m_{\{1,2\}}^{-1}(0)$ and the firm overproduces. Hence the commitment optimum is not implementable. It can be shown that the period 2 cutoff θ_2^* is increasing in period 1 cutoff θ_1^*. It follows that the optimal period 1 cutoff satisfies $\theta_1^* \geq m_1^{-1}(0)$.

References

Bass, F. M. (1969): "A New Product Growth Model for Consumer Durables," *Management Science*, 15(5), 215–227.

Biehl, A. R. (2001): "Durable-Goods Monopoly with Stochastic Values," *RAND Journal of Economics*, 32(3), 565–577.

Board, S. (2004): "Selling Options," Working paper, University of Toronto.

Bulow, J. (1982): "Durable-Goods Monopolists," *Journal of Political Economy*, 90(2), 314–332.

Butz, D. A. (1990): "Durable-Good Monopoly and Best-Price Provisions," *American Economic Review*, 80(5), 1062–1076.

Chow, Y. S., H. Robbins, and D. Siegmund (1971): *Great Expectations: The Theory of Optimal Stopping*. Houghton Mifflin.

Coase, R. H. (1972): "Durability and Monopoly," *Journal of Law & Economics*, 15(1), 143–149.

Conlisk, J. (1984): "A Peculiar Example of Temporal Price Discrimination," *Economics Letters*, 15(1–2), 121–126.

Conlisk, J., E. Gerstner, and J. Sobel (1984): "Cyclic Pricing by a Durable Goods Monopolist," *Quarterly Journal of Economics*, 99(3), 489–505.

Davidson, J. (1994): *Stochastic Limit Theory*. Oxford University Press, Oxford.

Dudine, P., I. Hendel, and A. Lizzeri (2005): "Storable Good Monopoly: The Role of Commitment," Working paper, NYU.

Hendel, I., and A. Lizzeri (1999): "Adverse Selection in Durable Goods Markets," *American Economic Review*, 89(5), 1097–1115.

Klass, M. (1973): "Properties of Optimal Extended–Valued Stopping Rules for S_n/n," *Annals of Probability*, 1(5), 719–757.

Laffont, J.-J., and J. Tirole (1996): "Pollution Permits and Compliance Strategies," *Journal of Public Economics*, 62(1–2), 85–125.

Landsberger, M., and I. Meilijson (1985): "Intertemporal Price Discrimination and Sales Strategy under Incomplete Information," *RAND Journal of Economics*, 16(3), 424–430.

Levhari, D., and R. S. Pindyck (1981): "The Pricing of Durable Exhaustible Resources," *Quarterly Journal of Economics*, 96(3), 365–378.

Milgrom, P., and I. Segal (2002): "Envelope Theorems for Arbitrary Choice Sets," *Econometrica*, 70(2), 583–601.

Myerson, R. (1981): "Optimal Auction Design," *Mathematics of Operations Research*, 6(1), 58–73.

Rotemberg, J. J., and M. Woodford (1999): "The Cyclical Behaviour of Prices and Costs," *in Handbook of Macroeconomics*, pp. 1051–1135, (J.B. Taylor and M. Woodford eds.).

Rust, J. (1985): "Stationary Equilibrium in a Market for Durable Assets," *Econometrica*, 53(4), 783–806.

——— (1986): "When is it Optimal to Kill off the Market for Used Durable Goods?," *Econometrica*, 54(1), 65–86.

Segal, I. (2003): "Awarding Monopoly Franchises," *American Economic Review*, 93(3), 509–529.

Sobel, J. (1991): "Durable Goods Monopoly with Entry of New Consumers," *Econometrica*, 59(5), 1455–1485.

Sobel, J., and I. Takahashi (1983): "A Multistage Model of Bargaining," *Review of Economic Studies*, 50(3), 411–426.

Stokey, N. L. (1979): "Intertemporal Price Discrimination," *Quarterly Journal of Economics*, 93(3), 355–371.

Tirole, J. (1988): *The Theory of Industrial Organization*. MIT Press, Cambridge, Massachusetts.

Topkis, D. M. (1998): *Supermodularity and Complimentarity*. Princeton University Press, New Jersey.

Waldman, M. (1996): "Durable Goods Pricing When Quality Matters," *Journal of Business*, 69(4), 489–510.

Wang, R. (2001): "Optimal Pricing Strategy for Durable-Goods Monopoly," *Journal of Economic Dynamics and Control*, 25(5), 789–804.