# Learning Dynamics in Social Networks

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# BMW i3

# Motivation

## How do societies learn about innovations?

- New products, e.g. electric cars.
- New production techniques, e.g. pineapples.
- New sources of finance, e.g. microfinance.

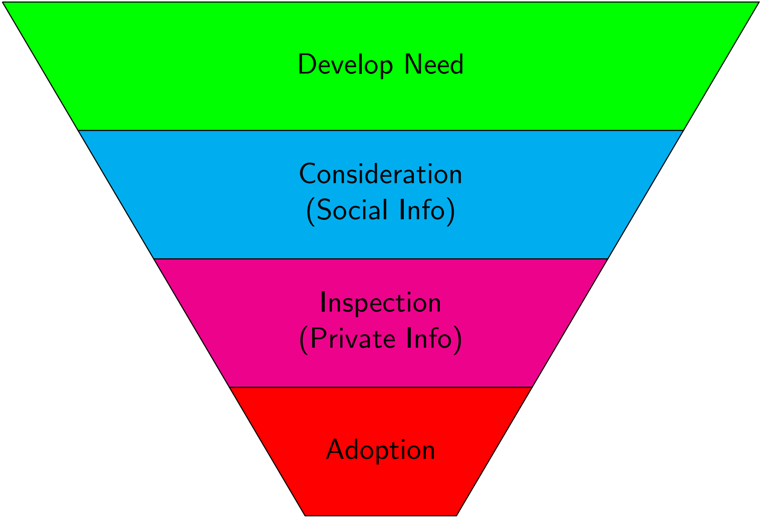
## Two sources of information

- Social information acquired from neighbors.
- Private information if inspect innovation.

## How does diffusion depend on the network?

- Is diffusion faster in more interconnected societies?
- Is diffusion faster in more centralized societies?

# The Social Purchasing Funnel



# Social Learning Curves

## Modeling approach

- Agents learn private information *after* inspection.
- Characterize social learning curves for any network via ODEs.

## Social learning in tree networks

- Learning from neighbors, and neighbors' neighbors
- Learning from direct vs. indirect links

## Network structure

- Learning from backward and correlating links.
- Characterize agent's favorite network.
- Compare centralized and decentralized networks.

# Literature

## Diffusion on networks

- Bass (1969)
- Morris (2000)
- Campbell (2013), Sadler (2017)

## Social learning on networks

- Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992)
- Smith and Sorensen (1996), Acemoglu et al (2011)
- Mueller-Frank and Pai (2016), Ali (2017), Lomys (2017)

## Social Learning and Adoption

- Guarino, Harmgart and Huck (2011)
- Hendricks, Sorensen and Wiseman (2012)
- Herrera and Horner (2013)

*"A significant gap in our knowledge concerns short-run dynamics and rates of learning in these models....The complexity of Bayesian updating in a network makes this difficult, but even limited results would offer a valuable contribution to the literature."*

Golub, Sadler, in Oxford Handbook 2016

# Model

# Model

## Players and Products

- $I$ players $i$ on exogenous, directed network $G$.
- Product quality $\theta \in \{L, H\}$, where $\Pr(H) = \pi_0$.

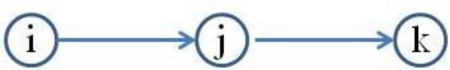
## Timing: Player $i$

- ...enters at iid "time" $t_i \sim U[0,1]$.
- ...observes which of her neighbors $N_i$ adopt product by $t_i$.
- ...can inspect product at iid cost $c_i \sim F$.
- ...adopts product iff inspected *and* $\theta = H$.

## Payoffs

- Player gets 1 if adopts; 0 otherwise, net of inspection cost $c_i$.

# The Inference Problem




$i$ sees $j$ has adopted

- Quality is high, $\theta = H$

$i$ sees $j$ has not adopted

- $j$ tried product, but quality is low, $\theta = L$?
- $j$ chose not to try product (maybe $k$ did not adopt)?
- $j$ has not yet entered, $t_j \geq t$?

# Example

# Directed Pair


## Definition: Adoption rate

$x_{i,t}$: Probability $i$ adopts product $H$ by time $t$

## Leader, $j$:

$$\dot{x}_{j,t} = \Pr(j \text{ inspect}) = F(\pi_0)$$

## Follower, $i$:

$$\dot{x}_{i,t} = \Pr(i \text{ inspect})$$

# Directed Pair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## Follower, $i$:

$$\dot{x}_{i,t} = 1 - \Pr(j \text{ not adopt}) \times \Pr(c_i \text{ high})$$

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$$\dot{x}_{j,t} = \Pr(j \text{ inspect}) = F(\pi_0)$$

## Follower, $i$:

$$\dot{x}_{i,t} = 1 - (1 - x_{j,t})(1 - F(\pi_t^{\varnothing}))$$

with posterior $\pi_t^{\varnothing} = \frac{\pi_0(1-x_{j,t})}{\pi_0(1-x_{j,t})+1-\pi_0}$

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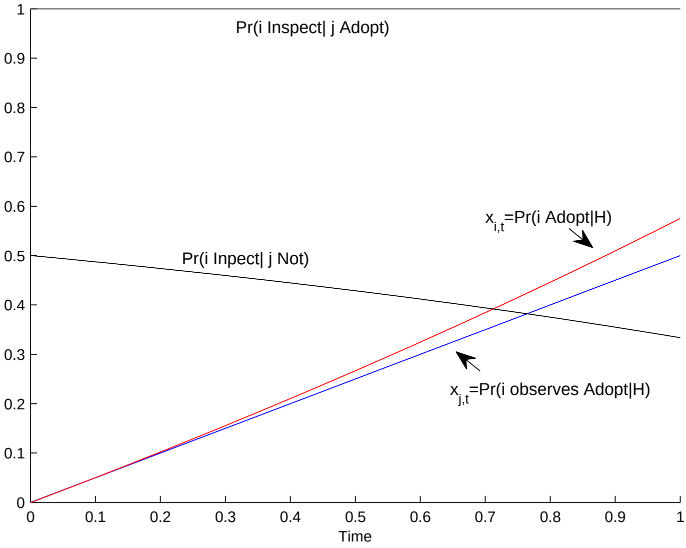
$$\dot{x}_{i,t} = 1 - (1 - x_{j,t})(1 - \tilde{F}(1 - x_{j,t}))$$

where $\tilde{F}(1-x) := F\left(\frac{\pi_0(1-x)}{\pi_0(1-x)+1-\pi_0}\right)$

# Agent $i$'s Social Learning Curve




Assumptions: $c \sim U[0,1]$, $\pi_0 = 1/2$.

# General Networks: Preliminaries

# Individual Adoption Rates . . . are not enough

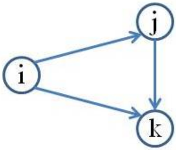
## A general formula for individual adoption rates

- $x^{-i}_{N_i,t}$: Probability some of $i$'s neighbors adopt $H$ by $t \leq t_i$.

$$\dot{x}_i = 1 - (1 - x^{-i}_{N_i})(1 - \tilde{F}(1 - x^{-i}_{N_i}))$$

## But cannot recover joint $x_{N_i}$ (or $x^{-i}_{N_i}$) from marginals $x_j$


Self-reference




Correlation

# The Social Learning Curve

## Definition: $i$'s social learning curve

- $x_{N_i,t}^{-i}$: Probability some of $i$'s neighbors adopts $H$ by $t \leq t_i$.

## Fact: $i$'s information Blackwell-increasing in $x_{N_i}^{-i}$

- $i$'s signal structure

| | $\geq 1$ adopt | $0$ adopt |
|---|---|---|
| $\theta = H$ | $x_{N_i}^{-i}$ | $1 - x_{N_i}^{-i}$ |
| $\theta = L$ | $0$ | $1$ |

- Signal $x < x'$ equiv. to "losing" adopt signal with prob. $\frac{x'-x}{x'}$.

# Social Learning and Adoption

**Assumption:** Costs have a bounded hazard rate (BHR) if

$$\frac{f(c)}{1-F(c)} \leq \frac{1}{(1-c)c} \quad \text{for } c \in [0, \pi_0] \tag{1}$$

- Satisfied if $f(c)$ weakly increasing, e.g. $c \sim U[0,1]$
- At bottom, when $c \approx 0$, always satisfied as RHS $\to \infty$.
- At top, holds with equality when $f(c) \propto 1/c^2$

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**Lemma 1.**
*Assume BHR. Adoption* $(x_{i,t})_t$ *rises in information* $(x^{-i}_{N_i,t})_t$.

## Idea

- Recall adoption probabilities are conditional on $\theta = H$
- Hence $E[\pi_t|H]$ exceeds $\pi_0$ and increases in information $x^{-i}_{N_i,t}$
- Compare: Increase in adoption given a neighbor adopts
  Decrease in adoption given no neighbor adopts

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## Counterexample

- Suppose $F \sim U[0, \pi_0]$
- Adoption maximized for zero social learning.

# Social Learning Improves over Time

**Lemma 2.**
*In any network, agent $i$'s information Blackwell improves over time. Under BHR, her adoption probability increases over time.*

## Idea

- Over time more people adopt, so $x_{N_i,t}^{-i}$ increases in $t$.
- Apply Lemma 1.

# Information Aggregation in Complete Networks

- Lowest cost type, $\underline{c} := \sup\{c | F(c) = 0\}$.

**Lemma 3 (HSW '12, HH '13).**

*In a complete network with $I \to \infty$ agents:*
*(a) Bad products fail, $\Pr_I^L(i \text{ inspects}) \to 0$.*
*(b) Good products succeed, $\Pr_I^H(i \text{ inspects}) \to 1$, iff $\underline{c} = 0$.*

## Proof

- Adoption: For all $t > 0$, as $I \to \infty$, $x_{N_i,t}^{-i}$ converges to

$$\bar{x} := \inf\{x : \tilde{F}(1 - x) = 0\}$$

- By definition $\frac{\pi(1-\bar{x})}{1-\pi\bar{x}} = \underline{c}$, and so $\bar{x} = 1$ iff $\underline{c} = 0$.

- Inspection: If $\theta = L$ at $t_i = t$:

$$\Pr_I^L(i \text{ inspects}) = \tilde{F}(1 - x_{N_i,t}^{-i}) \to 0$$

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- Inspection: If $\theta = H$ at $t_i = t$:

$$\mathrm{Pr}_I^H(i \text{ inspects}) = 1 - (1 - x_{N_i,t}^{-i})(1 - \tilde{F}(1 - x_{N_i,t}^{-i})) \to \bar{x}$$

# General Networks: Characterization

# A Larger State Space

## State of network $\lambda \in \{\emptyset, a, b\}^I$

- $\lambda_i = \emptyset$: $i$ hasn't moved yet, $t \leq t_i$.
- $\lambda_i = a$: $i$ has moved, tried, and adopted the product.
- $\lambda_i = b$: $i$ has moved, but not adopted the product.

## Agent $i$'s knowledge in state $\lambda$

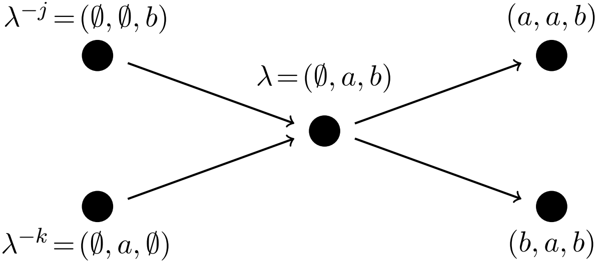
$$\Lambda(i, \lambda) := \{\lambda' : \lambda'_i = \lambda_i, \lambda_j = a \text{ iff } \lambda'_j = a \text{ for all } j \in N_i\}$$

## Additional notation

- Distribution $z = (z^\theta_\lambda)$, and $z^\theta_\Lambda := \sum_{\lambda \in \Lambda} z^\theta_\lambda$ for sets $\Lambda$.
- For $\lambda$ with $\lambda_i = a, b$, write $\lambda^{-i}$ for "same state with $\lambda_i = \emptyset$".

# State Transitions

Three agents $(i, j, k)$, state $\lambda = (\lambda_i, \lambda_j, \lambda_k)$



# ODE for General Networks

**Theorem 1.**
*Given quality $\theta = L, H$, the state evolves according to the ODE:*

$$
\begin{aligned}
\dot{z}_\lambda^H = & - \frac{1}{1-t} \sum_{i:\lambda_i=\emptyset} z_\lambda^H \\
& + \frac{1}{1-t} \sum_{i:\lambda_i=a} z_{\lambda^{-i}}^H \tilde{F}\left(\frac{z_{\Lambda(i,\lambda^{-i})}^H}{z_{\Lambda(i,\lambda^{-i})}^L}\right) \\
& + \frac{1}{1-t} \sum_{i:\lambda_i=b} z_{\lambda^{-i}}^H \left[1 - \tilde{F}\left(\frac{z_{\Lambda(i,\lambda^{-i})}^H}{z_{\Lambda(i,\lambda^{-i})}^L}\right)\right] \\
z_\lambda^L = & \ (1-t)^{\#\{i:\lambda_i=\emptyset\}} t^{\#\{i:\lambda_i=b\}} 0^{\#\{i:\lambda_i=a\}}
\end{aligned}
$$

## Implications

- Existence, uniqueness, discrete-time approximation ...
- But: ODE cannot be computed, since it is $3^I$-dimensional.

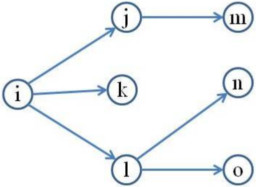
# Tree Networks

# Trees

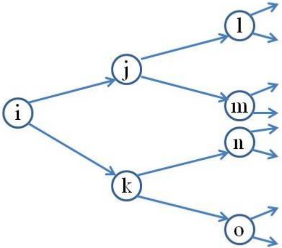
- Abstract from Self-reference and Correlation problems.
- Approximate large random network with finite degree.
- Resemble hierarchies seen in firms or on Twitter.

## Network $G$ is ...

- ... a *tree* if there is at most one path $i \to \ldots \to j$.
- ... *regular with degree* $d$ if every node has out-degree $d$.




Tree




Homogeneous Tree

# Adoption in Trees

## Conditional independence

- $(x_j)_{j \in N_i}$ independent of $\lambda_i = \emptyset$.
- Neighbors' adoption $(x_j)_{j \in N_i}$ conditionally independent.

## Probability some of $i$'s neighbors $N_i$ adopts:

$$x_{N_i}^{-i} = x_{N_i} = 1 - \prod_{j \in N_i} (1 - x_j)$$

## Individual adoption rates

$$\dot{x}_i = 1 - (1 - x_{N_i})(1 - \tilde{F}(1 - x_{N_i}))$$

- $I$-dimensional ODE.

# Adoption in Regular Trees

Probability some neighbor adopts

$$1 - (1 - x)^d$$

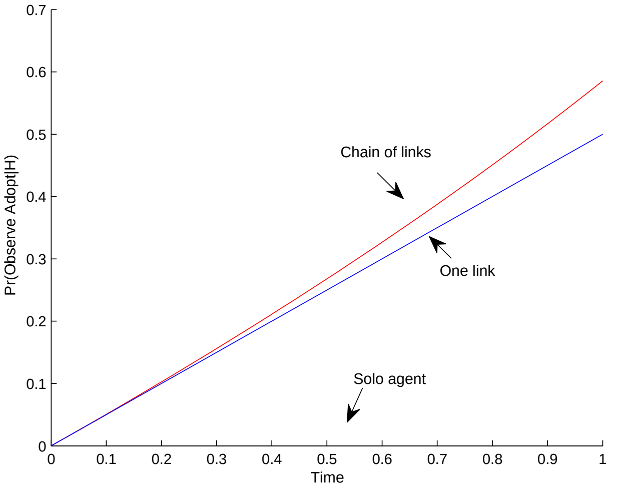
Evolution of individual adoption rates

$$\dot{x} = 1 - (1 - x)^d (1 - \tilde{F}((1 - x)^d))$$

- 1-dimensional ODE.

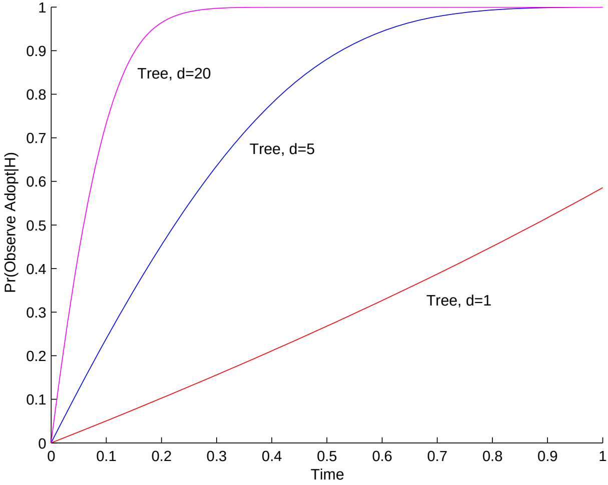
# Comparative Statics in Tree Networks

# Social Learning Curves: More Informed Neighbors




Assumptions: $c \sim U[0,1]$, $\pi_0 = 1/2$.

# Social Learning Curves: More Neighbors




Regular tree with $d$ neighbors, $c \sim U[0,1]$, $\pi_0 = 1/2$.

# Social Learning Improves in Links

- Consider tree $\hat{G}$ with subtree $G \subseteq \hat{G}$. Adoption rates: $\hat{x}_i, x_i$.

**Theorem 2.**

*Assume BHR. Social learning improves in links: For any agent $i$,*

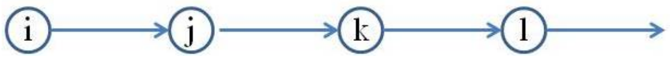
$$x_{N_i} \leq \hat{x}_{\hat{N}_i} \qquad (*)$$

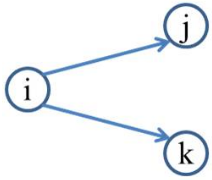
## Prove (*) by induction

- Leaves $i$ of $G$: $x_{N_i} = 0 \leq \hat{x}_{\hat{N}_i}$
- Fix any $i$ and assume (*) holds for all $j \in N_i$.
- By BHR, agent $j$ adopts more $x_j \leq \hat{x}_j$.
- Additionally, $i$ has more neighbors, $N_i \subseteq \hat{N}_i$. Thus:

$$x_{N_i} = 1 - \prod_{j \in N_i} (1 - x_j) \leq 1 - \prod_{j \in \hat{N}_i} (1 - \hat{x}_j) = \hat{x}_{\hat{N}_i}$$

# Direct vs Indirect Links



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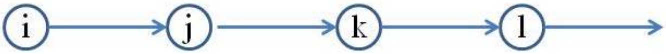
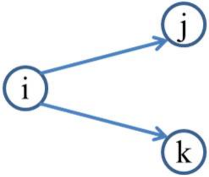



$$\dot{\vec{x}} = 1 - (1 - \vec{x})(1 - \tilde{F}(1 - \vec{x})) \leq 1 - (1 - \vec{x})(1 - \tilde{F}(1))$$



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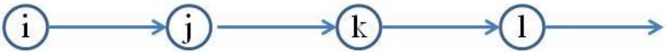
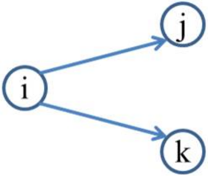



$$\vec{x}_t \leq \frac{\tilde{F}(1)}{1-\tilde{F}(1)} \exp((1-\tilde{F}(1))t-1)$$



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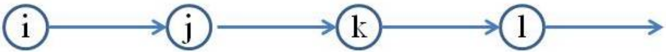
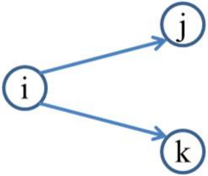
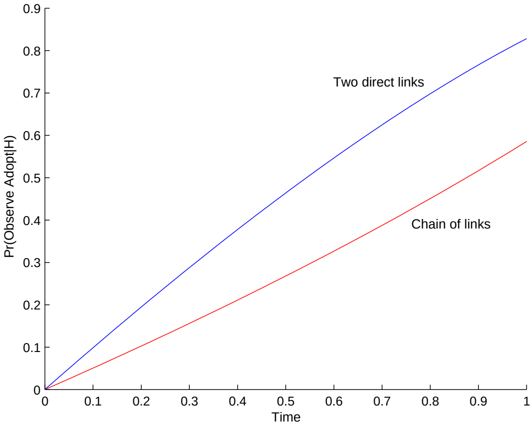



$$\vec{x}_t \leq \frac{\tilde{F}(1)}{1-\tilde{F}(1)} \exp((1-\tilde{F}(1))t-1)$$




$$\check{x}_t = x_{\{j,k\},t} = 1-(1-\tilde{F}(1)t)^2$$

# Direct vs Indirect Links




**Theorem 3.**
*Two direct links are superior to line of indirect ones:* $\check{x}_t > \vec{x}_t$.

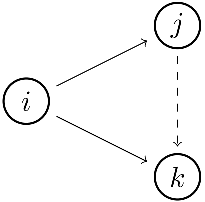
# Beyond Trees

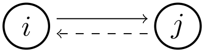
# Are All Links Beneficial?

## Rationale for Theorem 2

- Indirect links induce neighbors to inspect.
- Learn from neighbors' inspections and adoptions.

## How about correlating and backward links?



# Adding a Correlating Link


## The correlating link lowers $i$'s information and utility

- Agent $i$'s only learns from $j$ if $k$ has not adopted.
- In this event, adding $j \to k$ *reduces* $j$'s adoption.

# Adding a Backward Link

$$i \longrightarrow j \qquad\qquad\qquad i \rightleftarrows j$$

## The backward link lowers $i$'s information and utility

- Before $t_i$, agent $j$ never observes adoption by $i$.
- $x_{j,t}^{-i}$: Probability $j$ adopts product $H$ by $t \leq t_i$.

$$\dot{x}_{j,t}^{-i} = \Pr(j \text{ inspect} | i \text{ not})$$

# Adding a Backward Link

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## The backward link lowers $i$'s information and utility

- Before $t_i$, agent $j$ never observes adoption by $i$.
- $x_{j,t}^{-i}$: Probability $j$ adopts product $H$ by $t \leq t_i$.

$$\begin{aligned} \dot{x}_{j,t}^{-i} &= \tilde{F}(1 - x_{i,t}^{-j}) \leq \tilde{F}(1) \\ x_{j,t}^{-i} &\leq \tilde{F}(1)t = x_{j,t} \end{aligned}$$

where $x_{j,t}$ is $j$'s adoption probability in $i \to j$.

# Self-Referential and Correlating Links

$G$ is $i$-*tree* iff ...

1. $i$ has no backward links $B := \{j \to i\}$.
2. $i$'s neighbors $j, j' \in N_i$ are independent: $S_j \cap S_{j'} = \emptyset$; in particular, there are no correlating links $C := \{j \to j'\}$.

Adding self-referential and correlating links to an $i$-tree

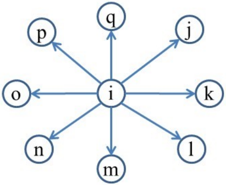
$$\hat{G} \text{ with } G \subsetneq \hat{G} \subset G \cup C \cup B.$$

**Theorem 4.**
*Backward and correlating links harm $i$'s learning:* $\hat{x}_{N_i}^{-i} < x_{N_i}$.

**Idea:** Links $C \cup B$ only matter when they convey bad news.

# Optimality of the Star Network

The $i$-Star




**Theorem 5.**
*The $i$-star maximizes $i$'s learning: For any $G \neq G^*$, $\hat{x}^*_{N_i^*} > x^{-i}_{N_i}$.*

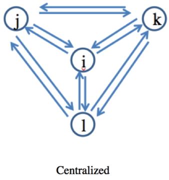
In $i$-Star

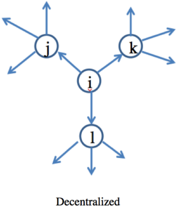
- $i$ observes no adoption at $t_i$ then $c_j > \pi \ \forall \{j : t_j < t_i\}$. (*)

In arbitrary network $G$, if (*) holds

- $j$ with lowest $t_j$ observes no adopt. $\Rightarrow$ does not inspect.
- $j'$ with next-lowest $t_{j'}$ observes no adopt. $\Rightarrow$ does not inspect.

# Centralized Networks vs. Decentralized Networks




**Theorem 6.**
*Assume BHR, and all agents have $d$ neighbors. All agents prefer large random network over complete network.*

## Idea

- Agent $i$'s optimal network is the $i$-star.
- Complete network worse: add correlated and reverse links.
- Random network better under BHR: add new information.

# Imperfect Information from Adoption

# Imperfect Learning

## Agents have idiosyncratic preferences

- Adopt with probability $q^\theta$ in state $\theta$.

## Social learning curves

- May see multiple adoptions
- Let $\{x_{A,t}^{-i}, y_{A,t}^{-i}\}$ be prob. $A \subset N_i$ adopt if $\theta \in \{H, L\}$.

## Adoption rates in general network

$$\dot{x}_i = q^H \sum_{A \subseteq N_i} x_A^{-i} \tilde{F}\left(\frac{x_A^{-i}}{y_A^{-i}}\right)$$

# Imperfect Learning

## Agents have idiosyncratic preferences

- Adopt with probability $q^\theta$ in state $\theta$.

## Social learning curves

- May see multiple adoptions
- Let $\{x_{A,t}^{-i}, y_{A,t}^{-i}\}$ be prob. $A \subset N_i$ adopt if $\theta \in \{H, L\}$.

## Adoption rates in tree

$$\dot{x}_i = q^H \sum_{A \subseteq N_i} x_A \tilde{F}\left(\frac{x_A}{y_A}\right) \quad \text{for} \quad x_A = \prod_{j \in A} x_j \prod_{j \in N_i \setminus A} (1 - x_j)$$

# Imperfect Learning

## Agents have idiosyncratic preferences

- Adopt with probability $q^\theta$ in state $\theta$.

## Social learning curves

- May see multiple adoptions
- Let $\{x_{A,t}^{-i}, y_{A,t}^{-i}\}$ be prob. $A \subset N_i$ adopt if $\theta \in \{H, L\}$.

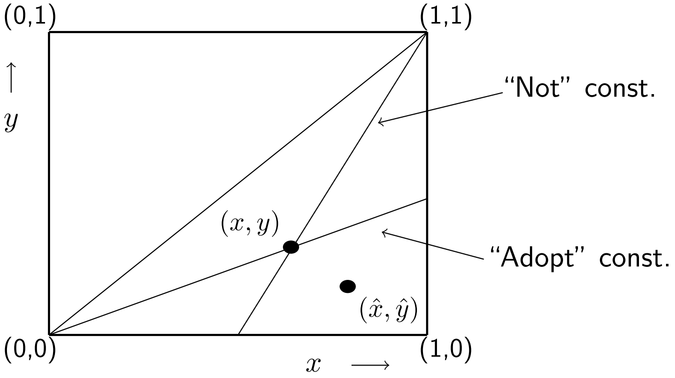
## Adoption rates in regular tree of degree $d$

$$\dot{x} = q^H \sum_{\nu=0}^{d} x_{(\nu,d)} \tilde{F}\left(\frac{x_{(\nu,d)}}{y_{(\nu,d)}}\right) \quad \text{for} \quad x_{(\nu,d)} := \begin{pmatrix} \nu \\ d \end{pmatrix} x^\nu (1-x)^{d-\nu}$$

# Comparative Statics: Blackwell Sufficiency

Experiment $(\hat{x}, \hat{y})$ is more informative than $(x, y)$ iff

$$\frac{\hat{x}}{\hat{y}} \geq \frac{x}{y} \quad \text{and} \quad \frac{1-\hat{x}}{1-\hat{y}} \leq \frac{1-x}{1-y}$$



# Comparative Statics for Trees

**Lemma 2.**
*Assume BHR. If social learning curves of $N_i$ are more informative then $i$'s adoption is more informative.*

## This implies that on trees

- Adoption is more informative over time.
- Adoption is more informative in direct and indirect links.
- Adoption is more informative if $q^H$ rises or $q^L$ falls.

## Also in some examples

- Self-referential links lower informativeness of adoption.

# Imperfect Information of Networks

# Imperfect Information Poisson Trees

## Known neighbors: Observes $A$ *and* $N_i$

- Probability of $\iota$ neighbors: $P(\iota|k) := e^{-k}k^{\iota}/\iota!$

$$\dot{x} = \sum_{\iota=0}^{\infty} P(\iota|k)[1-(1-x)^{\iota}(1-\tilde{F}((1-x)^{\iota}))]$$

## Unknown neighbors: Observes *only* $A$, not $N_i$

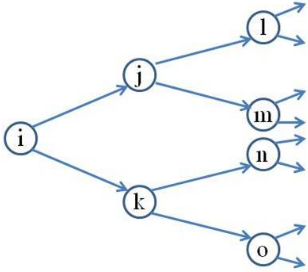
- Probability no neighbor adopts: $e^{-kx}$

$$\dot{x} = 1 - e^{-kx}(1-\tilde{F}(e^{-kx}))$$

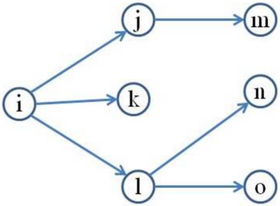
## Under BHR, more social learning with known neighbors.

- The number of neighbors $N_i$ is directly informative.
- This compounds and increases everyone's information.

# Deterministic vs Random Trees




DeterministicTree




Random Tree

# Deterministic vs Random Trees

- Random tree with $D$ links, known neighbors

$$\dot{\hat{x}} = E[1 - (1 - \hat{x})^D (1 - \tilde{F}((1 - \hat{x})^D))]$$

- Deterministic tree with $d = E[D]$ links

$$\dot{x} = 1 - (1 - x)^d (1 - \tilde{F}((1 - x)^d)) =: \phi(x, d)$$

If $\pi_0 \leq 1/2$, $c \sim U[0, 1]$, more social learning in Determ. tree.

- $\phi(x, d)$ concave in $d$, and so $E[\phi(x, D)] < \phi(x, d)$.
- Hence $\hat{x} \leq x$, and so $E[1 - (1 - \hat{x})^D] \leq 1 - (1 - x)^d$

# Undirected Poisson Networks

## Two Complications

- $i$'s neighbors $j$ have $P(\cdot|k) + 1$ neighbors.
- Before $t_i$, $j$ conditions on $\lambda_i = \emptyset$.

## Known neighbors

- $j$ has $\iota \sim P(\cdot|d)$ neighbors (and $i$, who has not adopted)

$$\dot{x} = \sum_{\iota=0}^{\infty} P(\iota|d)[1 - (1-x)^{\iota}(1 - \tilde{F}((1-x)^{\iota+1}))]$$

## Unknown neighbors

- Complications cancel, since $j$ can't see $i$ before $t_i$.
- $j$ observes $\nu \sim P(\cdot|dx)$ adoptions from $\iota \sim P(\cdot|d)$ neighbors.
- Same adoption rates $x_t$ as in directed Poisson network.

# Conclusion

## A tractable model of learning in networks

- Agents learn private information *after* inspection.
- Exogenous network, independent of timing.

## Social learning curves

- Describe full dynamics via ODEs.
- "Value function" of links in trees.
- Effects of undirected learning and correlation.
- Optimality of the star network.

## Future work

- Impact of network on aggregates: Welfare, diffusion.
- Policies: Pricing, advertising and seeding.
- And much more...