Reputation for Quality

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Overview

Investment and Reputation

- "Firm" can invest into future quality
- Moral hazard due to imperfect observability
- Reputation gives firm incentive to invest

Modeling Innovation

- Persistent quality: function of past investments
- Reputation: belief over endogenous state variable

Project Analyzes

- Reputational investment incentives
- Reputational dynamics

Learning Processes

Perfect Good News - Labor markets

- Market discovers high quality via "breakthroughs"
- Work-Shirk Equilibrium & Ergodic Dynamics

Perfect Bad News - Computer industry

- Market discovers low quality via "breakdowns"
- Shirk-Work Equilibria & Non-ergodic Dynamics

Imperfect Learning - Automotive

- Gradual market learning through consumer reports
- Work-Shirk Equilibrium & Ergodic Dynamics ...

Literature - Reputation

Theory

- Moral Hazard: Kreps (1990), ...
- Adverse Selection: Bar-Isaac (2003), ...
- Combination:
  - Kreps, Wilson (1982)
  - Holmstrom (1999)
  - Mailath, Samuelson (2001), ...

Empirical

- eBay: Cabral, Hortacsu (2008); Resneck et al. (2006)
- Airlines: Bosch et al. (1998); Chalk (1987)
- Restaurant Hygiene: Jin, Leslie (2009)

Outline

Bare-Bones Model

Players: One long-lived firm, many short-lived consumers

Timing: Continuous time $t \in [0, \infty)$, discount rate r

- Quality $\theta_t \in \{L = 0, H = 1\}$
- Invest $\eta_t \in [0, 1]$ at marginal cost c
- Expected consumption utility θ_t
- Reputation $x_t = \mathbb{E}[\theta_t]$

MPE: Beliefs $\widetilde{\eta} = \widetilde{\eta}(x)$, strategies $\eta = \eta(\theta, x)$ with

(1) $\eta(x_t, \theta_t)$ maximizes value $V_\theta(x) = \int e^{-rt}\mathbb{E}[x_t - c\eta_t]\, dt$

(2) Correct beliefs: $\widetilde{\eta}(x) = \mathbb{E}[\eta(\theta, x) | x]$

Fleshing out the Model

Technology: Poisson shocks with intensity λ

- At shock, effort determines quality $\Pr(\theta_t = H) = \eta_t$
- Otherwise, quality is constant $\theta_t = \theta_{t-dt}$

$$\Pr(\theta_t = H) = \int_0^t e^{\lambda(s-t)} \lambda \eta_s ds + e^{-\lambda t} \Pr(\theta_0 = H)$$

Information: Consumers update reputation x_t:

(1) Poisson signal with arrival rate μ_L, μ_H

(2) Believed effort $\tilde{\eta}_t$

$$dx_t = \text{"}Bayes\text{"} + \lambda(\tilde{\eta}_t - x_t)dt$$

Bayesian Learning from Poisson Signals

Perfect Good News: Product breakthrough with probability $\theta_t dt$

- Breakthrough: x_t jumps to 1
- Otherwise: $dx = -x(1-x)\,dt$

Perfect Bad News: Product breakdown with prob. $(1-\theta_t)dt$

- Breakdown: x_t jumps to 0
- Otherwise: $dx = x(1-x)\,dt$

Imperfect News: Signal with net arrival rate $\mu = \mu_H - \mu_L$

- Arrival: x_t jumps to $j(x) = x + \mu x(1-x)(\cdots)$
- Otherwise: $dx = -\mu x(1-x)\,dt$

First-Best Effort

Lemma: First-best effort $\eta \in [0,1]$ satisfies

$$\eta(x) = \begin{cases} 1 & \text{if } c < \frac{\lambda}{\lambda+r} \\ 0 & \text{if } c > \frac{\lambda}{\lambda+r} \end{cases}$$

Proof: Social benefit of effort is:

- ... social benefit of high quality 1, times
- ... probability ot technology shock λdt, annuitized by
- ... effective discount rate $r + \lambda$.

Always assume that effort is socially beneficial, i.e. $c < \frac{\lambda}{\lambda+r}$.

Equilibrium Characterization

Lemma: Optimal effort $\eta(x)$ is:

- Independent of quality θ,
- Bang-bang in reputation:

$$\eta(x) = \begin{cases} 1 & \text{if } c < \lambda\Delta(x), \\ 0 & \text{if } c > \lambda\Delta(x), \end{cases}$$

where $\Delta(x) := V_H(x) - V_L(x)$ is value of quality.

Proof:

- Probability of technology shock: λdt
- Benefit in case of shock: $\Delta(x)$

Asset Value of Quality

$$\Delta(x) = V_H(x) - V_L(x)$$

Theorem: In any MPE, Δ is present value of $D_H(x_t)$:

$$\Delta(x_0) = \int_0^\infty e^{-(r+\lambda)t}\mathbb{E}_{\theta_{\le t}=L}[D_H(x_t)]dt.$$

Specifically

$$D_H(x) = V_H(1) - V_H(x) \quad \text{(Good)}$$

$$D_L(x) = V_L(x) - V_L(0) \quad \text{(Bad)}$$

$$D_H(x) = \mu(V_H(j(x)) - V_H(x)) \quad \text{(Imperfect)}$$

Asset Value of Quality

$$\Delta(x) = (1-(r+\lambda)dt)\mathbb{E}[V_H(x+d_Hx) - V_L(x+d_Lx)]$$

Theorem: In any MPE, Δ is present value of $D_H(x_t)$:

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Asset Value of Quality

$$\Delta(x) = (1-(r+\lambda)dt)\mathbb{E}[V_H(x+d_H x) - V_H(x+d_L x)] + (1-(r+\lambda)dt)\mathbb{E}[V_H(x+d_L x) - V_L(x+d_L x)]$$

Theorem: In any MPE, Δ is present value of $D_H(x_t)$:

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Asset Value of Quality

$$\begin{aligned}\Delta(x) =& (1-(r+\lambda)dt)\mathbb{E}[V_H(x+d_Hx)-V_H(x+d_Lx)] \\ &+(1-(r+\lambda)dt)\mathbb{E}[\Delta(x+d_Lx)]\end{aligned}$$

Theorem: In any MPE, Δ is present value of $D_H(x_t)$:

$$\Delta(x_0) = \int_0^\infty e^{-(r+\lambda)t}\mathbb{E}_{\theta_{\leq t}=L}[D_H(x_t)]dt.$$

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Asset Value of Quality

$$\begin{aligned}\Delta(x) =& (1-(r+\lambda)dt)\mathbb{E}[V_H(x+d_H x) - V_H(x+d_L x)] \\ &+ (1-(r+\lambda)dt)\mathbb{E}[\Delta(x+d_L x)] \\ =& \textbf{Reputational Dividend} + \textbf{Cont Value}\end{aligned}$$

Theorem: In any MPE, Δ is present value of $D_H(x_t)$:

$$\Delta(x_0) = \int_0^\infty e^{-(r+\lambda)t}\mathbb{E}_{\theta_{\le t}=L}[D_H(x_t)]dt.$$

Specifically

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Asset Value of Reputation

Reputation x has asset value:

- Current revenue x
- Future revenue $x_t|_{x_0=x}$

Lemma: In MPE firm value $V_\theta(x)$ is strictly increasing in x.

Proof:

- Firm $x' > x$ can mimick x
- Same effort & quality $\Rightarrow x'_t \geq x_t$ for all t
- In MPE firm x' does at least as good

Perfect Good News

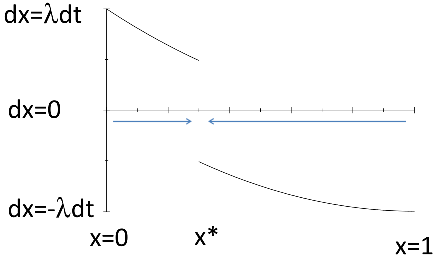
Updating & Dynamics

Reputational Updating: Breakthrough at rate $\mu = 1$ if $\theta = H$

- Breakthrough: x_t jumps to 1
- Otherwise: $dx = \lambda \left(\widetilde{\eta} \left(x \right) - x \right) dt - x \left(1 - x \right) dt$

"Work-Shirk" profile with cut-off x^*:

$$\eta \left(x \right) = \begin{cases} 1 & \text{for } x < x^* \\ 0 & \text{for } x > x^* \end{cases}$$



Work-Shirk

Proposition: Every equilibrium is work-shirk.

Proof:

$$\Delta\left(x_0\right) = \int e^{-(r+\lambda)t} D_H\left(x_t\right) dt$$

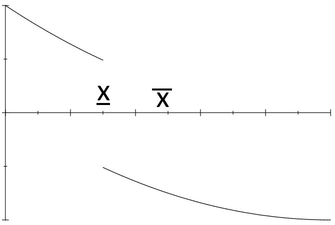
- Dividend $D_H(x) = V_H(1) - V_H(x)$ decreasing in x
- Future reputation x_t increasing in x_0 (conditional on $\theta_{\leq t} = L$)
- $\Delta\left(x\right)$ decreasing in x

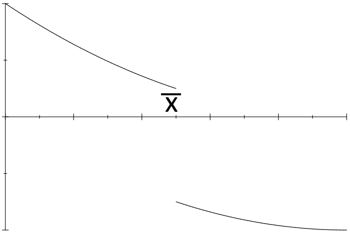
Corollary: Dynamics x_t are ergodic.

Unique Equilibrium

Proposition: Equilibrium is unique, if $\lambda > 1$.

Proof: Consider two cutoffs $\underline{x}$ and $\overline{x}$





- $\Delta_{\underline{x}}(\underline{x}) > \Delta_{\underline{x}}(\overline{x})$: Value of quality increasing in reputation
- $\Delta_{\underline{x}}(\overline{x}) > \Delta_{\overline{x}}(\overline{x})$: $\overline{x}$ has more to gain if he can drift further

Perfect Bad News

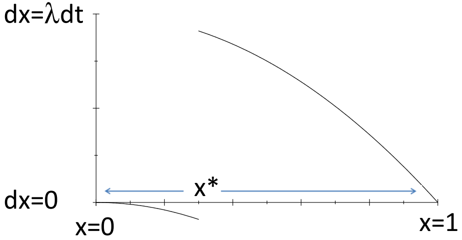
Updating & Dynamics

Reputational Updating: Breakdown with arrival rate $\mu_L = 1$

- Breakdown: x_t jumps to 0
- Otherwise: $dx = \lambda\left(\tilde{\eta}\left(x\right) - x\right) dt + x\left(1 - x\right) dt$

"Shirk-Work" profile with cut-off x^*:

$$\eta\left(x\right) = \begin{cases} 0 & \text{for } x < x^* \\ 1 & \text{for } x > x^* \end{cases}$$



Shirk-Work

Proposition: Every equilibrium is shirk-work.

Proof:

$$\Delta\left(x_0\right) = \int e^{-(r+\lambda)t} D_L\left(x_t\right) dt$$

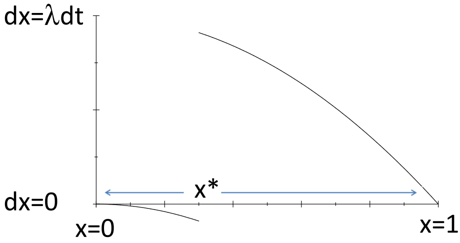
- Dividend $D_L(x) = V_L(x) - V_L(0)$ increasing in x
- Future reputation x_t increasing in x_0 (conditional on $\theta_{\leq t} = H$)
- $\Delta\left(x\right)$ increasing in x

Corollary: Dynamics x_t not ergodic.

Multiple Equilibria

Proposition: If $\lambda > 1$ and $c < \cdots$, there is $0 < \underline{x} < \overline{x} < 1$ s.t. every $x^* \in [\underline{x}, \overline{x}]$ can be equilibrium cutoff, if $\lambda > 1$.

Proof:



x^* is not indifferent:

- $x^* + \varepsilon$ drifts up, has lot to loose
- $x^* - \varepsilon$ drifts down, is lost anyway

$$\lambda \Delta_x^-(x) < c < \lambda \Delta_x^+(x)$$

Work vs. shirk is self-fulfilling prophecy

Bad News is Good

Theorem: For $\lambda > \cdots$ and $c < \cdots$

(Good) Pure shirking $\eta \equiv 0$ is only equilibrium.

(Bad) Shirk-work is equilibrium for any cutoff $x^* \in (0, 1)$

Mechanisms distinguishing bad news:

- Bounded likelihood ratios of defection (AMP)
- Divergent reputational dynamics (here)

Good News is Bad

Perfect Good & Bad news case

- Bad product has breakdown at rate μ_b
- Good product has breakthrough at rate $\mu_g > \mu_b$

-> Equilibria are work-shirk.

Corollary: For λ large:

(1) Effort sustainable with perfect bad news.

(2) Effort not sustainable with perfect good & bad news.

-> **More information can lead to less effort**

Idea:

- Breakthrough gives firm second chance
- Undermines incentives to avoid breakdowns

Imperfect Learning

Fundamental Asymmetry

Reputational Dividend

$$D_\theta(x) = \mu\left(V_\theta\left(x + \mu x(1-x)(\cdots)\right) - V_\theta(x)\right)$$

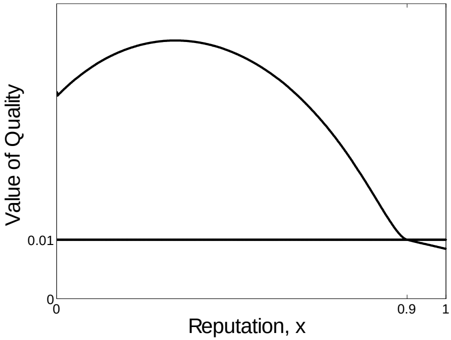
Imperfect learning: $\lim_{x \to 0;1} D_\theta(x) = 0$.

Fundamental Asymmetry

- Work at top $\eta(1) = 1$ not sustainable in MPE:
  $\rightarrow$ Reputation stuck at $x = 1$; dividend low
- Work at bottom $\eta(0) = 1$ sustainable in MPE:
  $\rightarrow$ Reputation drifts to $x \approx \frac{1}{2}$; dividend high

Work-Shirk Equilibrium

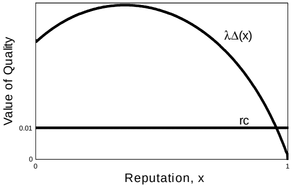
Theorem: Assume $\mu < 0$ (bad news) or $\lambda < \mu$ (fast learning):
For low c, a work-shirk equilibrium exists.



Corollary: Dynamics are ergodic.

Idea of Proof - Layer 1

$\Delta_1(x)$ has correct shape:



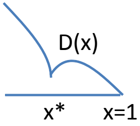
Looks like "by continuity":

$$\lambda\Delta_{x^*}(x) \begin{cases} > c & \text{for } x < x^* & \text{(Low types shirk),} \\ = c & \text{for } x = x^* & \text{(Cutoff type indifferent),} \\ < c & \text{for } x > x^* & \text{(High types work).} \end{cases}$$

Idea of Proof - Layer 2

Focus on $\mu < 0$. For $x^* < 1$:

- $V'(\cdot) = \int e^{-rt}\mathbb{E}\left[\frac{dx_t}{dx}\right] dt$ vanishes at x^*.
- $D(\cdot)$ increasing at x^*.
- $\Delta(\cdot)$ as well?



Lemma: If $x^* \approx 1$ and $x^* < x$, then $\Delta(x^*) > \Delta(x)$.

$$dx \approx \begin{cases} (\lambda - \mu)(1-x)\,dt & \text{for } x < x^* \\ -\lambda dt & \text{for } x > x^* \end{cases}$$

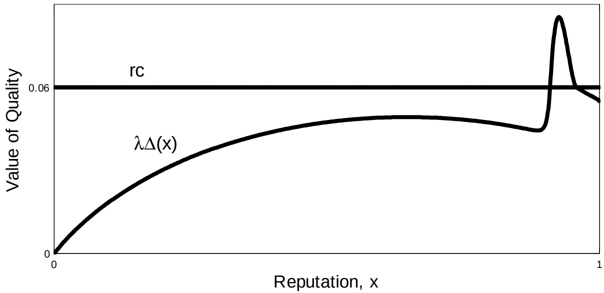
Proof: $\Delta_{x^*}(x)$ for $x > x^*$ convex combination of:

- Small dividends for $x' \in (x, x^*)$,
- $\Delta_{x^*}(x^*)$.

Shirk-Work-Shirk

Simulation Results:

For intermediate c, there exists a shirk-work-shirk equilibrium.



But for low c, there is no shirking in the middle

$$\lambda\Delta\left(\cdot\right) > c \text{ on } \left[\varepsilon; 1-\varepsilon\right]$$

Unique Equilibrium

HOPE: Market Learning satisfies

$$\Pr\left[x_t > x_0 \middle| x_0, \widetilde{\eta} = 0\right] > 0 \text{ for some } x_0$$

- Good news learning
- Bad news with drift $\mu_L - \lambda \geq 0$

Theorem: With imperfect learning, HOPE and low c, the work-shirk equilibrium is essentially unique.

Proof:

- $\lambda\Delta(\cdot) > c$ on $[\varepsilon, 1-\varepsilon]$
- HOPE: $\lambda\Delta(x_*) > c$ for shirk-work cutoff x_*

No HOPE: Two Types of Equilibria

Theorem: Assume no HOPE, and c small.
Work-Shirk equilibrium and Shirk-Work-Shirk equilibria co-exist.

Idea:

- Adding shirk-hole at bottom is incentive compatible
- Work vs. Shirk is self-fulfilling prophecy

Non-monotonic incentives in SWS equilibrium:

- One breakdown increases incentives: Hot-seat
- Multiple breakdowns destroy incentives: Shirk-hole

Conclusion

Modeling Innovation:

- Reputation as belief about *endogenous* quality
- Reputational drift driven by forward-looking incentives
- Reputation spent as well as built up

Role of learning process

- Perfect Good: Work-Shirk
- Perfect Bad: Shirk-Work
- Imperfect: Work-Shirk ...

Extensions

- Competition
- Entry & Exit

What Next?

Reputational Theory of Firm Dynamics (Board, MtV 2011)

- Market Entry and Exit driven by Reputational Capital
- Jointly determine Entry, Exit & Investment

Firm knows own quality

- Low quality firms exits when $x_t = x^E$
- Non-Exit signals high quality and ensures $x_t \geq x^E$
- Work-Shirk equilibrium: Fight till the bitter end

Firm does not know own quality

- Self-esteem $z = \mathbb{E}\left[\theta|\eta\right]$ vs. Reputation $x = \mathbb{E}\left[\theta|\tilde{\eta}\right]$
- Investment incentives: $\partial_z V\left(x, z\right)$
- Shirk-Work-Shirk equilibrium: Coast into liquidation