Relational Contracts in Competitive Labor Markets

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Motivation

Firms face incentive problems

- Employment contracts are typically incomplete.
- Firms motivate workers via long-term relationships.

Micro and macro interactions

- Longevity of firm's relationship depends on other firms' offers.
- We solve for equilibrium in optimal self-enforcing contracts.

Summary of Results

Firm-optimal self-enforcing contracts

- Stationary wage and effort.
- No back-loading.

Industry equilibrium

- Identical firms offer different contracts.
- Entry can lead to full employment.
- On-the-job search erodes productivity.

Applications

- Heterogeneous firms and firm location decision.
- Heterogeneous workers and over-qualification.
- Policy experiments.

Literature

Shapiro and Stiglitz (1984), MacLeod and Malcomson (1998).

- All firms offer same job.
- Unemployment necessary in equilibrium.

Burdett and Mortensen (1998)

- Wage posting with on-the-job search
- Higher wage attracts more employees.
- Non-degenerate wage distribution.

Empirics

Wage Distribution

Krueger, A. B., and L. H. Summers (1988): "Efficiency Wages and the Inter–Industry Wage Structure," *Econometrica*, 56(2), p. 261.

> *"If all firms were identical, one would not expect to see different firms paying different wages even if efficiency wages were important."*

Outline

Firm's Problem

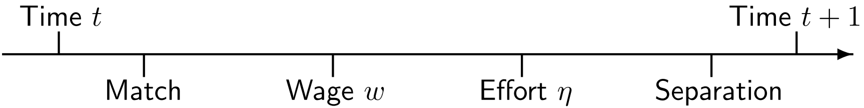
Model Overview

Economy

- Mass 1 identical workers and $n \leq 1$ identical firms.
- Firm has one job each period.
- Time $\{1, 2, \ldots\}$; discount rate $\delta \in (0, 1)$.

Job (stage game)

1. Worker receives outside offers; firm fills vacancy immediately.
2. Firm pays wage $w \in \Re_+$.
3. Worker exerts effort at cost $\eta \in \Re_+$ and produces output $\phi(\eta)$.
4. Separation with prob. $1 - \alpha$, and if either party terminates.



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Stage payoffs

- Utility $u := w - \eta$; Profit $\pi := \phi(\eta) - w$.
- Assume $\phi(0) = 0$, $\phi'(0) = \infty$, $\phi'(\infty) = 0$, $\phi''(\eta) < 0$.

Perspective of Single Firm

Matching Stage

- $W \sim F^e$ is cont. value of best offer; may have atom at 0.
- Firm fills vacancy instantly.

Restrictions:

- F^e stationary and anonymous.
- Contract $\langle w_t, \eta_t \rangle$ only depends on history within relationship.

Self-enforcing contracts

- SPNE in pure strategies.
- No voluntary terminations.
- Harshest penal code off equilibrium.

Firm's Problem

Firm's problem is to choose $\langle w_t, \eta_t \rangle$ to maximise Π_1 s.t.

$$w_t - \eta_t + \delta\alpha V_{t+1} + \delta(1-\alpha)V^{\varnothing} \geq w_t + \delta V^{\varnothing} \qquad \text{(IC)}$$

$$w_1 - \eta_1 + \delta\alpha V_2 + \delta(1-\alpha)V^{\varnothing} \geq \delta V^{\varnothing} \qquad \text{(IR)}$$

$$\Pi_t \geq \Pi_1 \qquad \text{(FIC)}$$

- Worker's pre- and post-matching value functions

$$V_t = \int \max\{W, W_t\} dF^e(W)$$

$$W_t = u_t + \delta\alpha V_{t+1} + \delta(1-\alpha)V^{\varnothing}$$

- Firm's pre-matching profit function

$$\Pi_t = F^e(W_t)[\phi(\eta_t) - w_t + \delta(\alpha\Pi_{t+1} + (1-\alpha)\Pi_1)] + [1 - F^e(W_t)]\Pi_1$$

Firm's Problem

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$$-\eta_t + \delta\alpha V_{t+1} \geq \delta\alpha V^{\varnothing} \tag{IC}$$

$$w_1 - \eta_1 + \delta\alpha V_2 \geq \delta\alpha V^{\varnothing} \tag{IR}$$

$$\Pi_t \geq \Pi_1 \tag{FIC}$$

- Worker's pre- and post-matching value functions

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Stationary Contracts

- Contract is stationary if independent of tenure, $\langle w, \eta \rangle$.

Theorem 1.
For any self-enforcing contract there is a stationary self-enforcing contract with weakly higher profits.

Idea

- Firm would like to backload to extract worker's rent.
- But firm would fire old workers, so not self-enforcing.

Notation

- Utility of job $u = w - \eta$ sufficient statistic for job.
- Value of job $V(u)$; outside offers $F^e(u)$.

Proof Sketch

Consider original contract $\langle w_t, \eta_t \rangle$

- Let $\phi(\eta^*) - \eta^* = \max_t\{\phi(\eta_t) - \eta_t\}$.
- Let $V^* = V_{\tau^*} = \max_t\{V_t\}$ and let w^* be corresponding wage.

New contract $\langle w^*, \eta^* \rangle$

- (IC): Follows from $\eta_t \leq \alpha\delta[V_{t+1} - V^{\varnothing}]$ for all t.
- (IR): Follows from $V^* \geq V_1$.
- (FIC): Follows from stationarity.

Profits are higher

- $\Pi_1^* \geq \Pi_{\tau^*}$: Higher surplus, same worker rents.
- $\Pi_{\tau^*} \geq \Pi_1$: Firm IC.

First-Order Conditions

Firm's problem is to choose $\langle u, \eta \rangle$ to maximize

$$\pi = \phi(\eta) - \eta - u$$

$$\text{s.t.} \quad -\eta + \delta\alpha V(u) \geq \delta\alpha V^{\varnothing} \tag{IC}$$

$$u + \delta\alpha V(u) \geq \delta\alpha V^{\varnothing} \tag{IR}$$

First-Order Conditions

Firm's problem is to choose η to maximize

$$\pi = \phi(\eta) - \eta - u_*(\eta)$$

$$\text{s.t.} \quad \eta = \delta\alpha[V(u_*(\eta)) - V^{\varnothing}] \qquad \text{(IC)}$$

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Value of job

$$V(u) = \int_{u}^{\overline{u}} \max\{u + \delta\alpha V(u); x + \delta\alpha V(x)\} dF^e(x) + \delta(1-\alpha)V^{\varnothing}$$

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$$V'(u) = (1 + \delta\alpha V'(u))F^e(u) = \frac{F^e(u)}{1 - \delta\alpha F^e(u)}$$

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First-order condition

$$\frac{d}{d\eta}(\eta + u_*(\eta)) = 1 + \frac{1}{\delta\alpha V'(u_*(\eta))} = \frac{1}{\delta\alpha F^e(u_*(\eta))}$$

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Job Market Matching

Job Market Matching

Initially: αn filled jobs, $(1-\alpha)n$ vacancies with cdf $F(u)$.
Axioms: Individual rationality, Anonymity, Market clearing.

- Offers to employed: $F^e(u)$
- Offers to unemployed: $F^{\varnothing}(u)$
- Market clearing:

$$\underbrace{(1-\alpha n)(1-F^{\varnothing}(u))}_{\text{unemployed}} + \underbrace{\alpha n F(u)(1-F^e(u))}_{\text{employed below } u} = \underbrace{(1-\alpha)n(1-F(u))}_{\text{vacancies above } u}$$

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Matching function $\psi : [0,1] \to [0,1]$, weakly increasing.

- Comparative advantage of employed: $F^{\varnothing}(u) = \psi(F^e(u))$.

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- Comparative advantage of employed: $F^{\varnothing}(u) = \psi(F^e(u))$.
- Retention rate $F^e(u) = \beta(F(u))$ on range of $F(\cdot)$.
- Let $q = F(u)$ and expand $\beta(q)$ to all $q \in [0,1]$.

More Matching

Examples

- Shapiro-Stiglitz: $\psi(z) = 0$.
- Fully anonymous: $\psi(z) = z$.
- Intern matching: $\psi(z) = 1$.

Properties

- We assume unemployed are better searchers: $\psi(z) \leq z$.
- $\psi(\cdot)$ obeys OJS if it is continuous (i.e. $\beta(q)$ strictly inc. in q).
- More OJS under $\tilde{\psi}(\cdot)$ than $\psi(\cdot)$ if $\tilde{\psi}(z) \geq \psi(z)$.

Equilibrium

Equilibrium

An industry equilibrium is mass n of contracts $\langle u, \eta \rangle$ s.t.

(a) Every contract $\langle u, \eta \rangle$ is firm-optimal w.r.t. F^e and $V^\varnothing$.

(b) F^e and $V^\varnothing$ derived from matching function ψ and rents F.

The value of unemployment is

$$V^\varnothing = \int (u + \delta \alpha V(u))\, dF^\varnothing(u) + \delta (1 - \alpha\theta) V^\varnothing$$

where $\theta = (1 - \alpha)n/(1 - \alpha n)$

Equilibrium Construction

Equilibrium $\langle u(x), \eta(x) \rangle_{x \in [0,1]}$ is defined by three conditions:

1. First-order condition (or marginal IC constraint)

$$\phi'(\eta(x)) = \frac{1}{\delta \alpha \beta(x)}$$

Uniquely determines $\eta(x)$.

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2. Constant profits: There is π such that

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3. IC constraint for highest firm

$$\eta(1) = \delta \alpha (V(u(1)) - V^{\varnothing}).$$

Uniquely determines π.

Equilibrium Characterization

Theorem 2.

(a) Industry equilibrium exists and is unique

(b) Equilibrium effort is determined by

$$\phi'(\eta(x)) = \frac{1}{\delta\alpha\beta(x)}$$

with support

$$\phi'(\eta(0)) = \frac{1}{\delta\alpha\beta(0)} \quad \textit{and} \quad \phi'(\eta(1)) = \frac{1}{\delta\alpha}$$

(c) With OJS, $F(u)$ is strictly increasing and continuous.

If $F(\cdot)$ has an atom and $\beta(\cdot)$ increasing

- Retention rate $\beta(F(u))$ jumps up, so MC jumps down.
- Profits kink upwards, contradicting local optimality.

Example: Shapiro-Stiglitz Matching

Shapiro-Stiglitz matching

- Only unemployed receive offers: $\psi \equiv 0$ and $\beta \equiv 1$.
- Theorem 2: All firms offer same job, with $\phi'(\overline{\eta}) := 1/\delta\alpha$.

Profits in Shapiro-Stiglitz

- Profit as function of effort:

$$\pi_*(\eta) = \phi(\eta) - \frac{1}{\delta\alpha}\eta - (1-\delta)V^{\emptyset}.$$

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- Overall profit

$$\pi_{\mathsf{SS}} = \phi(\overline{\eta}) - \frac{1}{\delta\alpha\,(1-\theta)}\overline{\eta}$$

with market tightness $\theta := (1-\alpha)\,n/\,(1-\alpha n)$.

Example: Fully Anonymous Matching

Fully anonymous matching

- Employed and unemployed receive same offers: $\psi(z) = z$.
- Retention rate: $\beta(q) = (1 - n(1-q))/(1 - \alpha n(1-q))$.

Lowest Job

- Retention rate $\beta\left(0\right) = 1 - \theta$.
- Theorem 2: Effort is $\phi'(\underline{\eta}) = 1/\delta\alpha(1-\theta)$.
- Profit as function of effort:

$$\pi_*(\eta) = \phi(\eta) - \frac{1}{\delta\alpha\left(1-\theta\right)}\eta$$

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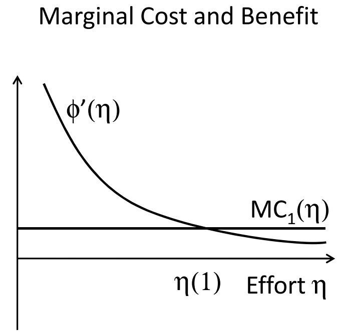
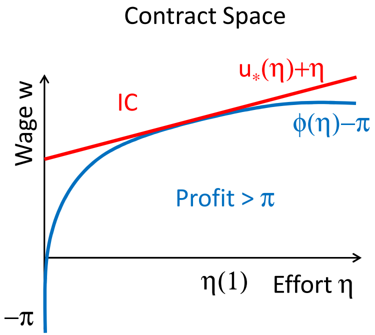
- Overall profit

$$\pi_{\mathsf{FA}} = \phi(\underline{\eta}) - \frac{1}{\delta\alpha(1-\theta)}\underline{\eta}.$$

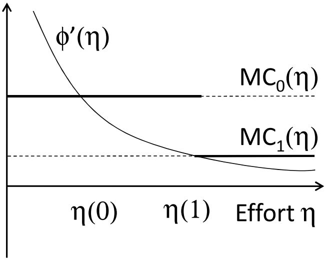
Shapiro-Stiglitz - No OJS



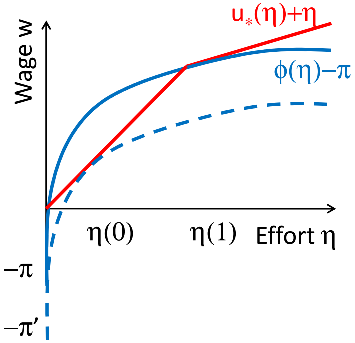


OJS - Downward Deviation

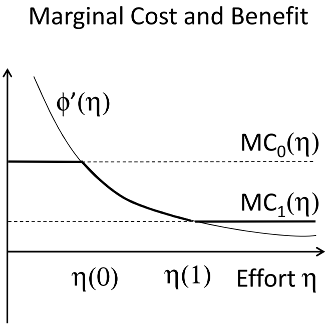
Marginal Cost and Benefit

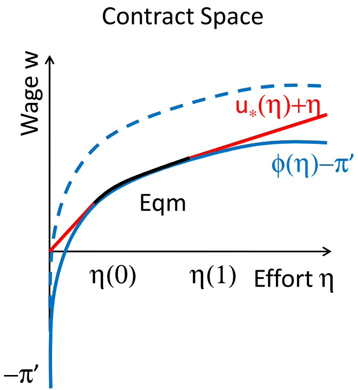


Contract Space



OJS - Equilibrium Contract Distribution





Comparative Statics of OJS

Theorem 3.
When on-the-job search increases:

(a) *Retention rates $\beta(x)$ decrease for all jobs.*

(b) *Effort $\eta(x)$, output and surplus decrease for all jobs.*

(c) *Rents $u(x)$ decrease for all employed/unemployed workers.*

(d) *Profits π increase for all firms.*

Idea

- Increase in OJS increases turnover and MC of effort.
- Firms substitute good jobs for bad.
- Lowers $V^{\varnothing}$ and introduces slack into IC.
- Firms lower wages until IC binds, raising profits.

Free Entry

Equilibrium

An industry equilibrium is a distribution of contract $\langle u, \eta \rangle$ s.t.

(a) Every contract $\langle u, \eta \rangle$ is firm-optimal w.r.t. F^e and $V^\varnothing$.

(b) Each contract yields zero profits, $\pi = 0$.

(c) F^e and $V^\varnothing$ derived from matching function ψ and rents $F(u)$.

- Retention rate $\beta_n(q)$ depends on n via market clearing

$$(1 - \alpha n)(1 - \psi(\beta_n(q))) + \alpha n q(1 - \beta_n(q)) = (1 - \alpha)n(1 - q)$$

Equilibrium Construction

Equilibrium $\{\langle u(x), \eta(x)\rangle\}_{x\in[0,1]}$ is defined by three conditions:

1. First-order condition (or marginal IC constraint)

$$\phi'(\eta(x)) = \frac{1}{\delta\alpha\beta_n(x)}$$

Uniquely determines $\eta(x)$ up to constant n.

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3. IC constraint for highest firm

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   Uniquely determines n.

Equilibrium Characterization

Theorem 4.

(a) *Equilibrium with free-entry exists and is unique.*

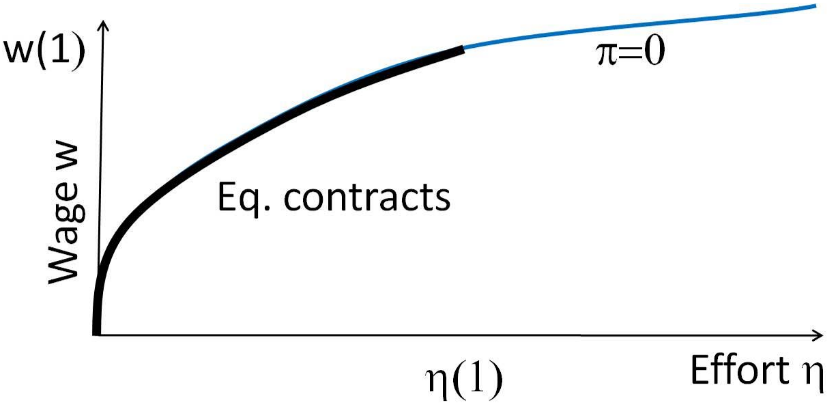
(b) *With FA matching, there is full employment* $n = 1$.

(c) *With less OJS, there is some unemployment,* $n < 1$.

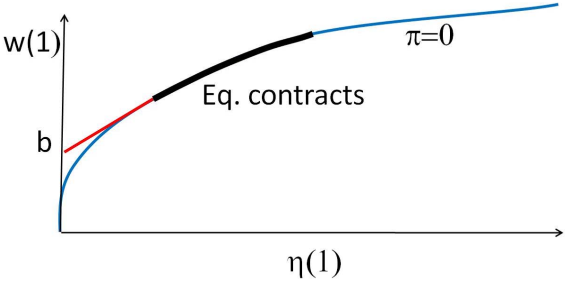
Idea

- Workers must be compensated for opp. cost of searching.
- Creates fixed cost to employ a worker.
- With fully anonymous matching, the fixed cost is zero.

Equilibrium Contracts - Fully Anonymous



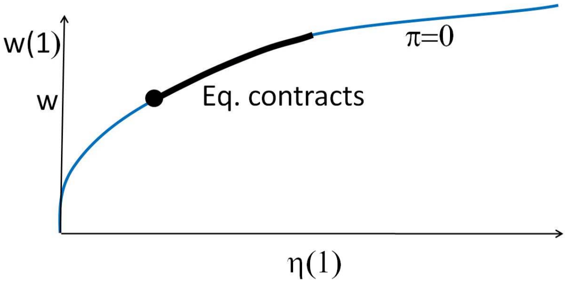
Policy Experiment - Unemployment Benefits



- Firms exit until IC is met.
- Equilibrium value of unemployment $V^{\varnothing}$ unaffected.

$$\eta(1) = \alpha\delta(V(u(1)) - V^{\varnothing})$$
$$V(u(1)) = u(1) + \delta(\alpha V(u(1)) + (1-\alpha)V^{\varnothing})$$

Policy Experiment - Minimum Wage



- Atom of jobs paying minimum wage.
- Increases variance at the bottom.

Comparative Statics of OJS

Theorem 5.
If $\eta \in [\eta(0), \eta(1)]$ then an increase in OJS:

(a) *Increases the number of jobs n.*

(b) *Decreases the number of good jobs with rent above $u(\eta)$.*

Increasing OJS ...

- Leads firms to replace good jobs with bad jobs.
- This lowers $V^{\varnothing}$ and leaves IC slack.
- Firms enter at bottom until IC tight.
- Welfare: loss of good jobs balanced by lower unemployment.

Intern Matching

Intern Matching (with $n < 1$)

Intern matching

- Employed prioritized: $\psi(z) \equiv 1$, so $\beta(q) = 0$ on $[0, 1 - \alpha]$.
- Internship with $w(0) = 0$, $\eta(0) > 0$ and $u(0) < 0$.
- Internships have mass $F(u(0)) > 1 - \alpha$.
- Entry jobs are gatekeepers for better jobs.

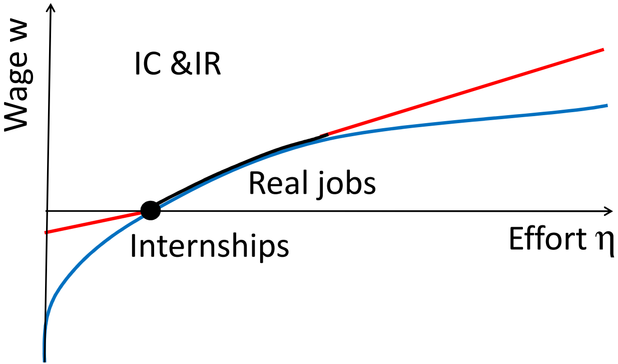
Characterizing job distribution

- For $\eta > \eta(0)$, $F(\cdot)$ characterized by FOC.
- Since IR binds in unemployed worker's first job, $V^{\varnothing} = 0$.
- Firms make monopoly profits: $\pi_{\mathsf{IM}} = \phi(\overline{\eta}) - \overline{\eta}/\alpha\delta$

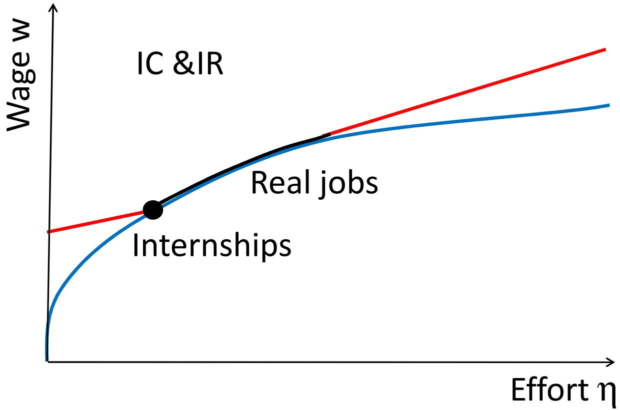
Increase in n

- Scales up distribution of contracts.
- Free entry leads to $n = 1$ and same effort distribution.

Intern Matching - Fixed n



Intern Matching - Free Entry



Heterogeneous Firms

Equilibrium Construction

Firm productivity $p \sim G[\underline{p}, \overline{p}]$ and $\phi(\eta, p)$ is supermodular.

1. First-order condition (or marginal IC constraint)

$$\frac{\partial}{\partial \eta}\phi(\eta, p) = \frac{1}{\delta\alpha\beta(G(p))}$$

Uniquely determines $\eta(p)$.

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2. Profits determined by envelope condition:

$$\pi(p) = \pi(\overline{p}) - \int_{p}^{\overline{p}} \frac{\partial}{\partial p}\phi(\eta(\hat{p}), \hat{p}) d\hat{p}$$

Utilities given by

$$u(p) = \phi(\eta(p), p) - \eta(p) - \pi(p)$$

Uniquely determines $u(p)$, with free parameter $\pi(\overline{p})$.

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   Uniquely determines $\eta(p)$.

2. Profits determined by envelope condition:

$$\pi(p) = \pi(\overline{p}) - \int_{p}^{\overline{p}} \frac{\partial}{\partial p}\phi(\eta(\hat{p}), \hat{p}) d\hat{p}$$

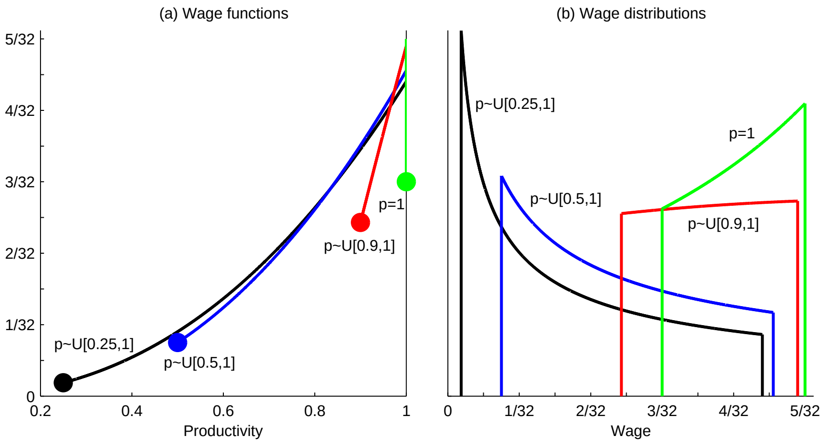
   Utilities given by

$$u(p) = \phi(\eta(p), p) - \eta(p) - \pi(p)$$

   Uniquely determines $u(p)$, with free parameter $\pi(\overline{p})$.

3. IC constraint for highest firm uniquely determines $\pi(\overline{p})$.

Wage as a Function of Productivity



Externalities

Shapiro-Stiglitz matching

- Increase in competitors prod. raises effort and rents.
- This tightens IC, reducing profits.
- Low productivity American firm should move to India.

Fully anonymous matching

- Increase in lower firms' prod. lowers $\pi(p)$.
- Increase in higher firms' prod. does not affect $\pi(p)$.

Intern matching

- Increase in lower firms' prod. does not affect $\pi(p)$.
- Increase in higher firms' prod. raises $\pi(p)$.
- Low productivity studio should move to LA.

Heterogeneous Workers

Equilibrium Construction

Workers have effort cost η/κ for $\kappa \in \{\kappa_L, \kappa_H\}$. Contracts $\langle u_\kappa(x), \eta_\kappa(x) \rangle$ offered by n_κ firms, where $n_L + n_H = n$.

1. First-order condition (or marginal IC constraint)

$$\phi'(\eta_\kappa(x)) = \frac{1}{\delta \alpha \kappa \beta(x)}$$

   Uniquely determines $\eta_\kappa(q)$.

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   Uniquely determines $\eta_\kappa(q)$.

2. Constant profits: There are $\{\pi, n_L, n_H\}$ such that

$$u_\kappa(x) = \phi(\eta_\kappa(x)) - \eta_\kappa(x) - \pi$$

   Uniquely determines $u_\kappa(x)$ up to $\{\pi, n_L, n_H\}$.

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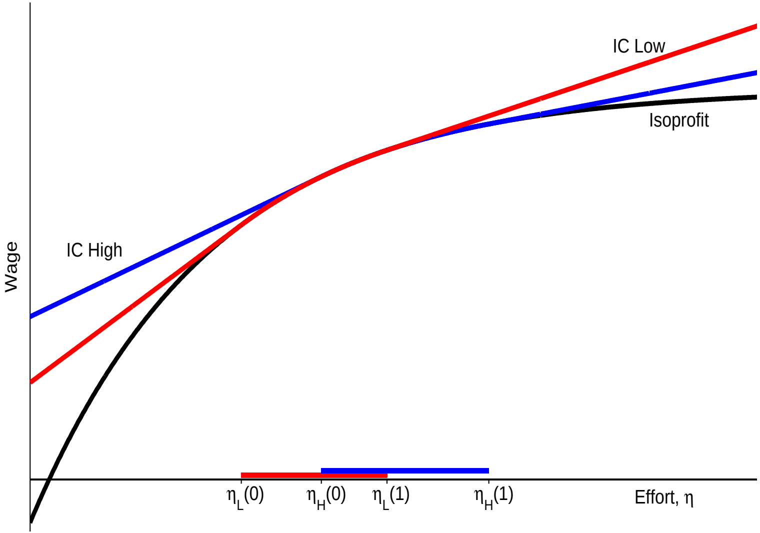
   Uniquely determines $u_\kappa(x)$ up to $\{\pi, n_L, n_H\}$.

3. IC constraint for highest firm for each type κ,

$$\eta_\kappa(1) = \delta\alpha(V(u_\kappa(1)) - V^{\varnothing}).$$

   Uniquely determines $\{\pi, n_L, n_H\}$.

Three Types of Contracts



Summary

Relational Contracts in Competitive Labor Markets

- Endogenous wage and productivity dispersion.
- Free entry can lead to full employment.
- On-the-job search erodes productivity.

Flexible Framework

- General class of matching technologies.
- Intern matching.
- Heterogeneous firms and workers.

Empirical Support for Model

Higher wages encourage effort

- High wage plants have fewer disciplinary actions.
- Wages are positively correlated with self-reported effort.
- Firms refuse to cut pay in order to sustain morale.

Relational contracts matter

- Effort declines at the end of a relationship
- Employment protection reduce effort.
- Unemployment increases effort

Empirics: Predictions

Predictions

- Large wage differentials across firms
- High wage firms have lower turnover and more applications.
- Large amount of wage growth occurs at job transitions.
- Wage jumps more frequent and larger at start of career.

Example: Professional industries

- Effort is more subjective and frequent job-to-job transitions.
- Explains higher levels of residual wage inequality.
- Job ladders common

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