Relational Contracts in Competitive Labor Markets

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Motivation

Firms face incentive problems

- Employment contracts are typically incomplete.
- Firms motivate workers via long-term relationships.

Micro and macro interactions

- Longevity of firm’s relationship depends on other firms’ offers.
- We solve for equilibrium in optimal self-enforcing contracts.
Summary of Results

Firm-optimal self-enforcing contracts
  ▶ Stationary wage and effort.
  ▶ No back-loading.

Industry equilibrium
  ▶ Identical firms offer different contracts.
  ▶ Entry can lead to full employment.
  ▶ On-the-job search erodes productivity.

Applications
  ▶ Heterogeneous firms and firm location decision.
  ▶ Heterogeneous workers and over-qualification.
  ▶ Policy experiments.

- All firms offer same job.
- Unemployment necessary in equilibrium.

Burdett and Mortensen (1998)

- Wage posting with on-the-job search
- Higher wage attracts more employees.
- Non-degenerate wage distribution.
Wage Distribution


“If all firms were identical, one would not expect to see different firms paying different wages even if efficiency wages were important.”
Outline

1. Introduction
2. Firm’s Problem
3. Matching
4. Industry Equilibrium
5. Free Entry
6. Internship Matching
7. Heterogeneous Firms and Workers
8. Conclusion
Firm’s Problem
Model Overview

Economy
- Mass 1 identical workers and \( n \leq 1 \) identical firms.
- Firm has one job each period.
- Time \( \{1, 2, \ldots \} \); discount rate \( \delta \in (0, 1) \).

Job (stage game)
1. Worker receives outside offers; firm fills vacancy immediately.
2. Firm pays wage \( w \in \mathbb{R}_+ \).
3. Worker exerts effort at cost \( \eta \in \mathbb{R}_+ \) and produces output \( \phi(\eta) \).
4. Separation with prob. \( 1 - \alpha \), and if either party terminates.

Time \( t \)  
\begin{align*}
\text{Match} & \quad \text{Wage } w & \quad \text{Effort } \eta & \quad \text{Separation} \\
\text{Time } t + 1
\end{align*}
Model Overview

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4. Separation with prob. \( 1 - \alpha \), and if either party terminates.

Stage payoffs

- Utility \( u := w - \eta \); Profit \( \pi := \phi(\eta) - w \).
- Assume \( \phi(0) = 0, \phi'(0) = \infty, \phi'(\infty) = 0, \phi''(\eta) < 0 \).
Perspective of Single Firm

Matching Stage

- $W \sim F^e$ is cont. value of best offer; may have atom at 0.
- Firm fills vacancy instantly.

Restrictions:

- $F^e$ stationary and anonymous.
- Contract $\langle w_t, \eta_t \rangle$ only depends on history within relationship.

Self-enforcing contracts

- SPNE in pure strategies.
- No voluntary terminations.
- Harshest penal code off equilibrium.
Firm’s Problem

Firm’s problem is to choose \( \langle w_t, \eta_t \rangle \) to maximise \( \Pi_1 \) s.t.

\[
\begin{align*}
  w_t - \eta_t + \delta \alpha V_{t+1} + \delta (1 - \alpha) V^\varnothing & \geq w_t + \delta V^\varnothing \\
  w_1 - \eta_1 + \delta \alpha V_2 + \delta (1 - \alpha) V^\varnothing & \geq \delta V^\varnothing \\
  \Pi_t & \geq \Pi_1
\end{align*}
\]  

(IC) \hspace{2cm} (IR) \hspace{2cm} (FIC)

- Worker’s pre- and post-matching value functions
  \[
  V_t = \int \max\{W, W_t\} dF^e(W)
  \]
  \[
  W_t = u_t + \delta \alpha V_{t+1} + \delta (1 - \alpha) V^\varnothing
  \]

- Firm’s pre-matching profit function
  \[
  \Pi_t = F^e(W_t)[\phi(\eta_t) - w_t + \delta (\alpha \Pi_{t+1} + (1 - \alpha) \Pi_1)] + [1 - F^e(W_t)]\Pi_1
  \]
Firm’s Problem

Firm’s problem is to choose \( \langle w_t, \eta_t \rangle \) to maximise \( \Pi_1 \) s.t.

\[
\begin{align*}
-\eta_t + \delta \alpha V_{t+1} & \geq \delta \alpha V^\emptyset \\
 w_1 - \eta_1 + \delta \alpha V_2 & \geq \delta \alpha V^\emptyset \\
 \Pi_t & \geq \Pi_1
\end{align*}
\]

\( (IC) \)

\( (IR) \)

\( (FIC) \)

- Worker’s pre- and post-matching value functions
  \[
  V_t = \int \max\{W, W_t\} dF^e(W)
  \\
  W_t = u_t + \delta \alpha V_{t+1} + \delta (1 - \alpha)V^\emptyset
  \]

- Firm’s pre-matching profit function
  \[
  \Pi_t = F^e(W_t)[\phi(\eta_t) - w_t + \delta(\alpha\Pi_{t+1} + (1 - \alpha)\Pi_1)] + [1 - F^e(W_t)]\Pi_1
  \]
Stationary Contracts

- Contract is stationary if independent of tenure, $\langle w, \eta \rangle$.

**Theorem 1.**

For any self-enforcing contract there is a stationary self-enforcing contract with weakly higher profits.

**Idea**

- Firm would like to backload to extract worker’s rent.
- But firm would fire old workers, so not self-enforcing.

**Notation**

- Utility of job $u = w - \eta$ sufficient statistic for job.
- Value of job $V(u)$; outside offers $F^e(u)$. 
Proof Sketch

Consider original contract $\langle w_t, \eta_t \rangle$

- Let $\phi(\eta^*) - \eta^* = \max_t \{ \phi(\eta_t) - \eta_t \}$.
- Let $V^* = V_{\tau^*} = \max_t \{ V_t \}$ and let $w^*$ be corresponding wage.

New contract $\langle w^*, \eta^* \rangle$

- (IC): Follows from $\eta_t \leq \alpha \delta [V_{t+1} - V^\emptyset]$ for all $t$.
- (IR): Follows from $V^* \geq V_1$.
- (FIC): Follows from stationarity.

Profits are higher

- $\Pi_1^* \geq \Pi_{\tau^*}$: Higher surplus, same worker rents.
- $\Pi_{\tau^*} \geq \Pi_1$: Firm IC.
Firm’s problem is to choose $\langle u, \eta \rangle$ to maximize

$$
\pi = \phi(\eta) - \eta - u
$$

s.t.

$$
- \eta + \delta \alpha V(u) \geq \delta \alpha V^\varnothing \quad \text{(IC)}
$$

$$
u + \delta \alpha V(u) \geq \delta \alpha V^\varnothing \quad \text{(IR)}
$$
First-Order Conditions

Firm’s problem is to choose $\eta$ to maximize

$$\pi = \phi(\eta) - \eta - u_*(\eta)$$

s.t.  $\eta = \delta \alpha [V(u_*(\eta)) - V^\emptyset]$  \hspace{1cm} (IC)
First-Order Conditions

**Firm’s problem** is to choose \( \eta \) to maximize

\[
\pi = \phi(\eta) - \eta - u_*(\eta)
\]

s.t. \( \eta = \delta \alpha [V(u_*(\eta)) - V^\emptyset] \) \hspace{1cm} (IC)

**Value of job**

\[
V(u) = \int_{\bar{u}} \max\{u + \delta \alpha V(u); x + \delta \alpha V(x)\} dF^e(x) + \delta (1 - \alpha) V^\emptyset
\]
First-Order Conditions

Firm’s problem is to choose $\eta$ to maximize

$$\pi = \phi(\eta) - \eta - u_*(\eta)$$

s.t.  

$$\eta = \delta \alpha [V(u_*(\eta)) - V^\emptyset]$$  \hspace{1cm} (IC)

Value of job

$$V'(u) = (1 + \delta \alpha V'(u)) F^e(u) = \frac{F^e(u)}{1 - \delta \alpha F^e(u)}$$
First-Order Conditions

Firm’s problem is to choose $\eta$ to maximize

$$\pi = \phi(\eta) - \eta - u_*(\eta)$$

s.t. $\eta = \delta \alpha [V(u_*(\eta)) - V^{\emptyset}]$ (IC)

Value of job

$$V'(u) = (1 + \delta \alpha V'(u))F^e(u) = \frac{F^e(u)}{1 - \delta \alpha F^e(u)}$$

First-order condition

$$\frac{d}{d\eta} (\eta + u_*(\eta)) = 1 + \frac{1}{\delta \alpha V'(u_*(\eta))} = \frac{1}{\delta \alpha F^e(u_*(\eta))}$$
First-Order Conditions

Firm’s problem is to choose \( \eta \) to maximize

\[
\pi = \phi(\eta) - \eta - u_*(\eta)
\]

s.t. \( \eta = \delta \alpha [V(u_*(\eta)) - V^\emptyset] \) (IC)

Value of job

\[
V'(u) = (1 + \delta \alpha V'(u))F^e(u) = \frac{F^e(u)}{1 - \delta \alpha F^e(u)}
\]

First-order condition

\[
\phi'(\eta) = \frac{1}{\delta \alpha F^e(u_*(\eta))}
\]
Job Market Matching
Job Market Matching

Initially: $\alpha n$ filled jobs, $(1 - \alpha)n$ vacancies with cdf $F(u)$.

Axioms: Individual rationality, Anonymity, Market clearing.

- Offers to employed: $F^e(u)$
- Offers to unemployed: $F^\emptyset(u)$
- Market clearing:

$$
(1 - \alpha n)(1 - F^\emptyset(u)) + \alpha n F(u)(1 - F^e(u)) = (1 - \alpha)n(1 - F(u))
$$

unemployed  
employed below $u$  
vacancies above $u$
Job Market Matching

Initially: \( \alpha n \) filled jobs, \((1 - \alpha)n\) vacancies with cdf \( F(u) \).

Axioms: Individual rationality, Anonymity, Market clearing.

- Offers to employed: \( F^e(u) \)
- Offers to unemployed: \( F^\varnothing(u) \)
- Market clearing:

\[
(1 - \alpha n)(1 - \psi(F^e(u))) + \alpha n F(u)(1 - F^e(u)) = (1 - \alpha n)(1 - F(u))
\]

unemployed  
employed below \( u \)  
vacancies above \( u \)

Matching function \( \psi : [0, 1] \rightarrow [0, 1] \), weakly increasing.

- Comparative advantage of employed: \( F^\varnothing(u) = \psi(F^e(u)) \).
Job Market Matching

Initially: $\alpha n$ filled jobs, $(1 - \alpha)n$ vacancies with cdf $F(u)$.

Axioms: Individual rationality, Anonymity, Market clearing.

- Offers to employed: $F^e(u)$
- Offers to unemployed: $F^\varnothing(u)$
- Market clearing:

$$(1 - \alpha n)(1 - \psi(\beta(F(u)))) + \alpha n F(u)(1 - \beta(F(u))) = (1 - \alpha)n(1 - F(u))$$

Matching function $\psi : [0, 1] \rightarrow [0, 1]$, weakly increasing.

- Comparative advantage of employed: $F^\varnothing(u) = \psi(F^e(u))$.
- Retention rate $F^e(u) = \beta(F(u))$ on range of $F(\cdot)$. 
Job Market Matching

Initially: \( \alpha n \) filled jobs, \((1 - \alpha)n\) vacancies with cdf \( F(u) \).

Axioms: Individual rationality, Anonymity, Market clearing.

- Offers to employed: \( F_e(u) \)
- Offers to unemployed: \( F_\emptyset(u) \)
- Market clearing:

\[
(1 - \alpha n)(1 - \psi(\beta(q))) + \alpha n q (1 - \beta(q)) = (1 - \alpha) n (1 - q)
\]

Matching function \( \psi : [0, 1] \rightarrow [0, 1] \), weakly increasing.

- Comparative advantage of employed: \( F_\emptyset(u) = \psi(F_e(u)) \).
- Retention rate \( F_e(u) = \beta(F(u)) \) on range of \( F(\cdot) \).
- Let \( q = F(u) \) and expand \( \beta(q) \) to all \( q \in [0, 1] \).
More Matching

Examples

- Shapiro-Stiglitz: $\psi(z) = 0$.
- Fully anonymous: $\psi(z) = z$.
- Intern matching: $\psi(z) = 1$.

Properties

- We assume unemployed are better searchers: $\psi(z) \leq z$.
- $\psi(\cdot)$ obeys OJS if it is continuous (i.e. $\beta(q)$ strictly inc. in $q$).
- More OJS under $\tilde{\psi}(\cdot)$ than $\psi(\cdot)$ if $\tilde{\psi}(z) \geq \psi(z)$. 
Equilibrium
Equilibrium

An industry equilibrium is mass $n$ of contracts $\langle u, \eta \rangle$ s.t.
(a) Every contract $\langle u, \eta \rangle$ is firm-optimal w.r.t. $F^e$ and $V^\emptyset$.
(b) $F^e$ and $V^\emptyset$ derived from matching function $\psi$ and rents $F$.

The value of unemployment is

$$V^\emptyset = \int (u + \delta \alpha V(u)) dF^\emptyset(u) + \delta (1 - \alpha \theta) V^\emptyset$$

where $\theta = (1 - \alpha)n/(1 - \alpha n)$
Equilibrium Construction

Equilibrium $\langle u(x), \eta(x) \rangle_{x \in [0,1]}$ is defined by three conditions:

1. First-order condition (or marginal IC constraint)

$$\phi'(\eta(x)) = \frac{1}{\delta \alpha \beta(x)}$$

Uniquely determines $\eta(x)$.
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Equilibrium $\langle u(x), \eta(x) \rangle_{x \in [0,1]}$ is defined by three conditions:

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   \[
   \phi'(\eta(x)) = \frac{1}{\delta \alpha \beta(x)}
   \]
   Uniquely determines $\eta(x)$.

2. Constant profits: There is $\pi$ such that
   \[
   u(x) = \phi(\eta(x)) - \eta(x) - \pi
   \]
   Uniquely determines $u(x)$ up to constant $\pi$. 

3.
**Equilibrium Construction**

Equilibrium \( \langle u(x), \eta(x) \rangle_{x \in [0,1]} \) is defined by three conditions:

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\]

Uniquely determines \( \eta(x) \).

2. **Constant profits:** There is \( \pi \) such that

\[
u(x) = \phi(\eta(x)) - \eta(x) - \pi
\]

Uniquely determines \( u(x) \) up to constant \( \pi \).

3. **IC constraint for highest firm**

\[
\eta(1) = \delta \alpha (V(u(1)) - V^{\emptyset}).
\]

Uniquely determines \( \pi \).
Equilibrium Characterization

Theorem 2.

(a) Industry equilibrium exists and is unique
(b) Equilibrium effort is determined by

\[ \phi'(\eta(x)) = \frac{1}{\delta \alpha \beta(x)} \]

with support

\[ \phi'(\eta(0)) = \frac{1}{\delta \alpha \beta(0)} \text{ and } \phi'(\eta(1)) = \frac{1}{\delta \alpha} \]

(c) With OJS, \( F(u) \) is strictly increasing and continuous.

If \( F(\cdot) \) has an atom and \( \beta(\cdot) \) increasing

- Retention rate \( \beta(F(u)) \) jumps up, so MC jumps down.
- Profits kink upwards, contradicting local optimality.
Example: Shapiro-Stiglitz Matching

Shapiro-Stiglitz matching

- Only unemployed receive offers: $\psi \equiv 0$ and $\beta \equiv 1$.
- Theorem 2: All firms offer same job, with $\phi'(\eta) := 1/\delta \alpha$.

Profits in Shapiro-Stiglitz

- Profit as function of effort:

$$\pi^*(\eta) = \phi(\eta) - \frac{1}{\delta \alpha} \eta - (1 - \delta)V^\phi.$$
Example: Shapiro-Stiglitz Matching

Shapiro-Stiglitz matching

- Only unemployed receive offers: $\psi \equiv 0$ and $\beta \equiv 1$.
- Theorem 2: All firms offer same job, with $\phi' (\bar{\eta}) := 1/\delta \alpha$.

Profits in Shapiro-Stiglitz

- Profit as function of effort:
  $$\pi^*_*(\eta) = \phi(\eta) - \frac{1}{\delta \alpha} \eta - (1 - \delta) V^\phi.$$  
- Overall profit
  $$\pi_{SS} = \phi(\bar{\eta}) - \frac{1}{\delta \alpha (1 - \theta)} \bar{\eta}$$

with market tightness $\theta := (1 - \alpha) n / (1 - \alpha n)$. 
Example: Fully Anonymous Matching

Fully anonymous matching

- Employed and unemployed receive same offers: $\psi(z) = z$.
- Retention rate: $\beta(q) = (1 - n(1 - q))/(1 - \alpha n(1 - q))$.

Lowest Job

- Retention rate $\beta(0) = 1 - \theta$.
- Theorem 2: Effort is $\phi'(\eta) = 1/\delta\alpha (1 - \theta)$.
- Profit as function of effort:
  \[
  \pi^*(\eta) = \phi(\eta) - \frac{1}{\delta\alpha (1 - \theta)} \eta
  \]
Example: Fully Anonymous Matching

Fully anonymous matching

- Employed and unemployed receive same offers: $\psi(z) = z$.
- Retention rate: $\beta(q) = (1 - n(1 - q))/(1 - \alpha n(1 - q))$.

Lowest Job

- Retention rate $\beta(0) = 1 - \theta$.
- Theorem 2: Effort is $\phi'(\eta) = 1/\delta \alpha (1 - \theta)$.
- Profit as function of effort:
  \[
  \pi^*(\eta) = \phi(\eta) - \frac{1}{\delta \alpha (1 - \theta) \eta}
  \]
- Overall profit
  \[
  \pi_{FA} = \phi(\eta) - \frac{1}{\delta \alpha (1 - \theta) \eta}.
  \]
Shapiro-Stiglitz - No OJS

Marginal Cost and Benefit

Contract Space

Marginal Cost and Benefit

Contract Space

\[ \phi'(\eta) \]

\[ \phi(\eta) - \pi \]

\[ u_*(\eta) + \eta \]

\[ \text{MC}_1(\eta) \]

\[ \text{Profit} > \pi \]

\[ \eta(1) \]

\[ \text{Effort} \eta \]

\[ \text{Wage} w \]
Marginal Cost and Benefit

\[ \phi'(\eta) \]

\[ \text{MC}_0(\eta) \]

\[ \text{MC}_1(\eta) \]

Effort \( \eta \)

Contract Space

\[ \phi(\eta) - \pi \]

\[ u_*(\eta) + \eta \]

\[ -\pi \]

\[ -\pi' \]
OJS - Equilibrium Contract Distribution

Marginal Cost and Benefit

Contract Space
Comparative Statics of OJS

Theorem 3.
When on-the-job search increases:
(a) Retention rates $\beta(x)$ decrease for all jobs.
(b) Effort $\eta(x)$, output and surplus decrease for all jobs.
(c) Rents $u(x)$ decrease for all employed/unemployed workers.
(d) Profits $\pi$ increase for all firms.

Idea
- Increase in OJS increases turnover and MC of effort.
- Firms substitute good jobs for bad.
- Lowers $V^\emptyset$ and introduces slack into IC.
- Firms lower wages until IC binds, raising profits.
Free Entry
Equilibrium

An industry equilibrium is a distribution of contract \( \langle u, \eta \rangle \) s.t.

(a) Every contract \( \langle u, \eta \rangle \) is firm-optimal w.r.t. \( F^e \) and \( V^\emptyset \).

(b) Each contract yields zero profits, \( \pi = 0 \).

(c) \( F^e \) and \( V^\emptyset \) derived from matching function \( \psi \) and rents \( F(u) \).

- Retention rate \( \beta_n(q) \) depends on \( n \) via market clearing

\[
(1 - \alpha n)(1 - \psi(\beta_n(q))) + \alpha n q (1 - \beta_n(q)) = (1 - \alpha) n (1 - q)
\]
Equilibrium Construction

Equilibrium \( \{ \langle u(x), \eta(x) \rangle \}_{x \in [0,1]} \) is defined by three conditions:

1. First-order condition (or marginal IC constraint)

\[
\phi'(\eta(x)) = \frac{1}{\delta \alpha \beta \gamma(x)}
\]

Uniquely determines \( \eta(x) \) up to constant \( n \).
Equilibrium Construction

Equilibrium $\{\langle u(x), \eta(x) \rangle \}_{x \in [0,1]}$ is defined by three conditions:

1. First-order condition (or marginal IC constraint)
   \[
   \phi'(\eta(x)) = \frac{1}{\delta \alpha \beta n(x)}
   \]
   Uniquely determines $\eta(x)$ up to constant $n$.

2. Zero Profits:
   \[
   u(x) = \phi(\eta(x)) - \eta(x)
   \]
   Uniquely determines $u(x)$ up to constant $n$. 
Equilibrium Construction

Equilibrium $\{\langle u(x), \eta(x) \rangle \}_{x \in [0, 1]}$ is defined by three conditions:

1. **First-order condition (or marginal IC constraint)**

   \[ \phi'(\eta(x)) = \frac{1}{\delta \alpha \beta n(x)} \]

   Uniquely determines $\eta(x)$ up to constant $n$.

2. **Zero Profits:**

   \[ u(x) = \phi(\eta(x)) - \eta(x) \]

   Uniquely determines $u(x)$ up to constant $n$.

3. **IC constraint for highest firm**

   \[ \eta(1) = \delta \alpha (V(u(1)) - V^\emptyset) \]

   Uniquely determines $n$. 
Equilibrium Characterization

**Theorem 4.**

(a) *Equilibrium with free-entry exists and is unique.*

(b) *With FA matching, there is full employment* $n = 1$.

(c) *With less OJS, there is some unemployment,* $n < 1$.

**Idea**

- Workers must be compensated for opp. cost of searching.
- Creates fixed cost to employ a worker.
- With fully anonymous matching, the fixed cost is zero.
Equilibrium Contracts - Fully Anonymous
Policy Experiment - Unemployment Benefits

- Firms exit until IC is met.
- Equilibrium value of unemployment $V^\varnothing$ unaffected.

$$\eta(1) = \alpha \delta (V(u(1)) - V^\varnothing)$$
$$V(u(1)) = u(1) + \delta (\alpha V(u(1)) + (1 - \alpha)V^\varnothing)$$
Policy Experiment - Minimum Wage

- Atom of jobs paying minimum wage.
- Increases variance at the bottom.
Comparative Statics of OJS

**Theorem 5.**
If $\eta \in [\eta(0), \eta(1)]$ then an increase in OJS:

(a) Increases the number of jobs $n$.
(b) Decreases the number of good jobs with rent above $u(\eta)$.

Increasing OJS . . .

- Leads firms to replace good jobs with bad jobs.
- This lowers $V^\emptyset$ and leaves IC slack.
- Firms enter at bottom until IC tight.
- Welfare: loss of good jobs balanced by lower unemployment.
Intern Matching
Intern Matching (with $n < 1$)

Intern matching

▶ Employed prioritized: $\psi(z) \equiv 1$, so $\beta(q) = 0$ on $[0, 1 - \alpha]$.
▶ Internship with $w(0) = 0$, $\eta(0) > 0$ and $u(0) < 0$.
▶ Internships have mass $F(u(0)) > 1 - \alpha$.
▶ Entry jobs are gatekeepers for better jobs.

Characterizing job distribution

▶ For $\eta > \eta(0)$, $F(\cdot)$ characterized by FOC.
▶ Since IR binds in unemployed worker’s first job, $V^{\emptyset} = 0$.
▶ Firms make monopoly profits: $\pi_{IM} = \phi(\bar{\eta}) - \bar{\eta}/\alpha \delta$

Increase in $n$

▶ Scales up distribution of contracts.
▶ Free entry leads to $n = 1$ and same effort distribution.
Intern Matching - Fixed $\eta$

Wage $w$

IC & IR

Real jobs

Effort $\eta$

Internships
Intern Matching - Free Entry

Wage $w$

Effort $\eta$

IC & IR

Real jobs

Internships
Heterogeneous Firms
Equilibrium Construction

Firm productivity $p \sim G[p, \bar{p}]$ and $\phi(\eta, p)$ is supermodular.

1. First-order condition (or marginal IC constraint)

$$\frac{\partial}{\partial \eta} \phi(\eta, p) = \frac{1}{\delta \alpha \beta(G(p))}$$

Uniquely determines $\eta(p)$. 
Equilibrium Construction

Firm productivity $p \sim G[p, \bar{p}]$ and $\phi(\eta, p)$ is supermodular.

1. First-order condition (or marginal IC constraint)

$$\frac{\partial}{\partial \eta} \phi(\eta, p) = \frac{1}{\delta \alpha \beta(G(p))}$$

Uniquely determines $\eta(p)$.

2. Profits determined by envelope condition:

$$\pi(p) = \pi(\bar{p}) - \int_p^{\bar{p}} \frac{\partial}{\partial \hat{p}} \phi(\eta(\hat{p}), \hat{p}) d\hat{p}$$

Utilities given by

$$u(p) = \phi(\eta(p), p) - \eta(p) - \pi(p)$$

Uniquely determines $u(p)$, with free parameter $\pi(\bar{p})$. 
Equilibrium Construction

Firm productivity $p \sim G[p, \bar{p}]$ and $\phi(\eta, p)$ is supermodular.

1. First-order condition (or marginal IC constraint)
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\frac{\partial}{\partial \eta} \phi(\eta, p) = \frac{1}{\delta \alpha \beta(G(p))}
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\pi(p) = \pi(\bar{p}) - \int_{p}^{\bar{p}} \frac{\partial}{\partial \hat{p}} \phi(\eta(\hat{p}), \hat{p}) d\hat{p}
\]
Utilities given by
\[
u(p) = \phi(\eta(p), p) - \eta(p) - \pi(p)
\]
Uniquely determines $u(p)$, with free parameter $\pi(\bar{p})$.

3. IC constraint for highest firm uniquely determines $\pi(\bar{p})$. 
Wage as a Function of Productivity

(a) Wage functions

(b) Wage distributions
Externalities

Shapiro-Stiglitz matching

- Increase in competitors prod. raises effort and rents.
- This tightens IC, reducing profits.
- Low productivity American firm should move to India.

Fully anonymous matching

- Increase in lower firms’ prod. lowers $\pi(p)$.
- Increase in higher firms’ prod. does not affect $\pi(p)$.

Intern matching

- Increase in lower firms’ prod. does not affect $\pi(p)$.
- Increase in higher firms’ prod. raises $\pi(p)$.
- Low productivity studio should move to LA.
Heterogeneous Workers
Equilibrium Construction

Workers have effort cost $\eta/\kappa$ for $\kappa \in \{\kappa_L, \kappa_H\}$. Contracts $\langle u_\kappa(x), \eta_\kappa(x) \rangle$ offered by $n_\kappa$ firms, where $n_L + n_H = n$.

1. First-order condition (or marginal IC constraint)

$$\phi'(\eta_\kappa(x)) = \frac{1}{\delta \alpha \kappa \beta(x)}$$

Uniquely determines $\eta_\kappa(q)$. 


Equilibrium Construction

Workers have effort cost $\eta/\kappa$ for $\kappa \in \{\kappa_L, \kappa_H\}$. Contracts $\langle u_\kappa(x), \eta_\kappa(x) \rangle$ offered by $n_\kappa$ firms, where $n_L + n_H = n$.

1. First-order condition (or marginal IC constraint)

$$\phi'(\eta_\kappa(x)) = \frac{1}{\delta\alpha\kappa\beta(x)}$$

Uniquely determines $\eta_\kappa(q)$.

2. Constant profits: There are $\{\pi, n_L, n_H\}$ such that

$$u_\kappa(x) = \phi(\eta_\kappa(x)) - \eta_\kappa(x) - \pi$$

Uniquely determines $u_\kappa(x)$ up to $\{\pi, n_L, n_H\}$. 
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3. IC constraint for highest firm for each type $\kappa$,

$$\eta_{\kappa}(1) = \delta\alpha(V(u_{\kappa}(1)) - V^\emptyset)$$

Uniquely determines $\{\pi, n_L, n_H\}$. 
Three Types of Contracts

Wage

IC High

IC Low

Isoprofit

Effort, $\eta$

$\eta_L(0)$ $\eta_H(0)$ $\eta_L(1)$ $\eta_H(1)$
Summary

Relational Contracts in Competitive Labor Markets

▶ Endogenous wage and productivity dispersion.
▶ Free entry can lead to full employment.
▶ On-the-job search erodes productivity.

Flexible Framework

▶ General class of matching technologies.
▶ Intern matching.
▶ Heterogeneous firms and workers.
Empirical Support for Model

Higher wages encourage effort

- High wage plants have fewer disciplinary actions.
- Wages are positively correlated with self-reported effort.
- Firms refuse to cut pay in order to sustain morale.

Relational contracts matter

- Effort declines at the end of a relationship
- Employment protection reduce effort.
- Unemployment increases effort
Empirics: Predictions

Predictions

- Large wage differentials across firms
- High wage firms have lower turnover and more applications.
- Large amount of wage growth occurs at job transitions.
- Wage jumps more frequent and larger at start of career.

Example: Professional industries

- Effort is more subjective and frequent job-to-job transitions.
- Explains higher levels of residual wage inequality.
- Job ladders common