

# Recruiting Talent

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# Motivation

## Talent is source of competitive advantage

- ▶ Universities: Faculty are key asset.
- ▶ Netflix: “We endeavor to have only outstanding employees.”
- ▶ Empirics: Managers (Bertrand-Schoar), workers (Lazear).

## Talent perpetuates via hiring

- ▶ Uni: Faculty responsible for recruiting juniors and successors.
- ▶ N: “Building a great team is manager’s most important task.”
- ▶ Empirics: Stars help recruit future talent (Waldinger)

## Key questions

- ▶ Can talent dispersion persist/avoid regression to mediocrity?
- ▶ Why don’t bad firms just compete advantage away?

# Overview

## Three ingredients for persistence

- ▶ High wages attract talented applicants
- ▶ Skilled management screens wheat from chaff.
- ▶ Today's recruits become tomorrow's managers.

## Static Model

- ▶ When talent is scarce, matching is positive assortative.
- ▶ Efficient matching is negative assortative.

## Dynamic model

- ▶ Persistent dispersion of talent, productivity and wages.
- ▶ Regression to mediocrity offset by PAM.
- ▶ Gradual adjustment to steady state.

# Literature

## Matching in labor markets

- ▶ Becker (1973), Lucas (1978), Garicano (2000), Levin & Tadelis (2005), Anderson & Smith (2010).

## Adverse selection

- ▶ Greenwald (1986), Lockwood (1991), Chakraborty et al (2010), Lauermann & Wolitzky (2015), Kurlat (2016).

## Wage & productivity dispersion

- ▶ Albrecht & Vroman (1992), Burdett & Mortensen (1998).

## Firm dynamics

- ▶ Prescott & Lucas (1971), Jovanovic (1982), Hopenhayn (1992), Hopenhayn & Rogerson (1993), Board & MtV (2014).

# STATIC MODEL

# Baseline Model

## Gameform

- ▶ Unit mass of firms  $r \sim F[\underline{r}, \bar{r}]$  post wages  $w(r)$ .
- ▶ Unit mass of workers apply from top to bottom wage.  
Proportion  $\bar{q}$  talented,  $1 - \bar{q}$  untalented.
- ▶ Firms sequentially screen applicants, hire one each.  
Proportion  $r$  skilled recruiters  $\theta = H$ ;  $1 - r$  unskilled  $\theta = L$ .

## Screening

- ▶ Talented workers pass test.
- ▶ Untalented screened out with iid prob.  $p_\theta$ ;  $0 < p_L < p_H < 1$ .
- ▶ Quality when recruiter  $\theta$  hires from applicant pool  $q$

$$\lambda(q; \theta) = q / (1 - (1 - q)p_\theta)$$

- ▶ Quality at firm  $r$ :  $\lambda(q; r) = r\lambda(q; H) + (1 - r)\lambda(q; L)$
- ▶ Profits  $\pi := \mu\lambda(q(w); r) - w - k$ .

# Preliminary Analysis

## Applicant Pool Quality

- ▶ Top wage: Proportion  $q(1) = \bar{q}$  talented workers.
- ▶ Wage rank  $x$ : Applicant pool quality  $q(x)$  obeys

$$q'(x) = \frac{\lambda(q(x); r(x)) - q(x)}{x}$$

- ▶ Quality  $q(x)$ :  $\begin{cases} \text{strictly increases in } x \\ \text{positive for } x > 0, \text{ but } q(0) = 0. \end{cases}$

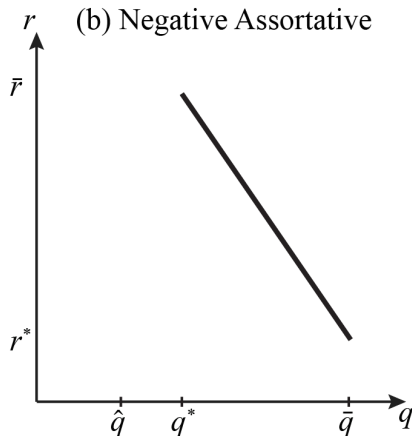
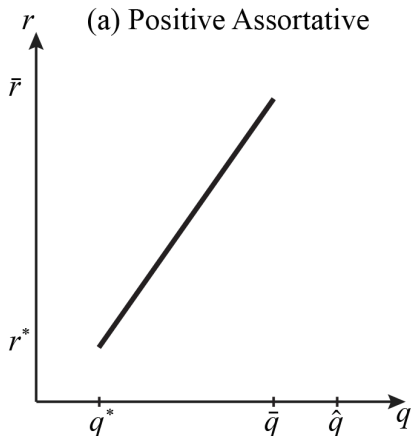
## Wage posting equilibrium

- ▶ Equilibrium wage distribution  $\{w(r)\}_r$  has no atoms or gaps.

# Firm-Applicant Matching

Wage profile  $\{w(r)\}_r$  induces firm-applicant matching

$$Q(r) = q(x(w(r)))$$





# Equilibrium Matching - Necessary Condition

## Incentive Compatibility in Equilibrium $\{w(r)\}_r$

- ▶ Firms  $r, \tilde{r}$  do not mimic each other:

$$\mu\lambda(Q(r); r) - w(r) \geq \mu\lambda(Q(\tilde{r}); r) - w(\tilde{r})$$

$$\mu\lambda(Q(\tilde{r}); \tilde{r}) - w(\tilde{r}) \geq \mu\lambda(Q(r); \tilde{r}) - w(r)$$

- ▶ Hence,  $\lambda(Q(\tilde{r}); r)$  supermodular in  $(\tilde{r}, r)$ .

## Return to Recruiter Quality

$$\Delta(q) := \lambda(q; H) - \lambda(q; L) = \frac{\partial}{\partial r} \lambda(q; r)$$

- ▶ IC:  $\Delta(Q(r))$  rises in  $r$ .
- ▶  $\Delta(\cdot)$  is single-peaked, with maximum  $\hat{q} \in (0, 1)$ .
- ▶  $\lambda(q; r)$  is super-modular for  $q < \hat{q}$ ; sub-modular for  $q > \hat{q}$ .

# Scarce Talent — Positive Assortative Matching

## Theorem 1.

*If  $\bar{q} \leq \hat{q}$ , there is a unique equilibrium. It exhibits PAM.*

## Proof

- ▶  $\Delta(q)$  increases for  $q \leq \bar{q}$ , and  $\Delta(Q(r))$  must increase.
- ▶ Hence,  $Q(r)$  must increase.

## Equilibrium described by

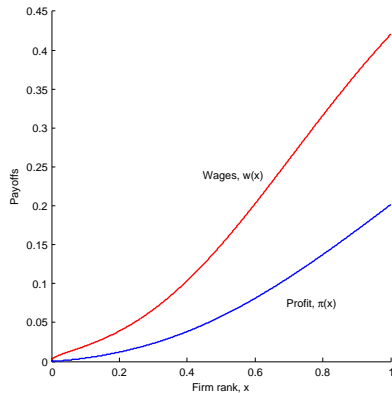
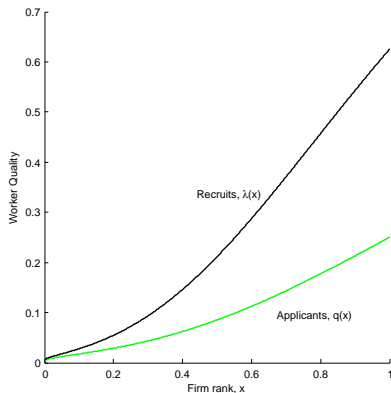
- ▶ Profits

$$\pi(r) = \mu \int_{\underline{r}}^r \Delta(Q(\tilde{r})) d\tilde{r}.$$

- ▶ Wages

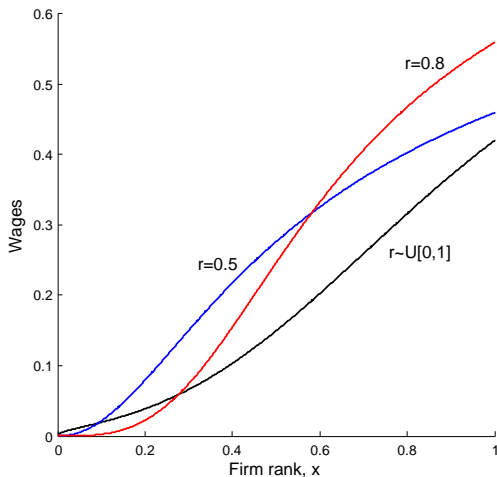
$$w(r) = \mu \int_{\underline{r}}^r \lambda'(Q(\tilde{r}); \tilde{r}) Q'(\tilde{r}) d\tilde{r}.$$

# PAM: Example



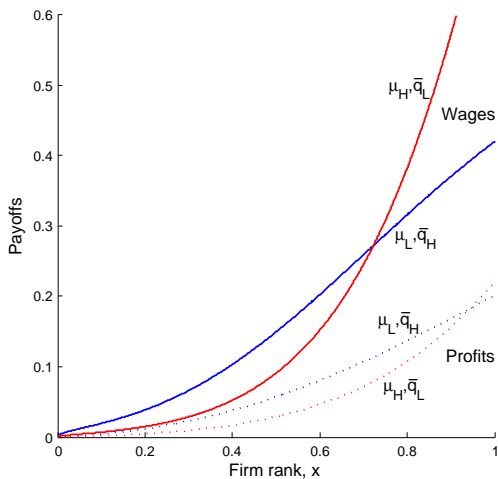
Assumptions:  $p_H = 0.8$ ,  $p_L = 0.2$ ,  $r \sim U[0, 1]$ ,  $\bar{q} = 0.25$ ,  $\mu = 1$ .

# Comparative Statics — Screening Skills



Assumptions:  $p_H = 0.8$ ,  $p_L = 0.2$ ,  $\bar{q} = 0.25$ ,  $\mu = 1$ .

# Comparative Statics — Technological Change



$$\bar{q}_H = .25, \mu_L = 1 \quad \rightarrow \quad \bar{q}_L = .05, \mu_H = 5.$$

# Abundant Talent

## Theorem 2.

*Assume  $\bar{q} > \hat{q}$ . There is a unique equilibrium. It has PAM on  $[q^*, \hat{q}]$  and NAM on  $[\hat{q}, \bar{q}]$ .*

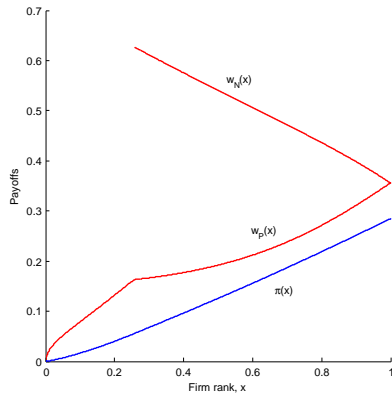
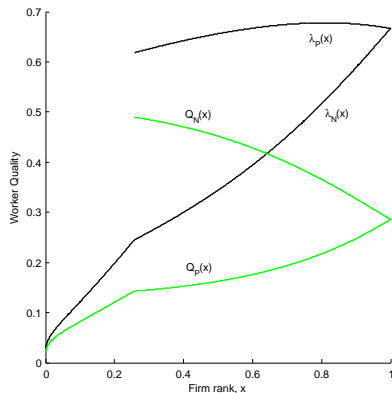
## Proof

- ▶ Key fact:  $\Delta(Q(r))$  increases in  $r$ .
- ▶ Top firm  $\bar{r}$  matches with  $\hat{q}$ .
- ▶ Below,  $r$  matches with  $Q_P(r) < \hat{q} < Q_N(r)$  s.t.

$$\Delta(Q_P(r)) = \Delta(Q_N(r))$$

and  $Q_P, Q_N$  obey the usual differential equations.

# PAM-NAM: Example



Assumptions:  $p_H = 0.8$ ,  $p_L = 0.2$ ,  $r \sim U[0, 1]$ ,  $\bar{q} = 0.5$ .

# DYNAMIC MODEL



# Model

## Basics

- ▶ Continuous time  $t$ , discount rate  $\rho$ .
- ▶ Workers enter and retire at flow rate  $\alpha$ .
- ▶ Talented workers become skilled recruiters.
- ▶ Assume talent is scarce,  $\bar{q} < \hat{q}$ .

## Firm's problem

- ▶ Firm's product  $\mu r_t$ ; initially,  $r_0$  exogenous.
- ▶ Attract applicants  $q_t$  with wage  $w_t(q_t)$  to manage talent  $r_t$

$$\dot{r}_t = \alpha(\lambda(q_t; r_t) - r_t).$$

- ▶ Firm value  $V_t(r)$ .

# Firm's Problem

- ▶ The firm solves

$$V_0(r_0) = \max_{\{q_t\}} \int_0^{\infty} e^{-\rho t} (\mu r_t - \alpha w_t(q_t)) dt,$$
$$\text{s.t. } \dot{r}_t = \alpha(\lambda(q_t; r_t) - r_t).$$

- ▶ Bellman equation

$$\rho V_t(r) = \max_q \{ \mu r - \alpha w_t(q) + \alpha V'_t(r) [\lambda(q; r) - r] + \dot{V}_t(r) \}.$$

- ▶ First order condition

$$\lambda'(q; r) V'_t(r) = w'_t(q).$$

# Positive Assortative Matching

## Theorem 3.

*Equilibrium exists and is unique. Firms with more talent post higher wages. The distribution of talent has no atoms at  $t > 0$ .*

## Idea

- ▶ The value function  $V_t(r)$  is convex.
- ▶ FOC implies matching is PAM.
- ▶ FOC also implies atoms immediately dissolve.

## Thus

- ▶ Time-invariant firm-rank  $x$ , s.t.  $r_t(x), q_t(x)$  increase in  $x$ .

# Constructing the Equilibrium

## Equilibrium Matching $r_t(x)$ , $q_t(x)$

- Talent evolution

$$\dot{r}_t(x) = \alpha(\lambda(q_t(x); r_t(x)) - r_t(x))$$

- Sequential Screening

$$q'_t(x) = (\lambda(q_t(x); r_t(x)) - q_t(x))/x$$

## Equilibrium Wages

$$w'_t(q) = V'_t(r)\lambda'(q; r)$$

where  $q = q_t(x)$ ,  $r = r_t(x)$  and

$$V'_t(r_t) = \frac{\partial}{\partial r_t} \int_t^\infty e^{-\rho(s-t)} [\mu r_s^* - \alpha w_s(q_s^*)] ds$$

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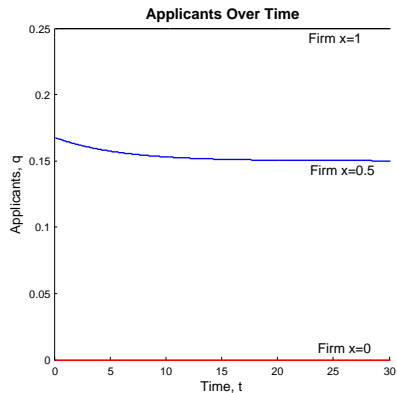
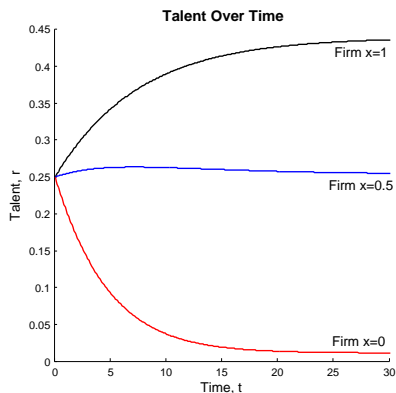
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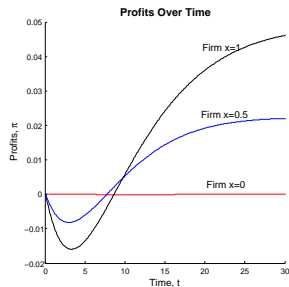
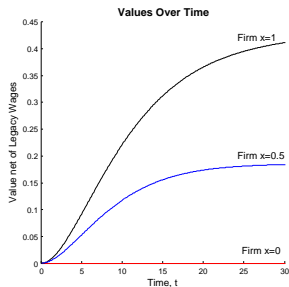
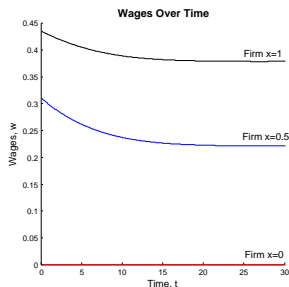
$$V'_t(r_t(x)) = \mu \int_t^\infty e^{-\int_t^s (\rho + \alpha(1 - \Delta(q_u(x)))) du} ds$$

# Equilibrium Firm Dynamics — Talent



Assumptions:  $p_H = 0.8$ ,  $p_L = 0.2$ ,  $\mu = 1$ ,  $\rho = 0.1$ ,  $\alpha = 0.2$ , and  $\bar{q} = 0.25$ .

# Equilibrium Firm Dynamics — Payoffs



Assumptions:  $p_H = 0.8$ ,  $p_L = 0.2$ ,  $\mu = 1$ ,  $\rho = 0.1$ ,  $\alpha = 0.2$ , and  $\bar{q} = 0.25$ .



# The Equilibrium Steady State

## Steady-state matching $r^*(x)$ , $q^*(x)$

- ▶ Constant quality,  $\lambda(q; r) = r$ , links  $q$  and  $r$ .
- ▶ Seq. screen.,  $q'(x) = (\lambda(q; r) - q)/x$ , determines  $r(x), q(x)$ .

## Steady state wages $w^*(q)$

- ▶ Marginal value of talent

$$V'(r) = \frac{\mu}{\rho + \alpha(1 - \Delta(q))}.$$

- ▶ Marginal wages

$$w'(q) = \frac{\mu}{\rho + \alpha(1 - \Delta(q))} \lambda'(q; r).$$

# Convergence to Steady State

## Theorem 4.

- a) *Steady State  $\{r^*(x), q^*(x), w^*(q)\}$  is unique; no gaps or atoms.*
- b) *For any initial talent distribution, equilibrium converges to SS.*

## Persistence of competitive advantage

- ▶ Random hiring: regression to mean at rate  $\alpha$ .
- ▶ Screening applicants  $q$ : regression to mean at  $\alpha(1 - \Delta(q))$ .
- ▶ But under PAM, high-quality firms pay more.
- ▶ Hence, talent is source of sustainable competitive advantage.

# Comparative Statics

## Talent dispersion rises in talent-skill correlation $\beta$

- ▶ Suppose recruiting skill is  $(1 - \beta)\bar{q} + \beta r$ .
- ▶ PAM if  $\beta > 0$ , but NAM if  $\beta < 0$ .
- ▶ Talent dispersion  $r^*(1) - r^*(0)$  rises in  $\beta$ .

## Wages rise in turnover $\alpha$

- ▶ Does not affect steady-state talent.
- ▶ Raises steady-state flow wages  $(\rho + \alpha)w_t$ .

$$(\rho + \alpha)w'(q(x)) = \mu\lambda'(q(x); r(x)) \frac{\rho + \alpha}{\rho + \alpha(1 - \Delta(q(x)))}$$

- ▶ Intuition: Effect of talent outlasts employment.

# DYNAMIC MODEL WITH HETEROGENOUS TECHNOLOGY

# Heterogeneous Technology

## Two types of heterogeneity

- ▶ Exogenous technology  $\mu \in \{\mu_L, \mu_H\}$ ; mass  $\nu$  low.
- ▶ Evolving talent  $r_t$ .
- ▶ Firms *stratified*, if  $r_t$  and  $\mu$  correlate perfectly.

## Wages increase in $\mu$ and $r$

- ▶ Recall FOC

$$w'_t(q) = V'_t(r; \mu) \lambda'(q; r)$$

- ▶ Higher  $r$  raises  $V'_t(r; \mu)$  and  $\lambda'(q; r)$ .
- ▶ Higher  $\mu$  raises  $V'_t(r; \mu)$ .

# Convergence to Steady State

## Theorem 5.

- a) There is a unique steady-state equilibrium.*
- b) The steady state is stratified.*
- c) Any equilibrium converges to this steady-state.*
- d) Distribution  $r^*(x), q^*(x)$  independent of  $\{\mu_L, \mu_H\}$ .*

## Idea

- ▶ Talent distribution becomes continuous.
- ▶ High-tech firms outbid low-tech firms when talent is close.

# Adjustment Dynamics of Single Firm

- ▶ Steady state with firms  $r \geq r^*$  high tech; wages  $w^*(r)$ .
- ▶ Low-tech firm with  $r < r^*$  becomes high-tech.

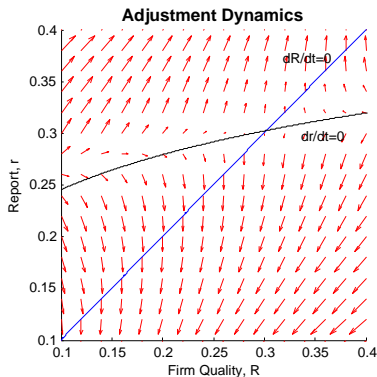
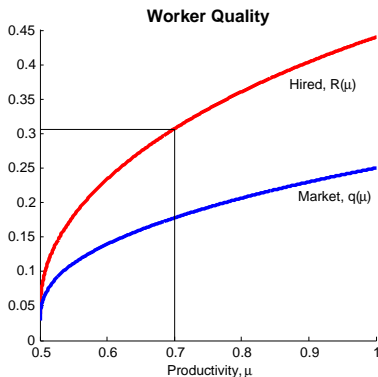
## Theorem 6.

- a) Wages satisfy  $w_t \in (w^*(r_t), w^*(r^*)]$
- b) Talent  $r_t$  converges to  $r^*$  as  $t \rightarrow \infty$ .

## Idea

- ▶  $w_t > w^*(r_t)$  since firm has higher tech.
- ▶  $w_t \leq w^*(r^*)$  since firm has less talent.
- ▶ Since  $w_t > w^*(r_t)$ , talent  $r_t$  rises over time.

# Saddle-point Stable Adjustment Path



- $r_0$  chosen to hit  $r^*$ . Near steady state,

$$\begin{bmatrix} r_t - r^* \\ r_t - r^* \end{bmatrix} = (r_0 - r^*) \begin{bmatrix} 0.2032 \\ 1 \end{bmatrix} e^{-0.2281t}.$$



# WELFARE

# Welfare

## Introducing Welfare

- ▶ Entry cost  $k > 0$ .
- ▶ Marginal firm:  $\mu\lambda(Q(\tilde{r}); \tilde{r}) = k$ .
- ▶ Welfare  $\int_{\tilde{x}}^1 (\mu\lambda(q(x); r(x)) - k) dx$ .

## Maximize Aggregate Sorting

- ▶ Planner chooses entry and rank  $x$  for every firm  $r$ .
- ▶ Equilibrium entry threshold  $\tilde{x}$  is efficient (given PAM).
- ▶ But, does PAM for  $x \in [\tilde{x}, 1]$  maximize employed talent?

# Efficient Matching

## Theorem 7.

*For any entry threshold  $\check{x}$ , NAM maximizes employed talent.*

## Two economics forces

- ▶ Becker: PAM maximizes comparative advantage (if  $q < \hat{q}$ ).
- ▶ Akerlof: PAM also maximizes adverse selection.
- ▶ And...

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- ▶ Akerlof: PAM also maximizes adverse selection.
- ▶ And... Akerlof wins!

# Proof Sketch

## Marginal Employed Talent

- ▶ Employed talent  $\omega(\hat{x})$ , where  $\omega(x) = \bar{q} - xq(x)$
- ▶ Effect of better screening skills at rank  $x$

$$\begin{aligned}\zeta(x) &:= \frac{\partial \omega(\hat{x})}{\partial r(x)} = \frac{\partial \omega(\hat{x})}{\partial \omega(x)} \frac{\partial \omega(x)}{\partial r(x)} \\ &= \exp\left(-\int_{\hat{x}}^x \frac{\lambda'(q(x); r(x))}{x} dx\right) \Delta(q(x))\end{aligned}$$

## Shifting Screening Skills Up

$$\zeta'(x) \simeq \underbrace{\Delta'(q(x))q'(x)}_{\text{Becker}} - \underbrace{\lambda'(q(x); r(x))\Delta(x)/x}_{\text{Akerlof}} < 0.$$

# Dynamic Efficiency

- ▶ Upfront entry cost  $k/\rho > 0$ .
- ▶ Present surplus  $\int_0^\infty e^{-\rho t}(\mu R_t - k)dt$ , with  $R_t = \int_{\check{x}}^1 r_t(x)dx$ .
- ▶ Choose entry and wage ranks to maximize surplus.

## Theorem 8.

*For any  $\check{x}$ , NAM surplus exceeds PAM surplus at all times.*

## Idea

- ▶ For fixed recruiting skills  $R_t$ , NAM maximizes talent input.
- ▶ Additional talent under NAM helps recruit even more talent.

# EXTENSIONS

# Model of Hierarchies

## Hierarchy

- ▶  $N + 1$  layers: Level  $n = 0$  directors; level  $n = N$  workers.
- ▶ Mass 1 of firms; each has  $\alpha^n$  positions at level  $n$ .
- ▶ Mass  $\alpha^n$  of job seekers at each level; proportion  $\bar{q}$  skilled.

## Firms

- ▶ Director quality  $r_0$  exogenous.
- ▶ Level  $n$  agents hire level  $n + 1$  agents,  $r_{n+1} = \lambda(q_{n+1}, r_n)$ .
- ▶ Only workers produce,  $v_N = \mu\alpha^N r_N$ .



# Hierarchies: Equilibrium Wages

## Equilibrium with $\bar{q} < \hat{q}$

- ▶ Assume  $r_0 \sim F_0$  steady state; then  $r_{n+1} = r_n$ .
- ▶ Level- $(n-1)$  value  $v_{n-1}(r) := v_n(\lambda(q_n; r)) - \alpha^n w_n$ ; then

$$v'_n(r) = \mu \alpha^N \Delta(q)^{N-n}$$

- ▶ Marginal level- $n$  wages

$$w'_n(q) = \lambda'(q; r) \mu (\alpha \Delta(q))^{N-n}.$$

- ▶ Assume  $\alpha \Delta(q) > 1$ ; then wages increase in rank.

## Wage dispersion across firms $q > \tilde{q}$ and levels $n < \tilde{n}$

- ▶ Inter-firm dispersion greater at high levels:  $\frac{w_n(q)}{w_n(\tilde{q})} \geq \frac{w_{\tilde{n}}(q)}{w_{\tilde{n}}(\tilde{q})}$ .
- ▶ Intra-firm dispersion greater at high firms:  $\frac{w_n(q)}{w_{\tilde{n}}(q)} \geq \frac{w_n(\tilde{q})}{w_{\tilde{n}}(\tilde{q})}$ .

# Conclusion

We've proposed a model in which

- ▶ Firms compete to identify and recruit talent.
- ▶ Today's recruits become tomorrow's recruiters.

## Main results

- ▶ Positive assortative matching.
- ▶ Persistent productivity dispersion.
- ▶ Equilibrium inefficiency due to adverse selection.

## Next steps

- ▶ Characterize dynamic matching with  $\bar{q} > \hat{q}$ .
- ▶ Characterize dynamic and steady state dispersion.
- ▶ Study dynamics when  $\mu_t$  are stochastic.