Recruiting Talent

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Motivation

Talent is source of competitive advantage

- Universities: Faculty are key asset.
- Netflix: “We endeavor to have only outstanding employees.”
- Empirics: Managers (Bertrand-Schoar), workers (Lazear).

Talent perpetuates via hiring

- Uni: Faculty responsible for recruiting juniors and successors.
- N: “Building a great team is manager’s most important task.”
- Empirics: Stars help recruit future talent (Waldinger)

Key questions

- Can talent dispersion persist/avoid regression to mediocrity?
- Why don’t bad firms just compete advantage away?
Overview

Three ingredients for persistence

- High wages attract talented applicants
- Skilled management screens wheat from chaff.
- Today’s recruits become tomorrow’s managers.

Static Model

- When talent is scarce, matching is positive assortative.
- Efficient matching is negative assortative.

Dynamic model

- Persistent dispersion of talent, productivity and wages.
- Regression to mediocrity offset by PAM.
- Gradual adjustment to steady state.
Literature

Matching in labor markets


Adverse selection


Wage & productivity dispersion


Firm dynamics

Static Model
Baseline Model

Gameform

- Unit mass of firms $r \sim F[r, \bar{r}]$ post wages $w(r)$.
- Unit mass of workers apply from top to bottom wage.
  Proportion $\bar{q}$ talented, $1 - \bar{q}$ untalented.
- Firms sequentially screen applicants, hire one each.
  Proportion $r$ skilled recruiters $\theta = H$; $1 - r$ unskilled $\theta = L$.

Screening

- Talented workers pass test.
- Untalented screened out with iid prob. $p_\theta$; $0 < p_L < p_H < 1$.
- Quality when recruiter $\theta$ hires from applicant pool $q$
  \[ \lambda(q; \theta) = q/(1 - (1 - q)p_\theta) \]
- Quality at firm $r$:
  \[ \lambda(q; r) = r\lambda(q; H) + (1 - r)\lambda(q; L) \]
- Profits $\pi := \mu\lambda(q(w); r) - w - k$. 
Preliminary Analysis

Applicant Pool Quality

- Top wage: Proportion $q(1) = \bar{q}$ talented workers.
- Wage rank $x$: Applicant pool quality $q(x)$ obeys

$$q'(x) = \frac{\lambda(q(x); r(x)) - q(x)}{x}$$

- Quality $q(x)$: \begin{align*}
\text{strictly increases in } x \\
\text{positive for } x > 0, \text{ but } q(0) = 0.
\end{align*}

Wage posting equilibrium

- Equilibrium wage distribution $\{w(r)\}_r$ has no atoms or gaps.
Firm-Applicant Matching

Wage profile \( \{w(r)\}_r \) induces firm-applicant matching

\[ Q(r) = q(x(w(r))) \]

(a) Positive Assortative

(b) Negative Assortative
Equilibrium Matching - Necessary Condition

Incentive Compatibility in Equilibrium \( \{w(r)\}_r \)

- Firms \( r, \tilde{r} \) do not mimic each other:
  \[
  \mu \lambda(Q(r); r) - w(r) \geq \mu \lambda(Q(\tilde{r}); r) - w(\tilde{r})
  
  \mu \lambda(Q(\tilde{r}); \tilde{r}) - w(\tilde{r}) \geq \mu \lambda(Q(r); \tilde{r}) - w(r)
  \]

- Hence, \( \lambda(Q(\tilde{r}); r) \) supermodular in \((\tilde{r}, r)\).

Return to Recruiter Quality

\[
\Delta(q) := \lambda(q; H) - \lambda(q; L) = \frac{\partial}{\partial r} \lambda(q; r)
\]

- IC: \( \Delta(Q(r)) \) rises in \( r \).
- \( \Delta(\cdot) \) is single-peaked, with maximum \( \hat{q} \in (0, 1) \).
- \( \lambda(q; r) \) is super-modular for \( q < \hat{q} \); sub-modular for \( q > \hat{q} \).
Theorem 1.
If $\bar{q} \leq \hat{q}$, there is a unique equilibrium. It exhibits PAM.

Proof
- $\Delta(q)$ increases for $q \leq \bar{q}$, and $\Delta(Q(r))$ must increase.
- Hence, $Q(r)$ must increase.

Equilibrium described by
- Profits
  \[ \pi(r) = \mu \int_r^\infty \Delta(Q(\tilde{r}))d\tilde{r}. \]
- Wages
  \[ w(r) = \mu \int_r^\infty \lambda'(Q(\tilde{r}); \tilde{r})Q'(\tilde{r})d\tilde{r}. \]
Assumptions: $p_H = 0.8$, $p_L = 0.2$, $r \sim U[0, 1]$, $\bar{q} = 0.25$, $\mu = 1$. 
Comparative Statics — Screening Skills

Assumptions: \( p_H = 0.8, \ p_L = 0.2, \ \bar{q} = 0.25, \ \mu = 1. \)
Comparative Statics — Technological Change

\[ \bar{q}_H = 0.25, \, \mu_L = 1 \quad \rightarrow \quad \bar{q}_L = 0.05, \, \mu_H = 5. \]
Theorem 2.
Assume $\bar{q} > \hat{q}$. There is a unique equilibrium. It has PAM on $[q^*, \hat{q}]$ and NAM on $[\hat{q}, \bar{q}]$.

Proof

- Key fact: $\Delta(Q(r))$ increases in $r$.
- Top firm $\bar{r}$ matches with $\hat{q}$.
- Below, $r$ matches with $Q_P(r) < \hat{q} < Q_N(r)$ s.t.
  \[
  \Delta(Q_P(r)) = \Delta(Q_N(r))
  \]
  and $Q_P, Q_N$ obey the usual differential equations.
Assumptions: $p_H = 0.8, p_L = 0.2, r \sim U[0, 1], \bar{q} = 0.5.$
Dynamic Model
Model

Basics

- Continuous time $t$, discount rate $\rho$.
- Workers enter and retire at flow rate $\alpha$.
- Talented workers become skilled recruiters.
- Assume talent is scarce, $\bar{q} < \hat{q}$.

Firm’s problem

- Firm’s product $\mu r_t$; initially, $r_0$ exogenous.
- Attract applicants $q_t$ with wage $w_t(q_t)$ to manage talent $r_t$

\[
\dot{r}_t = \alpha(\lambda(q_t; r_t) - r_t).
\]

- Firm value $V_t(r)$.
Firm’s Problem

- The firm solves
  \[ V_0(r_0) = \max_{\{q_t\}} \int_0^\infty e^{-\rho t} (\mu r_t - \alpha w_t(q_t)) dt, \]
  \text{s.t.} \quad \dot{r}_t = \alpha (\lambda(q_t; r_t) - r_t). \]

- Bellman equation
  \[ \rho V_t(r) = \max_q \left\{ \mu r - \alpha w_t(q) + \alpha V_t'(r)[\lambda(q; r) - r] + \dot{V}_t(r) \right\}. \]

- First order condition
  \[ \lambda'(q; r)V_t'(r) = w_t'(q). \]
Positive Assortative Matching

**Theorem 3.**
Equilibrium exists and is unique. Firms with more talent post higher wages. The distribution of talent has no atoms at $t > 0$.

Idea
- The value function $V_t(r)$ is convex.
- FOC implies matching is PAM.
- FOC also implies atoms immediately dissolve.

Thus
- Time-invariant firm-rank $x$, s.t. $r_t(x), q_t(x)$ increase in $x$. 
Constructing the Equilibrium

Equilibrium Matching $r_t(x), q_t(x)$

- Talent evolution

$$\dot{r}_t(x) = \alpha(\lambda(q_t(x); r_t(x)) - r_t(x))$$

- Sequential Screening

$$q'_t(x) = (\lambda(q_t(x); r_t(x)) - q_t(x))/x$$

Equilibrium Wages

$$w'_t(q) = V'_t(r)\lambda'(q; r)$$

where $q = q_t(x)$, $r = r_t(x)$ and

$$V'_t(r_t) = \frac{\partial}{\partial r_t} \int_t^\infty e^{-\rho(s-t)}[\mu r_s^* - \alpha w_s(q_s^*)]ds$$
Constructing the Equilibrium

Equilibrium Matching \( r_t(x), q_t(x) \)

- Talent evolution

\[
\dot{r}_t(x) = \alpha(\lambda(q_t(x); r_t(x)) - r_t(x))
\]

- Sequential Screening

\[
q_t'(x) = (\lambda(q_t(x); r_t(x)) - q_t(x))/x
\]

Equilibrium Wages

\[
w_t'(q) = V_t'(r)\lambda'(q; r)
\]

where \( q = q_t(x), r = r_t(x) \) and

\[
V_t'(r_t) = \mu \int_t^\infty e^{-\rho(s-t)} \frac{\partial r_s^*}{\partial r_t} ds
\]
Constructing the Equilibrium

Equilibrium Matching \( r_t(x), q_t(x) \)

- Talent evolution
  \[ \dot{r}_t(x) = \alpha(\lambda(q_t(x); r_t(x)) - r_t(x)) \]

- Sequential Screening
  \[ q'_t(x) = \frac{(\lambda(q_t(x); r_t(x)) - q_t(x))}{x} \]

Equilibrium Wages

\[ w'_t(q) = V'_t(r)\lambda'(q; r) \]

where \( q = q_t(x), r = r_t(x) \) and

\[ V'_t(r_t(x)) = \mu \int_t^\infty e^{-\int_t^s (\rho + \alpha(1 - \Delta(q_u(x)))) \, du} \, ds \]
Assumptions: \( p_H = 0.8, \ p_L = 0.2, \ \mu = 1, \ \rho = 0.1, \ \alpha = 0.2, \) and \( \bar{q} = 0.25. \)
Equilibrium Firm Dynamics — Payoffs

Assumptions: \( p_H = 0.8, \ p_L = 0.2, \ \mu = 1, \ \rho = 0.1, \ \alpha = 0.2, \) and \( \bar{q} = 0.25. \)
The Equilibrium Steady State

Steady-state matching $r^*(x), q^*(x)$

- Constant quality, $\lambda(q; r) = r$, links $q$ and $r$.
- Seq. screen., $q'(x) = (\lambda(q; r) - q)/x$, determines $r(x), q(x)$.

Steady state wages $w^*(q)$

- Marginal value of talent
  \[ V'(r) = \frac{\mu}{\rho + \alpha(1 - \Delta(q))} \cdot \]
- Marginal wages
  \[ w'(q) = \frac{\mu}{\rho + \alpha(1 - \Delta(q))} \lambda'(q; r). \]
Convergence to Steady State

**Theorem 4.**

a) *Steady State* \( \{r^*(x), q^*(x), w^*(q)\} \) is unique; no gaps or atoms.

b) For any initial talent distribution, equilibrium converges to SS.

Persistence of competitive advantage

- Random hiring: regression to mean at rate \( \alpha \).
- Screening applicants \( q \): regression to mean at \( \alpha(1 - \Delta(q)) \).
- But under PAM, high-quality firms pay more.
- Hence, talent is source of sustainable competitive advantage.
Comparative Statics

Talent dispersion rises in talent-skill correlation $\beta$

- Suppose recruiting skill is $(1 - \beta)\bar{q} + \beta r$.
- PAM if $\beta > 0$, but NAM if $\beta < 0$.
- Talent dispersion $r^*(1) - r^*(0)$ rises in $\beta$.

Wages rise in turnover $\alpha$

- Does not affect steady-state talent.
- Raises steady-state flow wages $(\rho + \alpha)w_t$.

\[
(\rho + \alpha)w'(q(x)) = \mu \lambda'(q(x); r(x)) \frac{\rho + \alpha}{\rho + \alpha(1 - \Delta(q(x)))}
\]

- Intuition: Effect of talent outlasts employment.
Dynamic Model with Heterogenous Technology
Heterogeneous Technology

Two types of heterogeneity

▶ Exogenous technology $\mu \in \{\mu_L, \mu_H\}$; mass $\nu$ low.
▶ Evolving talent $r_t$.
▶ Firms stratified, if $r_t$ and $\mu$ correlate perfectly.

Wages increase in $\mu$ and $r$

▶ Recall FOC

$$w'_t(q) = V'_t(r; \mu) \lambda'(q; r)$$

▶ Higher $r$ raises $V'_t(r; \mu)$ and $\lambda'(q; r)$.
▶ Higher $\mu$ raises $V'_t(r; \mu)$. 
Convergence to Steady State

Theorem 5.

a) There is a unique steady-state equilibrium.
b) The steady state is stratified.
c) Any equilibrium converges to this steady-state.
d) Distribution $r^*(x), q^*(x)$ independent of $\{\mu_L, \mu_H\}$.

Idea

- Talent distribution becomes continuous.
- High-tech firms outbid low-tech firms when talent is close.
Adjustment Dynamics of Single Firm

- Steady state with firms $r \geq r^*$ high tech; wages $w^*(r)$.
- Low-tech firm with $r < r^*$ becomes high-tech.

**Theorem 6.**

a) Wages satisfy $w_t \in (w^*(r_t), w^*(r^*))$

b) Talent $r_t$ converges to $r^*$ as $t \to \infty$.

Idea

- $w_t > w^*(r_t)$ since firm has higher tech.
- $w_t \leq w^*(r^*)$ since firm has less talent.
- Since $w_t > w^*(r_t)$, talent $r_t$ rises over time.
Saddle-point Stable Adjustment Path

- $r_0$ chosen to hit $r^*$. Near steady state,

$$ \begin{pmatrix} r_t - r^* \\ r_t - r^* \end{pmatrix} = (r_0 - r^*) \begin{pmatrix} 0.2032 \\ 1 \end{pmatrix} e^{-0.2281t}. $$
WELFARE
Welfare

Introducing Welfare

- Entry cost $k > 0$.
- Marginal firm: $\mu \lambda(Q(\tilde{r}); \tilde{r}) = k$.
- Welfare $\int_{\tilde{x}}^{1} (\mu \lambda(q(x); r(x)) - k) dx$.

Maximize Aggregate Sorting

- Planner chooses entry and rank $x$ for every firm $r$.
- Equilibrium entry threshold $\tilde{x}$ is efficient (given PAM).
- But, does PAM for $x \in [\tilde{x}, 1]$ maximize employed talent?
Efficient Matching

**Theorem 7.**

*For any entry threshold \( \tilde{x} \), NAM maximizes employed talent.*

**Two economics forces**

- Becker: PAM maximizes comparative advantage (if \( q < \hat{q} \)).
- Akerlof: PAM also maximizes adverse selection.
- And...
Efficient Matching

**Theorem 7.**

*For any entry threshold $\tilde{x}$, NAM maximizes employed talent.*

**Two economics forces**

- Becker: PAM maximizes comparative advantage (if $q < \hat{q}$).
- Akerlof: PAM also maximizes adverse selection.
- And... Akerlof wins!
Proof Sketch

Marginal Employed Talent

- Employed talent $\omega(\hat{x})$, where $\omega(x) = \bar{q} - xq(x)$
- Effect of better screening skills at rank $x$

\[
\zeta(x) := \frac{\partial \omega(\hat{x})}{\partial r(x)} = \frac{\partial \omega(\hat{x})}{\partial x} \cdot \frac{\partial x}{\partial r(x)}
\]

\[
= \exp \left( - \int_{\hat{x}}^{x} \frac{\lambda'(q(x); r(x))}{x} \, dx \right) \Delta(q(x))
\]

Shifting Screening Skills Up

\[
\zeta'(x) \simeq \Delta'(q(x))q'(x) - \lambda'(q(x); r(x))\Delta(x)/x < 0.
\]
Dynamic Efficiency

- Upfront entry cost \( \frac{k}{\rho} > 0 \).
- Present surplus \( \int_{0}^{\infty} e^{-\rho t} (\mu R_t - k) dt \), with \( R_t = \int_{\tilde{x}}^{1} r_t(x) dx \).
- Choose entry and wage ranks to maximize surplus.

**Theorem 8.**
For any \( \tilde{x} \), NAM surplus exceeds PAM surplus at all times.

Idea

- For fixed recruiting skills \( R_t \), NAM maximizes talent input.
- Additional talent under NAM helps recruit even more talent.
EXTENSIONS
Model of Hierarchies

Hierarchy

- \(N + 1\) layers: Level \(n = 0\) directors; level \(n = N\) workers.
- Mass 1 of firms; each has \(\alpha^n\) positions at level \(n\).
- Mass \(\alpha^n\) of job seekers at each level; proportion \(\bar{q}\) skilled.

Firms

- Director quality \(r_0\) exogenous.
- Level \(n\) agents hire level \(n + 1\) agents, \(r_{n+1} = \lambda(q_{n+1}, r_n)\).
- Only workers produce, \(v_N = \mu\alpha^N r_N\).
Hierarchies: Equilibrium Wages

Equilibrium with $\bar{q} < \hat{q}$

- Assume $r_0 \sim F_0$ steady state; then $r_{n+1} = r_n$.
- Level-$(n-1)$ value $v_{n-1}(r) := v_n(\lambda(q_n; r)) - \alpha^n w_n$; then
  $$v'_n(r) = \mu\alpha^N \Delta(q)^{N-n}$$
- Marginal level-$n$ wages
  $$w'_n(q) = \lambda'(q; r)\mu(\alpha\Delta(q))^{N-n}.$$
- Assume $\alpha\Delta(q) > 1$; then wages increase in rank.

Wage dispersion across firms $q > \tilde{q}$ and levels $n < \tilde{n}$

- Inter-firm dispersion greater at high levels: $\frac{w_n(q)}{w_n(\tilde{q})} \geq \frac{w_{\tilde{n}}(q)}{w_{\tilde{n}}(\tilde{q})}$.
- Intra-firm dispersion greater at high firms: $\frac{w_n(q)}{w_{\tilde{n}}(q)} \geq \frac{w_n(\tilde{q})}{w_{\tilde{n}}(\tilde{q})}$.
Conclusion

We’ve proposed a model in which

- Firms compete to identify and recruit talent.
- Today’s recruits become tomorrow’s recruiters.

Main results

- Positive assortative matching.
- Persistent productivity dispersion.
- Equilibrium inefficiency due to adverse selection.

Next steps

- Characterize dynamic matching with $\bar{q} > \hat{q}$.
- Characterize dynamic and steady state dispersion.
- Study dynamics when $\mu_t$ are stochastic.