

Recruiting Talent^{*}

Simon Board[†], Moritz Meyer-ter-Vehn[‡] and Tomasz Sadzik[§]

August 30, 2023

Abstract

We study a parsimonious model of a competitive labor market in which firms privately screen workers to identify talent. The equilibrium exhibits dispersion in wages and productivity, whereby firms with superior screening skills post higher wages, attract better applicants, and recruit more talented workers. High-wage firms impose a compositional externality on low-wage firms, leading to equilibrium inefficiency: Welfare would be higher if low-skilled firms posted high wages and selected first. We also provide a micro-foundation for firms' heterogeneous screening skills. When talented workers are better at screening (e.g. via superior referrals), a dynamic version of the economy converges to a unique steady-state in which differences in talent, profits and screening skills persist forever.

^{*}We are particularly grateful to Steve Tadelis for a conversation that inspired this project. We have received useful comments from Carlos Akkar, Jim Albrecht, Stepan Aleksenko, Alp Atakan, Roland Benabou, Alan Benson, Dirk Bergemann, V Bhaskar, Bob Gibbons, Marina Halac, Johannes Hörner, Peter Kondor, Alessandro Lizzeri, Guido Menzio, Benny Moldovanu, Stephen Morris, Volker Nocke, Kevin Roberts, Larry Samuelson, Susan Vroman, Leeat Yariv, and Diego Zúñiga. We thank seminar audiences at Berlin, Bonn, Cambridge, Edinburgh, ESWM 2017, Games 2016, Gerzensee 2016, ITAM, Leicester, Mannheim, Miami, MIT, NBER Firms 2016, Northwestern/Duke/MIT IO conference 2016, Oxford, Penn, Princeton, Queen Mary, UBC, UCL, UCLA, and Yale. Keywords: adverse selection, assortative matching, firm dynamics, labor market competition, productivity dispersion, recruitment, referrals, wage dispersion. JEL codes: D21, D82, M51.

[†]UCLA, <http://www.econ.ucla.edu/sboard/>

[‡]UCLA, <http://www.econ.ucla.edu/mtv/>

[§]UCLA, <https://sites.google.com/view/tsadzik>

1 Introduction

The success of most firms is built upon hundreds of individuals who take thousands of decisions, making it critical to identify and recruit the best talent. For example, the Netflix human resource manual states “One outstanding employee gets more done and costs less than two adequate employees. We endeavor to have only outstanding employees” (Hastings and McCord, 2009). Similarly, Google’s former head of human resources writes “Hiring is the single most important people activity in any organization. [...] Our greatest single constraint on growth has always, always been our ability to find great people” (Bock, 2015). Within economics, there is a large literature that measures the importance of employee talent, from top executives (Bertrand and Schoar, 2003) to blue-collar workers (Lazear, 2000), and from salespeople (Benson et al., 2019) to bureaucrats (Fenizia, 2022).

The market for talent is plagued by imperfect information.¹ As a result, firms carefully screen applicants, obtaining referrals and conducting interviews. In Behrenz’s (2001) survey, employers report that their most important source of information when hiring are methods of “private screening” in the form of interviews (41%) and personal contacts (25%), as compared to “public information” like references from past employers (21%), references from schools (5%) and the application (3%). Indeed, referrals account for over a third of US jobs (Holzer, 1987) and more than half the jobs at Google (Bock, 2015).

Firms differ in their skill at screening applicants. An exemplar is Google that is said to have “built the world’s first self-replicating talent machine” by turning “every employee into a recruiter” (Bock, 2015). On the flip side, VW struggled to copy Tesla’s battery operations because of “difficulty hiring qualified engineers in fields it knew very little about,” where one recruiter noted “you can only become an expert if you do it yourself.”²

We capture these ideas with a parsimonious model of a competitive labor market, where firms privately screen workers (e.g. interviews, referrals). Equilibrium gives rise to dispersion in wages and productivity across firms, whereby firms with superior screening skills complement them by posting high wages that attract better applicants. We show the market is inefficient in that equilibrium minimizes firms’ aggregate effectiveness in sorting talented workers into the industry. Motivated by the Google and VW examples, we also endogenize firms’ screening skills by considering a dynamic model in which firms with more talented employees are more skilled at recruiting. The economy converges to

¹For example, economists have examined how measures of ability filter into wages (Farber and Gibbons (1996), Fredriksson et al. (2018)) and how certification affects wages and employment (Stanton and Thomas (2015), Pallais (2014)).

²See “To Beat Tesla, Volkswagen Bets on Making Its Own EV Batteries,” *WSJ*, 8th Aug 2021.

a steady state featuring persistent dispersion in talent, screening skills, and profits.

In Section 2, we introduce a static model of labor market competition in which firms have private information about workers’ talent. Specifically, a continuum of firms competes for a continuum of workers who have high or low ability. Talented workers have positive value added, while untalented workers would be better employed outside the industry. Firms attract applicants by posting wages and then receive independent noisy signals about each applicant, with more skilled firms obtaining more accurate signals. The market is non-exclusive (workers can apply to all firms) and anonymous (firms do not know anything about their applicants, other than the fact they apply). We model this by assuming the highest-paying firm chooses from all workers while other firms hire from the remaining, adversely selected pool of workers. The applicant pool quality thus endogenously declines with the wage rank, giving rise to equilibrium dispersion of wages and productivity. Assuming that talent is scarce, we show that firm-worker matching is *positive assortative*. That is, firms with more skilled recruiters post higher wages, attract better applicants, and hire more talented recruits. Intuitively, skilled firms have a comparative advantage in hiring from a high-wage applicant pool with a balance of talented workers, rather than hiring from a low-wage pool in which few talented applicants remain.

The model is stylized but is highly tractable and speaks to important features of labor markets. In equilibrium, wages and productivity are dispersed across firms and positively correlated (e.g. Card et al. (2018)). The model describes how talent is allocated across firms and how wages differ from workers’ marginal product (e.g. Card et al. (2013)).³ It also generates mismatch, with some untalented workers employed and other, talented workers excluded (e.g. Fredriksson et al. (2018)). Firms with better management pay higher wages, recruit better workers, and have higher productivity (e.g. Bender et al. (2018)). The model also predicts an increase in screening skills (e.g. due to increasing job tests and referrals) increases the dispersion of productivity and wages, as well as segregation, with talented workers increasingly working together, consistent with recent trends (e.g. Barth et al. (2016), Song et al. (2019)).

In Section 3 we examine the welfare consequences of the model. In sharp contrast to classic matching models (e.g. Shapley and Shubik (1971), Becker (1973)), the positive assortative matching seen in equilibrium is inefficient. Intuitively, high-wage firms screen applicants first and exert a negative *compositional externality* on low-wage firms

³The typical way to identify worker talent is via the “AKM-decomposition” of wage dispersion into worker- and firm-effects (Abowd et al., 1999). However, this approach is problematic when firms imperfectly observe workers’ ability. For example, in our model, identical firms can systematically pay different wages; differences between workers thus look like firm-effects.

by extracting talent from the applicant pool. Positive assortative matching maximizes this externality since the high-wage firms are skilled at extracting talent; this outweighs the private gain from positive assortative matching. Indeed, we show that negative assortative matching, whereby low-skill firms offer high wages and screen first, minimizes the externality and optimally selects talent into the industry. Standard labor market policies (e.g. minimum wages, progressive taxes) do not help restore efficiency, but a wage cap can induce a random screening order and raise welfare.

In Section 4 we introduce a dynamic version of the model in which talented workers are better at recruiting and show that persistent differences in firms’ talent and recruiting skills arise endogenously. The talent of each firm evolves as workers retire and today’s recruits become tomorrow’s recruiters. In the unique equilibrium, talented firms post high wages, attract the best applicants and hire talented recruits, reinforcing their initial advantage. If all firms start off with similar talent, the better endowed accumulate talent over time, while the worse endowed hire from poor, deteriorating applicant pools and lose talent. The economy converges to a steady-state with persistent talent differentials, balancing two countervailing forces: Imperfect screening which leads to mean reversion and equalizes firms, and positive assortative matching that amplifies differences across firms. While low-quality firms could in principle catch up by posting higher wages and hiring more talented workers, it is not profitable for them to do so. Thus, talent becomes a source of sustainable competitive advantage. Importantly, steady-state talent differences are sustained by firms’ endogenous wage choices and thus arise for any degree of correlation between productivity and recruiting skills, however weak.

The key premise behind the dynamic model is that firms with more talented employees are more skilled at recruiting.⁴ This is seen in Gupta (2018), where more productive sales managers hire more productive salespeople who remain highly productive even after they switch teams. Similarly, Waldinger (2012, 2016) finds that the loss of star professors in Nazi Germany led to a permanent reduction in the quality of hires, while Huber et al. (2021) show losing Jewish managers led to a permanent reduction in firm performance. Given this premise, our results show that small initial differences in talent are amplified over time and generate persistent differences in talent and productivity. These dynamics resemble Giorcelli (2019) where a management training program generated productivity

⁴There are two natural mechanisms for talented employees to be better at recruiting. First, they may provide better referrals, as is seen in the field experiments of Beaman and Magruder (2012) and Pallais and Sands (2016). Second, talented employees may be better at recruiting, as illustrated by the famous Dunning-Krueger effect: “The skills you need to produce a right answer are exactly the skills you need to recognize what a right answer is”. (See David Dunning in “The Anosognosic’s Dilemma: Something’s Wrong but You’ll Never Know What It Is,” *New York Times*, 10th June, 2010.)

gains that grew from 15% in the first year to 49% after 15 years. One channel was that “better managed firms paid higher average wages to their workers, which may indicate that trained managers were able to hire/retain better workers.”

1.1 Literature

The static model of sequential screening is most closely related to Broecker (1990), Montgomery (1991), and Kurlat (2016). Broecker considers a version of our model with a finite number of homogenous firms with unlimited capacity. As in our model, adverse selection induces wage dispersion, but our key questions of sorting and its inefficiency do not arise because firms are homogeneous.

In Montgomery’s classic model of referrals, each talented employee refers one positively selected applicant; firms then make *applicant-specific* wage offers. In equilibrium, firms post random high wages for referred workers; if they strike out, they pay deterministic low wages in the non-referred market. The model displays mean-reversion, so differences in talent and wages across firms erode over time (see Section 4.4). In our model, firms receive positive signals about a proportion of the applicants and then make *firm-specific* wage offers. This is consistent with the idea that firms reward employees according to a pay-scale that differs across firms (e.g. Netflix pays more than Disney).⁵ As a result, our dynamic model generates persistent differences in talent and wages; intuitively, high-talent firms pay high wages, so a talented worker hires better recruits if their colleagues are also talented. Additionally, we study equilibrium efficiency which Montgomery does not address.

Like us, Kurlat (2016) studies non-exclusive markets where buyers screen sellers of uncertain quality. In Kurlat’s financial market, sellers’ assets are *perfectly divisible* and, in his baseline model, buyers’ signals about the assets are *nested*, in that a more informed buyer knows everything that a less informed buyer knows. In our labor market, workers look for one, *indivisible* job and firms’ signals are (conditionally) *independent*. In the labor market context, nested signals capture public information such as reference letters, while independent signals capture private information such as personal referrals or interviews.⁶ These two modeling assumptions are crucial. Our firms post different wages, low-wage firms are subject to adverse selection, and matching is positive assortative and inefficient;

⁵See http://www.econ.ucla.edu/sboard/papers/Indeed_Netflix_Disney_Aug23.pdf. This difference between firm-level wages and applicant-level wages is also what differentiates Burdett and Mortensen (1998) from Postel-Vinay and Robin (2002a, 2002b).

⁶Either modeling assumption seems plausible. Indeed, Kurlat and Scheuer (2021)’s labor market adaptation of Kurlat (2016) to Spencian signaling, studies nested signals for indivisible jobs.

in contrast, Kurlat’s buyers purchase at the same price, avoid adverse selection, and matching is negative assortative and efficient. When assets are divisible, sellers can predict how much they will sell on any given market, allowing them to simultaneously cross-list their assets across markets with no market facing an adversely selected left-over pool of assets. When signals are nested, high-skill firms can screen out all applicants who failed tests of low-skill firms and so do not mind hiring last. We elaborate on these differences in Appendix A and argue that adverse selection and wage dispersion arise whenever goods exhibit some indivisibility and signals exhibit some independence.

Adverse selection has also been introduced into models of random search (Lockwood (1991), Lauermann and Wolinsky (2016), Kaya and Kim (2018)) and directed search (Guerrieri et al. (2010), Li and Shimer (2019)). With random search, selective hiring dilutes the applicant pool at later firms, but firms do not control the order of applications. With directed search, firms do control their applicant pool by posting contract terms, but turned-down workers cannot apply to other firms. We combine firms’ control over their applicants with sequential applications into a model of “directed sequential search” that abstracts from search frictions and focuses on information frictions.

Our model complements the wider literature on wage dispersion. In Burdett and Judd (1983) and Burdett and Mortensen (1998), dispersion derives from firms competing for more workers in an economy with search frictions, whereas our dispersion derives from firms competing for better workers in an economy with adverse selection. In many labor markets, the pertinent search friction is in evaluating the quality of applicants, rather than finding them in the first place. Van Ours and Ridder (1992) find that “76% of all vacancies are filled by applicants who arrived during an application period that lasts for about 2 weeks”, leading them to write that “vacancy durations should be interpreted as selection periods and not as search periods for applicants.” This view is consistent with the extensive evidence of imperfect information in the labor market (e.g. Farber and Gibbons (1996)), the widespread use of referrals (e.g. Holzer (1987)), and the significant amount firms spend on screening candidates (e.g. Barron et al. (1985), Blatter et al. (2012)).⁷

Our heterogeneous-firm model contributes to the literature on firm-worker matching. Becker (1973) observes that more productive firms hire more talented workers when productivity is supermodular. In a dynamic model, Anderson and Smith (2010) and Anderson (2015) suppose agents match each period and evolve as a function of the match; they show

⁷In recent years, online job search has further reduced search frictions: For example, Davis and Samaniego (2020) report that 45% of job applications arrive in the first 48 hours and half the job postings are taken down in a week. The mean vacancy duration exceeds 40 days, so the average firm then spends over four weeks screening and selecting applicants.

that equilibrium is efficient, and derive sufficient conditions for matching to be positive assortative. In contrast to this literature that focuses on complementary production, our paper focuses on asymmetric information in the hiring process, and shows that complementarities arise endogenously, with more skilled firms posting higher wages. Our model has different implications from Becker. On the normative side, we prove that equilibrium is inefficient; surprisingly, this inefficiency can prevail even with production complementarities (see Section 3.1). On the positive side, we examine how wage dispersion and mismatch depend on screening skills (see Section 2.2 and 3.2).

Our dynamic model provides a theory of firm evolution in which a firm's stock of talent is its key strategic asset. In addition to Montgomery (1991), this relates to a broader set of papers on firm dynamics in which competitive advantage stems from technology (Lucas and Prescott (1971), Hopenhayn (1992)), reputation (Jovanovic, 1982) or the stock of labor (Hopenhayn and Rogerson, 1993). By focusing on talent and recruiting, our paper provides a new channel through which firms can sustain a competitive advantage and gives rise to predictions concerning the inter- and intratemporal relationship between productivity, wages and employee quality.

2 Competitive Screening

This Section introduces our static model and characterizes equilibrium. Section 2.1 presents the model. Section 2.2 describes the outcomes of firms' sequential screening and shows that wages are dispersed in equilibrium. Section 2.3 provides sufficient conditions for positive assortative matching. Section 2.4 justifies the market clearing mechanism.

2.1 Model

A unit mass of firms, each with one vacancy, compete for a unit mass of workers. Workers differ in their talent θ , with proportion $\bar{q} \in (0, 1)$ talented, $\theta = H = 1$, and the remainder untalented, $\theta = L = 0$. Firms select among applicants by administering a pass/fail test to each applicant. Talented workers pass all tests, while untalented workers are screened out with probability $p \in (0, 1)$, independently across firms and workers. Firm screening skills p are distributed according to $F[\underline{p}, \bar{p}]$, where $0 < \underline{p} \leq \bar{p} < 1$; for simplicity we assume throughout that the cdf F is invertible, except for a few clearly marked parts in the paper where we consider homogeneous screening skills.

We seek to model a labor market that is non-exclusive and anonymous. Firms simultaneously post wages and offer their job to any worker who passes their test. Workers

accept their highest wage offer above their outside option. To operationalize this, order firms in terms of their wages and suppose workers apply to firms from highest to lowest wage, breaking ties at random. The highest firm screens applicants in a random order and hires the first who passes their test; the adversely selected remainder then apply to the “second” firm, and so on until all firms and workers are matched. We describe the outcome of this procedure in Section 2.2 and provide a micro-foundation in Section 2.4.^{8,9}

When a recruiter screens an applicant pool with expected talent q , proportion $1 - (1 - q)p$ of the applicants pass the test. Bayes’ rule implies that the fraction of recruits who are talented equals

$$\lambda(q, p) := \frac{q}{1 - (1 - q)p}. \quad (1)$$

Expected talent $\lambda(q, p)$ increases in both the applicant quality q and the screening skill p .

Payoffs are as follows. Workers only care about wages and so accept any job paying more than their outside option, $\underline{w} \geq 0$. We call this outside option “unemployment”, but it could be a job in a different industry.¹⁰ Productivity is normalized to 1 for talented workers and 0 for untalented workers. Thus, when a firm posts wage w and attracts applicants q , its expected profits are

$$\pi := \lambda(q, p) - w. \quad (2)$$

We solve for Nash equilibrium in wages.¹¹

⁸The assumption that firms use binary, pass-fail tests and that talented workers pass the test with certainty is without loss. Indeed, consider a more general information structure with finitely many signals s that arise with probability $p^\theta(s)$. A firm that attracts any applicants attracts a continuum of them, and thus some applicant with the signal \bar{s} that maximizes the odds-ratio $\bar{\ell} = p^H(\bar{s})/p^L(\bar{s})$. Recruit quality is then $\frac{q\bar{\ell}}{q\bar{\ell} + (1-q)}$ which collapses to (1) when $\bar{\ell} = p^H(\bar{s})/p^L(\bar{s}) = 1/(1 - p)$ for \bar{s} = “pass”.

⁹We can reinterpret firms’ screening as referrals. Assume that each firm is connected to each talented worker via a referral with probability ϵ , and to each untalented worker with probability $\epsilon(1 - p)$. Each firm extends provisional wage offers to their referrals, and the market clears from the top with workers accepting their best offers and firms rescinding their remaining offers once their position is filled.

¹⁰Alternatively, \underline{w} could be an operating cost for the firm; under this formulation, equilibrium allocations and payoffs are the same while wages are shifted down by \underline{w} .

¹¹Our firms post wages rather than contracts. If output were contractible and workers knew their types, firms might be able to induce self-selection by paying for output. Such self-selection is rare in practice, perhaps because individual output is non-contractible (e.g. workers may be part of a team, measures of output may be manipulated), because workers’ types are multi-dimensional (e.g. contingent contracts attract risk-seekers), or because contingent contracts signal the firm’s private information (e.g. equity contracts are cheap for low-performing firms). Given our assumption that firms post wages and workers non-strategically accept the highest wage offer, the question whether workers know their types is moot.

2.2 Sequential Screening

To characterize equilibrium, we first study how the quality of the applicant pool depends on the firm's rank in the wage distribution. Suppose all firms post different wages, write $x \in [0, 1]$ for the resulting wage quantiles, and $w(x), Q(x), P(x)$ for wage, applicant quality, and screening skills at wage quantile x .¹² Wages $w(x) < \underline{w}$ do not attract applicants and result in zero profits.

The highest ranked firm faces applicant pool $Q(1) = \bar{q}$; thus, proportion $\lambda(Q(1), P(1))$ of its recruits are talented. Since firms select talented workers disproportionately, lower-ranked firms face an adversely selected applicant pool, meaning that $Q(x)$ falls as the firm rank x declines. Specifically, at rank x there is a total of $xQ(x)$ talented workers, of which firms $[x, x + dx]$ hire $\lambda(Q(x), P(x))dx$; hence $d[xQ(x)] = \lambda(Q(x), P(x))dx$. Rearranging, the talent pool evolves according to the *sequential screening equation*

$$Q_x(x) = \phi(Q(x), P(x), x) \quad \text{where} \quad \phi(q, p, x) := \frac{\lambda(q, p) - q}{x}. \quad (3)$$

Since screening is imperfect, some talent remains, $Q(x) > 0$, for all $x > 0$. However, firms pick over the applicants so many times that eventually no talent remains, $Q(0) = 0$, assuming every worker is employed.¹³

In equilibrium, wages are distributed continuously. To see this, observe that if an atom of firms offered the same wage, then a firm could attract discretely better job applicants with a marginal wage raise. Similarly, there can be no gap $[w, w']$ in the wage distribution, since firms offering w' could attract the same applicants at the lower wage w . By the same argument, the lowest wage in the wage distribution equals the outside option \underline{w} , so the share of unemployed workers $\underline{x} \geq 0$ solves $w(\underline{x}) = \underline{w}$. Since expected talent (1) and profits (2) rise in p , firms above some $\underline{p} = F^{-1}(\underline{x})$ enter the market with a wage above \underline{w} while those below \underline{p} exit (or, equivalently, post a wage below \underline{w}). To summarize:

Lemma 1. *Equilibrium wages and productivity are continuously distributed with a minimum of \underline{w} . There is unemployment $\underline{x} > 0$ if and only if $\underline{w} > 0$. Firms $p \in [\underline{p}, \bar{p}]$ enter.*

Example 1 (Homogeneous Firms). Wage dispersion does not require heterogeneous

¹²This analysis assumes that firms post different wages. Otherwise, in case of a tie with an atom in the wage distribution $G(w)$, assume for concreteness that all workers break the tie in the same way, as if the firms were infinitesimally differentiated. The rank x of a firm with wage w is then drawn uniformly from $[\lim_{\epsilon \rightarrow 0} G(w - \epsilon), G(w)]$. Lemma 1, below, implies that this complication does not arise in equilibrium.

¹³To see $Q(x) > 0$ for $x > 0$, note that $(\log Q(x))_x = (\lambda(Q(x), P(x))/Q(x) - 1)/x = (1/(1 - (1 - Q(x))P(x)) - 1)/x$ is bounded for any fixed $x > 0$ since $\bar{p} < 1$. To see $Q(0) = 0$, note that $\underline{p} > 0$ implies that $(\log Q(x))_x$ is of order $1/x$, and so the integral $\log Q(x) = \log \bar{q} - \int_x^1 (\log Q)_x$ diverges as $x \rightarrow 0$, implying $\log Q(0) = -\infty$, or $Q(0) = 0$.

screening skills, and also obtains when all firms have the same skill p . In this case, competition depletes profits and wages coincide with expected productivity, $w(x) = \lambda(Q(x), p)$. The worst applicant quality \underline{q} that firms are willing to consider is given by $\lambda(\underline{q}, p) = \underline{w}$ and unemployment \underline{x} solves $Q(\underline{x}) = \underline{q}$.

In Appendix B.1, we show that wage and productivity dispersion rises when screening skills p rise (say, due to improvements in algorithmic hiring or employment networks). Intuitively, the top firms extract more talented workers from the applicant pool, lowering the productivity and wages of low-wage firms.¹⁴ \triangle

2.3 Positive Assortative Matching

In this section, we characterize the equilibrium of our wage-posting game. We say there is *positive assortative matching* (PAM) between firms and applicants if $P(x)$ is increasing, meaning firms with high screening skills p attract applicant pools of high quality, q . From Becker (1973), we know that equilibrium features PAM if skilled recruiters have a comparative advantage in screening applicants with higher expected talent, i.e. if expected recruit quality $\lambda(q, p)$ is supermodular. To ensure supermodularity, we assume that talent is scarce, in that a worker from the unselected pool who passes the most stringent test is more likely untalented than talented:

$$\lambda(\bar{q}, \bar{p}) \leq 1/2. \quad (4)$$

This is a joint condition on the distributions of talent \bar{q} and screening skills \bar{p} . This assumption is necessary for equilibrium sorting, but not for the welfare results in Section 3. This assumption is satisfied in industries with relatively few highly productive individuals, such as technology or sales.¹⁵

Theorem 1. *Assume scarce talent (4). Equilibrium exists and is unique. The equilibrium wage distribution is unique. Matching is positive assortative.*

Proof. Note the partial derivatives

$$\lambda_p(q, p) = \frac{q(1-q)}{(1-p(1-q))^2} \quad \text{and} \quad \lambda_{qp}(q, p) = \frac{1-2\lambda(q, p)}{(1-p(1-q))^2}. \quad (5)$$

¹⁴We also show that wage and productivity dispersion rises when average talent \bar{q} falls (say, due to skill-biased technological change that gives rise to a few superstars).

¹⁵Writing about Google and Netflix, Bock (2015, p. 62) says that “Only 10% of your applicants (at best) will be top performers,” while Hastings and McCord (2009) say “In creative/inventive work, the best are 10 times better than average.” Based on salespeople at 131 firms, Benson et al. (2019) confirm “a well-known heuristic [...] that the top 20% of the sales force is responsible for 80% of sales.”

Scarce talent (4) implies that $\lambda_{qp} > 0$ for all $q \leq \bar{q}, p \leq \bar{p}$, and matching is positive assortative. That is, higher skill firms post higher wages, and so $P(x)$ increases.

We can now construct the equilibrium. Given PAM, a firm's rank in the skill distribution equals its equilibrium wage rank, $F(p) = x$, and we can identify a firm by this rank x . The skill of the firm with wage-rank x is then given by $P(x) = F^{-1}(x)$, its applicant quality $Q(x)$ is determined by the sequential screening equation (3), and the recruit quality $\lambda(Q(x), P(x))$ by Bayes' rule, (1).

From this we can derive the entry threshold \underline{p} , wages, and profits. Denote the equilibrium wage required to attract applicants of quality q by $W(q)$. The marginal firm pays the outside option, so employment \underline{x} is given by $\lambda(Q(\underline{x}), P(\underline{x})) = \underline{w}$, which determines the entry threshold $\underline{p} = F^{-1}(\underline{x})$. Wages are determined by firms' first-order conditions,

$$W_q(Q(x)) = \lambda_q(Q(x), P(x)). \quad (6)$$

Finally, profits $\Pi(p)$ follow by the envelope condition

$$\Pi_p(P(x)) = \lambda_p(Q(x), P(x)). \quad (7)$$

Intuitively, productivity $\lambda(q, p)$ depends on both the applicant pool quality and the screening ability; workers capture the marginal benefit of the former and firms capture the marginal benefit of the latter.

The so constructed wage profile $W(Q(x))$ is the only possible candidate for an equilibrium. To verify that the wages are indeed optimal, it suffices to note that marginal profits $\lambda_q(q, p) - W_q(q)$ single-cross in p and matching is positive assortative; hence the FOC (6) implies global optimality. \square

Theorem 1 captures a natural complementarity in the recruiting function λ . Intuitively, recruiting skills do not matter if all applicants are either talented or untalented, but they do matter when applicant quality is intermediate. Since $Q(x)$ is bounded above by \bar{q} , our scarce-talent assumption (4) implies that skilled firms have a comparative advantage at screening better applicants. Thus, skilled firms pay high wages, attract high-quality applicants, recruit talented workers, and achieve high productivity and profits.¹⁶

¹⁶The comparative advantage of skilled recruiters in screening high-quality applicant pools relies on our assumption that firms can screen applicants until one passes the test. In contrast, Li and Shimer (2019)'s directed search model assumes that each firm screens a single applicant. Multiplying (2) with the hiring probability yields $\tilde{\pi} = q - (1 - (1 - q)p)w$ with cross-partial $\partial^2 \tilde{\pi} / \partial p \partial q = -w < 0$. Intuitively skilled recruiters hire fewer untalented workers and thus have a comparative advantage at screening bad applicant pools with lots of untalented workers.

Without the scarce talent assumption (4), PAM fails at the top of the distribution for $q = \bar{q}$ and $p = \bar{p}$ by (5). It is not clear how to characterize equilibrium when the sign of λ_{qp} changes over the domain of (q, p) , but PAM still obtains at the bottom of the distribution since $\lim_{x \rightarrow 0} Q(x) = 0$.¹⁷

In Appendix B.2 we extend our model to continuous types with linear hiring propensity. When types are exponentially distributed, equilibrium is positive assortative.

2.4 Justification of the Market Clearing Mechanism

We now justify the clearing mechanism that underlies the sequential screening equation (3). In our model, firms choose wages w_i strategically; given these wages, the market then clears mechanically, “top-to-bottom”. A potential concern with this reduced-form approach is that it doesn’t allow workers to “jump the queue” in an attempt to avoid adverse selection. To understand the idea, consider a two-firm version of our model with $w_1 > w_2$. If a worker applies to Firm 2 in “round 1”, they have higher expected talent than a worker who applies to Firm 2 in “round 2”, after being rejected by Firm 1. This argument assumes that (i) the worker can prove she did not apply to Firm 1, and (ii) the worker can commit not to leave Firm 2 if Firm 1 makes her an offer later in the game. We justify our “top-to-bottom” application order by arguing that these two assumptions are unreasonable for the anonymous, non-exclusive labor markets we have in mind. Each involves extending the model in a different way; since this is not the central contribution of the paper, we keep the discussion informal.

Our preferred justification is that our static model represents the steady state in a larger, dynamic game. At each time, mass 1 of workers enter the market and each firm x has an evergreen job post that pays wage $w(x)$. With a single generation of applicants, a worker could apply to Firm 2 in “round 1” and avoid adverse selection; with multiple generations, a worker who jumps over Firm 1 will reach Firm 2 at the same time as all the rejected workers from the “prior generation”. Indeed, in many real-world labor markets, firms entertain applications on a continuous basis and treat those arriving on June 1st and June 2nd symmetrically. We say a firm has *passive beliefs*, in that its acceptance decisions depend only on its signals and its wage-rank, as in the steady-state of a dynamic game. Given passive beliefs, workers optimally apply top-to-bottom.¹⁸

¹⁷This assumes \underline{w} small, so enough firms enter the market. In a previous version of this paper, with a slightly different parametrization of firms’ screening skills (Board et al. (2017)), we fully characterize equilibrium sorting and show wages $w(p)$ are hump-shaped when talent is abundant.

¹⁸Even in a one-generation model, passive beliefs can arise if application times are subject to noise, so that all application times are on-path. Top-to-bottom applications then constitute an equilibrium,

Another justification is that application orders other than top-to-bottom give rise to “unstable” matches. Consider a model with finite workers and firms; workers do not know their own types, and firms observe private signals about workers. Workers choose application orders, and the market clears according to Gale-Shapley’s worker-proposing deferred acceptance mechanism. Now, suppose worker A ranks Firm 2 ahead of Firm 1, even though Firm 1 pays higher wages. With positive probability A gets accepted by Firm 2. While Firm 1 cannot directly observe Firm 2’s offer to A in our anonymous labor market, it does observe that A never applies for its job and infers that A found some other job. If A also scores highly on Firm 1’s test, this positive inferences renders worker A and Firm 1 a blocking pair to the resulting matching; the matching thus fails Chakraborty et al.’s (2010) notion of weak stability. One problem with such non-cooperative notions of stability is that equilibrium inferences may be overly sensitive to fine details of the game. A promising avenue for future research is thus to extend cooperative notions of stability with incomplete information (e.g. Liu et al. (2014), Liu (2020)), to our setting where wages are fixed before the matching. One challenge with this approach is to capture the anonymity of our market, which prevents it from aggregating firms’ information.

These arguments provide support for our top-to-bottom application order. It is also highly tractable, allowing us to study sorting and welfare, to which we now turn.

3 Welfare

In the model, a worker’s productivity equals θ in our industry and \underline{w} elsewhere. The social value of screening consists in sorting talented workers, $\theta = H$, into the industry and untalented workers, $\theta = L$, out of the industry. In other words, mismatch arises from unemployed talented workers, and employed untalented workers. Section 3.1 studies firms’ screening order while Section 3.2 studies the level of firms’ screening skills.

3.1 Screening Order

Equilibria in matching models with transferable utility are typically efficient (e.g. Shapley and Shubik (1971), Becker (1973)). Surprisingly, this welfare theorem fails in our model. In particular, Theorem 2 shows that welfare is minimized by positive assortative matching and maximized by negative assortative matching (NAM). The key difference to standard

since firms’ inferences depend only on their equilibrium beliefs and not on application times; this in turn justifies top-to-bottom applications.

matching models is the compositional externality: The applicant pool quality is endogenous and depends on the screening skill of higher-paying firms. Intuitively, high-skilled firms pick out more talented workers than low-skilled firms and introduce more adverse selection. This externality is maximized by PAM and minimized by NAM.¹⁹ The following example illustrates the idea.

Example 2 (Two Types of Firms). Suppose mass η_H of firms are skilled, $p > 0$ and mass η_L are unskilled, $p = 0$. To abstract from entry, assume firms are on the short side of the market and the minimum wage \underline{w} does not bind, so all firms enter. Under PAM, skilled firms first hire from the unselected pool \bar{q} ; unskilled firms then hire from an adversely selected pool with quality $q < \bar{q}$. In contrast, under NAM, unskilled firms hire from the unselected pool \bar{q} ; they do not change the quality of the talent pool, and so the skilled firms also hire from a pool of quality \bar{q} . Since the unskilled firm impose no compositional externality on the skilled firms, more talented workers are hired under NAM than under PAM, raising social surplus.

The welfare loss from incorrect sorting can be substantial: For example, $(\eta_H, \eta_L) = (0.09, 0.90)$ firms screening with NAM has the same welfare as $(\eta'_H, \eta'_L) = (0.90, 0.09)$ firms screening under PAM. To see this, note that with unemployment fixed at $\underline{x} = 1 - \eta_H - \eta_L$, welfare is decreasing in the talent remaining among the unemployed $Q(\underline{x})$. By (3), Q_x scales inversely with the number of remaining applicants x . Thus, mass η_H skilled recruiters screening at wage ranks $x \in [\underline{x}, 1 - \eta_L]$ under NAM reduces applicant quality by the same amount as mass $\eta_H/(1 - \eta_L)$ skilled recruiters at wage ranks $x \in [1 - \eta_H/(1 - \eta_L), 1]$ under PAM. Thus, switching from PAM to NAM has the same welfare consequence as sticking to PAM while scaling up the number of skilled firms by a factor $1/(1 - \eta_L)$. \triangle

To argue the inefficiency of the competitive equilibrium more generally, consider a planner who can direct all firms' entry and wage decisions but is subject to the same informational frictions.²⁰ Her problem is to choose employment $1 - \underline{x}$ and a measure-preserving matching function, or equivalently *screening order*, $P : [\underline{x}, 1] \rightarrow [\underline{p}, \bar{p}]$ to maximize welfare

¹⁹Compositional externalities of a different nature are seen in models of directed search. In Albrecht et al. (2023), workers applying to recent job postings exert an externality on later applicants if employers fail to remove their filled "phantom vacancies" from the market. In Athey and Ellison (2011)'s model of position auctions, high-quality advertisers serve more searchers than low-quality advertisers when winning first position in the ad auction; this order minimizes search costs, meaning equilibrium is efficient.

²⁰In particular, the planner cannot communicate firms' test results to each other. With a continuum of firms and independent tests, allowing such communication would trivially solve any mismatch.

$$S := \int_{\underline{x}}^1 [\lambda(Q(x), P(x)) - \underline{w}] dx. \quad (8)$$

For example, in equilibrium, matching is positive assortative $P^{\text{PAM}}(x) = F^{-1}(x)$, with entry cutoff \underline{x} satisfying $\lambda(Q(\underline{x}), P^{\text{PAM}}(\underline{x})) = \underline{w}$; assuming the same entry behavior, NAM is characterized by $P^{\text{NAM}}(x) = F^{-1}(\underline{x} + 1 - x)$. In contrast to the standard assignment model, the surplus of the x -ranked firm in the integrand of (8) depends on $P(x')$ for $x' > x$ via the applicant quality $Q(x)$.

We first argue that the planner wants the highest skilled firms to enter; hence the matching function P has range $[\underline{p}, \bar{p}]$, where $\underline{p} = F^{-1}(\underline{x})$. Increasing skills $P(x)$ at wage rank x increases the surplus at that rank, $\lambda(Q(x), P(x))$; however, it reduces applicant quality $Q(\tilde{x})$ at lower ranks $\tilde{x} \in [\underline{x}, x]$. This negative indirect effect diminishes but does not overturn the positive direct effect. Formally, the proof of Theorem 2 shows that the indirect effect scales down the direct effect by a factor

$$\delta := \exp \left(- \int_{\underline{x}}^x \frac{\lambda_q(Q(\tilde{x}), P(\tilde{x}))}{\tilde{x}} d\tilde{x} \right). \quad (9)$$

Turning to the screening order $P(\cdot)$, we claim that surplus (8) increases whenever a low-skill firm p is promoted from wage rank x to $x' > x$ past firms with higher skills $P(\hat{x}) > p$ for $\hat{x} \in [x, x']$. Thus PAM minimizes surplus, and NAM maximizes it. In a discrete analogue of our model, consider two firms with screening skills p and $p' = p + dp$ at adjacent wage ranks x and $x' = x + dx$ under PAM. The corresponding applicant quality equals $q = q' - \phi(q', p', x')dx$ and q' , recalling the compositional externality ϕ from (3). How does switching their screening order affect their joint recruited talent and hence surplus? First, the low-skill firm p now screens the better pool; since $\lambda(q, p)$ is supermodular, this lowers total surplus of the two firms by $\lambda_{qp}\phi dpdx$. In Figure 1 this is represented by the vertical shift from white circles representing PAM to the black circles representing NAM. If the pool quality was exogenous as in Becker, this would be the end of the story. In our model, there is a second effect: firm p extracts $\phi_p dpdx$ less talent, increasing surplus by $\lambda_q \phi_p dpdx$. In Figure 1, this is represented by the applicant quality of firm p' rising from q' to q'' , and the associated shift of the black circle to the black square. The next inequality shows that the second term outweighs the first, and so moving the low-skill firm p ahead of the high-skill firm p' raises surplus

$$\frac{\lambda_{qp}}{\lambda_q} = 2 \frac{1 - q}{1 - p(1 - q)} - \frac{1}{1 - p} < \frac{1 - q}{1 - p(1 - q)} = \frac{\lambda_p}{\lambda} < \frac{\lambda_p}{\lambda} \frac{\lambda}{\lambda - q} = \frac{\phi_p}{\phi}. \quad (10)$$

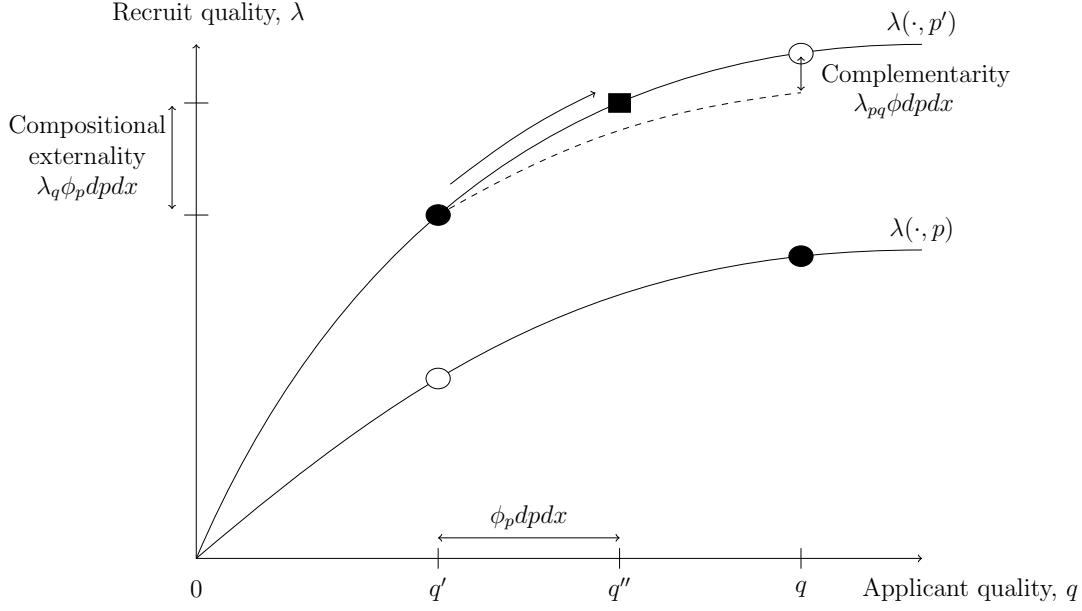


Figure 1: This figure illustrates the supermodularity and compositional externality that arise in Theorem 2. The empty dots represent PAM, the solid dots represent NAM if applicant quality were exogenous (the “Becker effect”), and the shift to the square comes from the endogeneity of the applicant pool (the “Akerlof effect”).

In words, marginal recruiting success λ_q is less sensitive to p than absolute recruiting success λ , which in turn is less sensitive to p than the compositional externality ϕ . Thus, total surplus of the two firms p, p' is maximized by NAM; the effect on aggregate surplus (8) must be discounted by (9), but the sign remains unchanged. The compositional externality overturns one of the most fundamental insights of the assignment model, namely the First Welfare Theorem.

Theorem 2. *For any level of unemployment $\underline{x} > 0$, PAM minimizes surplus (8) and NAM maximizes surplus.*

Proof. See Appendix C.2. □

From a policy perspective, it is hard to solve the inefficiency seen in equilibrium. In some rare cases, the planner might be able to directly implement negative assortative matching. For example, in the NFL “reverse” draft the lowest-ranked football teams pick first. To induce negative assortative matching indirectly through Pigouvian taxes on wages, the planner would need to charge higher wage taxes to firms with higher screening skills p . If the planner cannot observe the skill of different firms but can restrict the

set of admissible wages, her best policy is a single industry-level wage, inducing firms to select in a random order.²¹ The same outcome can be achieved by a wage cap (see Appendix C.3). More common labor market policies do not address the inefficiency. Unemployment benefits or minimum wages raise \underline{w} but don't change the equilibrium screening order; these policies lower welfare since equilibrium entry is optimal given the screening order. Progressive taxes on wages or profits flatten net wages $w(x)$ without changing the screening order; this policy may also change the number of firms \underline{x} , and thus lower welfare.

The positive assortative matching that arises in equilibrium (Theorem 1) and its inefficiency (Theorem 2) are robust to natural generalizations of our model. First, consider a model with production complementarities, where a firm's revenue is multiplicative in firm-type and expected worker-type $p \cdot \lambda(q, p)$, so high-type firms also have a higher marginal product. Since the (exogenous) productive-complementarity reinforces the (endogenous) screening-complementarity, equilibrium sorting remains positive assortative. More surprisingly, the compositional externality overcomes both the productive complementarity and the screening complementarity, and PAM remains inefficient (see Appendix C.4).

Second, the theorems are robust to screening costs. If there is a cost κ to screen each applicant and firms screen applicants until one passes their test, profit (2) becomes $\pi := \lambda(q, p) - \kappa / (1 - p(1 - q)) - w$. Since skilled firms are more selective, they incur higher screening costs and are more sensitive to applicant quality; thus equilibrium continues to exhibit PAM. Moreover, this equilibrium continues to be inefficient, with the planner improving on PAM by swapping neighboring firms (see Appendix C.5).

3.2 Implications of the Compositional Externality

In the last section, we saw that the compositional externality generates inefficient sorting of firms. In this section we show that it impairs the market's ability to aggregate information and induces firms to over-invest in their screening skills.

Limits of Information Aggregation. In the model, each firm has independent signals about each worker. Intuition from multi-unit auctions might suggest a "wisdom of crowds" results, whereby the market perfectly aggregates information (e.g. Pesendorfer and Swinkels (1997)). However, our model works very differently because, unlike common-

²¹To see this, note that high-skill firms always offer weakly higher wages than low-skill firms (by Theorem 1), while welfare increases with any promotion of low-skill over high-skilled firms (by Theorem 2).

value auctions, each worker’s quality is independent. To study this question we will, for simplicity, focus on the case of homogeneous firms with screening skills p (Example 1). We quantify information aggregation in two ways:

First, we observe that imprecise signals are of no value to society. Formally, the impact of screening skills p on surplus $S(p)$ is initially zero, $S_p(0) = 0$, even though such skills improve the selection of individual firms, $\lambda_p(q, 0) > 0$.²² That is, a continuum of imprecise, independent signals is perfectly informative when aggregated by an auction for homogeneous goods, but perfectly uninformative when aggregated by our market clearing mechanism for heterogeneous workers. Intuitively, imprecise signals enable firms to weed out some untalented applicants, but workers get so many attempts at different firms that every worker passes one of the tests, resulting in no increase in social surplus.

Second, we argue that social surplus is higher when firms have access to one public signal per worker that screens out bad workers with probability p (akin to Phelps (1972)), rather than each firm having a (conditionally) independent signal. That is, the independence of information across firms induces adverse selection rather than contributing new information. To see this, note that with the public signal, posterior expected talent equals $\lambda(\bar{q}, p)$ for the $1 - (1 - q)p$ applicants who pass the test, and 0 for the $(1 - q)p$ applicants who fail the test. This distribution of expected talent is a mean-preserving spread of the distribution of expected talent with private signals, which consists of a continuous distribution from \underline{w} to $\lambda(\bar{q}, p)$ for employed workers and an atom at \underline{q} for the unemployed. Since hiring decisions are efficient conditional on the available information, the superior information with public signals implies higher welfare. Intuitively, failing a public test black-lists an untalented job applicant, while failing a private test allows him to re-enter the pool, inducing adverse selection for other firms. Since firms earn zero profits, workers are better off, on average, if they each have a single, public, standardized test rather than lots of independent tests that give them “a second chance”.²³

Investment Inefficiency. In the model, firms’ screening skills p are exogenous. In practice, firms can raise the skills by improving their recruiting infrastructure (e.g. developing

²²There are two cases. If $\bar{q} < \underline{w}$ then no-one is hired for small p and the result trivially follows. If $\bar{q} > \underline{w}$, then everyone is hired when $p = 0$, and unemployment is $\underline{x} = \underline{X}(p) = 0$. The result then follows because the integral in the discount factor (9) diverges.

²³This Blackwell-comparison between perfectly correlated and independent signals extends to heterogeneous firms, $p \sim F[\underline{p}, \bar{p}]$. In particular, welfare is higher when agents have nested signals than when they have independent signals. To see this, recall Kurlat’s (2016) insight that equilibrium matching with nested signals is negative assortative and features no adverse selection. One can then argue that the induced distribution of expected talent is a mean preserving spread of the distribution induced by independent signals and any screening order, so in particular of equilibrium PAM.

referral systems, training recruiters, engaging headhunters). If firms can choose their screening skill p at cost $c(p)$, then wages and screening skills are dispersed and positively correlated across firms, with $p(x)$ solving $c'(p) = \lambda_p(Q(x), p) > 0$ by (7). From society’s perspective, firms over-invest in screening skills. Using equation (9), the marginal gain to society is $\delta \cdot \lambda_p(Q(x), p)$, where $1 - \delta$ reflects the compositional externality on lower-ranked firms. In the extreme case when $\underline{w} = 0$, we have full employment $\underline{x} = 0$, the screening game is constant-sum, and all investment is wasteful. This overinvestment contrasts with the finding of efficient investment found in classic matching models (e.g. Cole et al. (2001)). Overinvestment can be important in practice: Bock (2015, p. 60) urges companies to spend more on screening recruits, writing that “you can find a way to hire the very best, or you can hire average performers and try to turn them into the best. [...] At Google, we front-load our people investment. This means the majority of our time and money spent on people is invested in attracting, assessing, and cultivating new hires.” Alas, not every firm can hire the best.

4 Endogenous Screening Skills and Firm Dynamics

We now embed our static labor market into a model of firm dynamics to endogenize firms’ screening skills and study the evolution of talent over time. The key premise is that firms with more talented employees are more skilled at recruiting. As discussed in the introduction, this might happen because talented employees provide better referrals, or because they are better at identifying talented applicants. Firms thus desire talented workers both for the immediate increase in productivity, and for the benefit of having skilled recruiters in the future.

Section 4.1 describes the model. Section 4.2 solves for a firm’s optimal wage path. Section 4.3 shows equilibrium is unique with high-talent firms offering high wages that complement their superior screening skills; they thus attract superior applicants, amplifying their talent advantage. Section 4.4 characterizes firm dynamics, and shows that the economy converges to a steady state that exhibits persistent dispersion in talent, wages and productivity. In equilibrium, the positive assortative firm-applicant matching offsets the regression to mediocrity that results from imperfect screening.

4.1 Model

Time $t \geq 0$ is continuous. There is a unit mass of firms, each with a unit mass of jobs. At time t , a firm is described by its proportion of talented workers $r(t)$; initially, the

distribution of $r(0)$ is exogenous. At every instant $[t, t + dt]$, proportion αdt workers retire, leaving firms with vacancies. In the job market, there are then αdt open jobs and αdt applicants, of whom fraction \bar{q} are talented. Analogous to the static model, firms compete for these applicants by posting life-time wages $w(t)$.

Talented workers have an advantage in recruiting. We describe this relationship by a skill function $\psi(r)$ satisfying $\psi_r > 0$ and $\psi_{rr} \geq 0$. For example, if an employee with talent $\theta \in \{L, H\}$ has screening skill p^θ and firms ask random employees to act as recruiters, then the skill function is linear, $\psi(r) = p^L + r(p^H - p^L)$. As before, we assume that talent is scarce for all firms, which holds if²⁴

$$\lambda(\bar{q}, \psi(1)) < 1/2. \quad (11)$$

The job market is analogous to that in Section 2.1. If a firm with talent r posts wage w with rank x at time t , it attracts applicants with quality $Q(x, t)$ and hires recruits of quality $\lambda(Q(x, t), \psi(r))$. Writing $R(x, t)$ for the talent of the firm with wage rank x at time t , $Q(x, t)$ is determined by the sequential screening equation,

$$Q_x(x, t) = \phi(Q(x, t), \psi(R(x, t)), x) \text{ and } Q(1, t) = \bar{q}, \quad (12)$$

as in (3). In turn, the evolution of a given firm's talent $r(t)$ with wage rank $x(t)$ is given by the difference between its inflow λ and outflow r ,

$$r_t(t) = \alpha \left(\lambda(Q(x(t), t), \psi(r(t))) - r(t) \right). \quad (13)$$

Turning to payoffs, workers maximize lifetime wages $w(t)$, while firms' revenue equals $r(t)$. To abstract from entry and exit, suppose that there is no outside option, $\underline{w} = 0$. A firm's problem is to choose wages to maximize total discounted profits. Denoting the discount rate by $\beta > 0$, its value function is

$$V(r, s) = \max_{\{w(t)\}_{t \geq s}} \int_s^\infty e^{-\beta(t-s)} (r(t) - \alpha w(t)) dt, \quad (14)$$

where $r(t)$ evolves according to (13) with initial condition $r(s) = r$.²⁵

²⁴This assumption is stronger than necessary. It states that a firm exclusively endowed with talented workers has less than 50% talented recruits. In Section 4.3 we consider an example where firms are initially similar; the assumption can then be weakened to $\lambda(\bar{q}, \psi(R^*(1))) < 1/2$, where $R^*(x)$ is the steady-state talent of firm x .

²⁵Note that the "wages" $w(t)$ are really one-time payments to each of the α newly hired employees at time t . More realistically, one could model worker compensation as constant flow wage $(\beta + \alpha)w(s)$ until retirement, but it simplifies the accounting to have firms incur these costs up-front.

An equilibrium is given by a wage path $\{w(t)\}_{t \geq 0}$ for every firm,²⁶ so that given the induced wage ranks $x(w, t)$ and applicant qualities $Q(x, t)$, every firm's wage path is optimal. We say an equilibrium is *essentially unique*, if the induced distribution over equilibrium trajectories $\{r(t)\}_{t \geq 0}$ is unique.²⁷

Remarks. We assume that employment is for life. If firms identify and fire low types at a constant rate, the analysis would be qualitatively unchanged. The possibility of firing would introduce a new term into the evolution of talent (13) and lower the quality of workers in the market, \bar{q} . However, the firms' FOCs (16) are unchanged, meaning we still obtain positive assortative matching and steady-state dispersion, as in Theorems 3-4 below.

Unlike Section 3, there are no welfare margins in the dynamic model. In comparison to Section 3.1, the outside option is zero, $\underline{w} = 0$, so all workers are hired and the game is constant sum. And in comparison to Section 3.2, investments into screening skills are transfers to workers, rather than real economic costs.

4.2 Firm's Problem

First, we study a firm's optimal wage path $\{w(t)\}_{t \geq 0}$ for any given applicant function $Q(x(w, t), t)$ without imposing equilibrium restrictions on other firms. As in Section 2.3, it is convenient to write $W(q, t)$ for the wage required to attract applicants q at time t , and let the firm optimize directly over the applicant pools $\{q_t\}_{t \geq 0}$. After this change of variable, the firm's Bellman equation becomes

$$\beta V(r, t) = \max_q \{r - \alpha W(q, t) + \alpha(\lambda(q, \psi(r)) - r)V_r(r, t) + V_t(r, t)\}. \quad (15)$$

Firm value is determined by its flow profits plus appreciation due to talent acquisition and a secular trend. Assuming wages are differentiable, the first-order condition is

$$W_q(q, t) = \lambda_q(q, \psi(r))V_r(r, t). \quad (16)$$

²⁶As in the static model, the restriction to deterministic wages is without loss. In principle, a firm might mix between two wages by switching between them arbitrarily fast. To avoid measurability issues associated with such strategies, we allow for "distributional wage strategies" but show in Section 4.3 that equilibrium strategies are almost always pure.

²⁷This definition avoids two spurious notions of multiplicity. First, in continuous time, any firm's optimal strategy $\{w_t\}_{t \geq 0}$ can be unique only almost always. Second, if two or more firms are initially identical but then drift apart, only the distribution of trajectories can be determined uniquely.

Intuitively, the cost of attracting better applicants (the LHS) must balance the gains of a higher quality applicant pool which increases the recruit quality and thereby firm value (the RHS). The RHS of (16) is increasing in r , so firms with more talent have a higher marginal benefit from attracting better applicants, yielding positive assortative matching. Intuitively, firms with more talent have higher marginal benefit from better applicants $\lambda_q(q, \psi(r))$ because of the supermodularity of λ (see Theorem 1) and the convexity of ψ , and such firms have a higher marginal value of talent $V_r(r, t)$ because $V(r, t)$ is convex in r (see Appendix D.1). Hence, wages are dynamic complements: an increase in today's wage raises tomorrow's talent, and thereby tomorrow's optimal wage.

To compute equilibrium wages from the first-order condition (16), we write $r(u)$ and $q(u)$ for the equilibrium trajectory of talent and applicant quality, and apply the envelope theorem and the law of motion of firm talent (13) to compute (see Appendix D.1 for details)

$$V_r(r(t), t) = \int_t^\infty e^{-\int_t^s \beta + \alpha (1 - \lambda_p(q(u), \psi(r(u))) \psi_r(r(u)))} du ds. \quad (17)$$

Intuitively, the future benefit of better employees is discounted both at the interest rate β and the retirement rate α . But selective recruiting raises the persistence of firm talent or, equivalently, reduces the talent decay rate by a factor $1 - \lambda_p \psi_r$.

4.3 Equilibrium

Given the single-firm analysis, it is straightforward to characterize equilibrium. Firms with more talent post higher wages and attract better applicants. More strongly, even if firms share the same talent $r(0)$ initially, they post different wages (as in the static model), recruit different types of workers, and diverge immediately (see Appendix D.2). Thus, in equilibrium, each firm is characterized by a rank x , which describes the firm's position in the talent, applicant, and wage distribution at all times $t > 0$.

Equilibrium is then characterized in two steps

- (1) Allocations. At time t , applicant quality $Q(x, t)$ is determined by sequential screening (12). The evolution of firm x 's talent $R(x, t)$ is then given by the firm dynamics equation (13) with $x(t) \equiv x$.
- (2) Payoffs. Firm x 's marginal value of talent is determined by (17), with $q(u) = Q(x, u)$. Using this, wages $W(q, t)$ are given by the first-order condition (16), with $r = R(x, t)$, $q = Q(x, t)$, and $W(0, t) = \underline{w} = 0$.

Given these wages, the FOC (16) implies global optimality because the net benefit of better applicants $\lambda_q(q, \psi(r))V_r(r, t) - W_q(q, t)$ single-crosses in r . Standard verification theorems then imply that the policy functions are indeed optimal. To summarize:

Theorem 3. *Assume talent is scarce (11). Equilibrium exists and is essentially unique. Firm-applicant matching is positive assortative and the distribution of talent has no atoms at any $t > 0$.*

Proof. Only the claim about no atoms remains to be shown. See Appendix D.2. \square

Thus, even if firms start off with identical talent, some post higher wages than others, attract better applicants, and hire better recruits. These firms accumulate talent, continue to pay high wages, and the distribution of talent disperses over time.

Example 3 (Initially Homogeneous Firms). To understand the evolution of talent $R(x, t)$ and applicant quality $Q(x, t)$, consider Figure 2. At $t = 0$, all firms employ average workers, with quality $\bar{q} = 0.25$. In panel (a), the “vertical” lines represent the cross-sectional distribution of (r, q) at different times, while the “horizontal” lines represent the sample-paths of selected firms. The top-ranked firm recruits from the constant applicant pool $Q(1, t) = \bar{q}$, and so (13) implies that its talent grows monotonically and converges to a steady state. For lower-ranked firms, the dynamics are more subtle. For instance, firm $x = 0.5$ initially improves as its recruits are more talented than its retirees. However, as higher-ranked firms become better at identifying talent, its applicant pool deteriorates and its quality eventually falls back. These time paths are shown in panel (b).

Turning to payoffs, note that firms earn zero lifetime value since they start homogeneous. The top firms initially lose money as they post high wages and invest in talent; this raises both their productivity and their screening skill, giving them an advantage in the labor market, and delivering a steady stream of profits. Over time, rents shift from workers to firms: early workers are paid more than their productivity as firms invest; later workers are paid less than their productivity as differentiation softens competition. \triangle

4.4 Steady-State Allocations

Steady-state talent among recruits and applicants $\{R^*(x), Q^*(x)\}$ is easily characterized. First, the talent of each firm’s recruits and retirees balance,

$$\lambda(q, \psi(r)) = r. \tag{18}$$

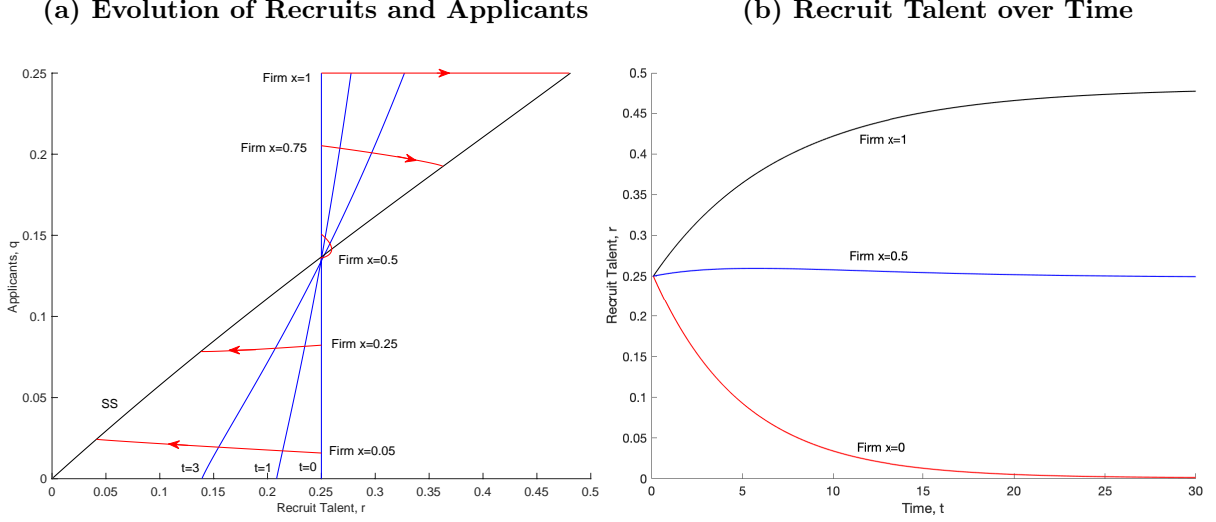


Figure 2: **Equilibrium Dynamics of Applicant and Recruit Quality.** All firms start with talent $\bar{q} = 0.25$, and choose wages optimally as characterized in the text. The skill function is $\psi(r) = 0.4 + 0.5r$ and (annual) turnover is $\alpha = 0.2$.

This has a unique fixed point, $r = \rho(q)$, since $\lambda(q, \psi(r)) - r$ is convex (see Appendix D.1), positive at $r = 0$, negative at $r = 1$, and hence crosses zero exactly once. Naturally, firms with better applicants have higher talent; formally, $\rho(q)$ increases since $\lambda(q, \psi(r)) - r$ rises in q and single-crosses from above in r .

Second, substituting $R^*(x) = \rho(Q(x))$ into the sequential screening equation (12), steady-state applicant quality $Q^*(x)$ is given by

$$Q_x^*(x) = \phi(Q^*(x), \psi(\rho(Q^*(x))), x). \quad (19)$$

Together (18) and (19) pin down firm x 's talent $R^*(x)$ and applicant quality $Q^*(x)$ in steady state. Differentiating and dropping arguments for legibility, steady-state talent dispersion is given by²⁸

$$R_x^* = \rho_q Q_x^* = \frac{\lambda_q}{1 - \lambda_p \psi_r} Q_x^*. \quad (20)$$

Indeed, the economy converges to this steady state from any initial condition.

Theorem 4. *Assume talent is scarce (11). The steady-state talent distribution $R^*(x)$ is unique and has no gaps or atoms. For any distribution of initial talent $r(0)$, firm x 's equilibrium talent $R(x, t)$ converges to $R^*(x)$.*

²⁸Note that the denominator in this expression is positive around the steady state: While $1 - \lambda_p(q, \psi(r))\psi_r(r)$ may be negative for arbitrary values of r , this cannot happen in steady state where $r = \rho(q)$, since we know that $\rho(q)$ increases in q and, by the Implicit Function Theorem, $R_q(q) = \lambda_q(q, \psi(\rho(q)))/(1 - \lambda_p(q, \psi(\rho(q)))\psi_r(\rho(q)))$.

Proof. See Appendix D.3 □

To show convergence, Figure 2 suggests a “proof by induction.” The top firm recruits from a pool of constant quality, so equation (13) implies that its talent converges to steady state. Then consider firm x close to 1. As the talent of higher firms converges, firm x ’s applicant pool converges, and then firm x ’s talent converges, too, by (13).

In steady state, talent $R(x)$ are dispersed. If all firms were hiring from the same applicant pool q , talent differences from the steady state level $r - \rho(q)$ would decay exponentially. The link between talent and recruiting skills slows this decay but cannot stop it. Rather, the decay is halted by positive assortative matching: firms with skilled employees post higher wages and recruit from a better pool. As a result, the steady state supports permanent heterogeneity in firm quality, productivity and profits.

Talent thus generates a sustainable competitive advantage. One might wonder why a firm with untalented workers doesn’t compete more aggressively in wages to build its talent over time. This strategy is feasible, but too expensive. High-talent firms have a higher marginal benefit from raising wages because recruiting skills and applicant quality are complements in the job market.

This logic relies on our assumption that firms have *firm-specific* wage policies. In contrast, Montgomery (1991)’s model of *applicant-specific* wage offers does not generate persistent dispersion.²⁹ If we interpret the skill function $\psi(r)$ as the firm sampling its current employees for referrals, our model predicts that a talented worker recruits better referrals if their colleagues are also talented. This is because high-talent firms pay high wages, and help convert the recommendation of the talented worker into a talented recruit. This does not happen in Montgomery, where each referral receives a separate wage offer.

²⁹Montgomery (1991) only has two periods. For an apples-to-apples comparison with our model, assume an infinite time horizon, continuous time, and that firms employ a unit mass of workers with aggregate talent $r(t)$ who retire at rate α . When a firm has a vacancy, a random current employee makes a referral. Write $\bar{\lambda}$ (resp. $\underline{\lambda}$) for the equilibrium expected talent of a recruit who was referred by a talented (resp. untalented) employee. Since wage offers are applicant specific, $\bar{\lambda}$ and $\underline{\lambda}$ do not depend on the talent of the firm’s other employees. Talent evolution is then governed by $r_t = \alpha(\underline{\lambda} + (\bar{\lambda} - \underline{\lambda})r - r)$ and converges to $r^* = \underline{\lambda}/(1 - \bar{\lambda} + \underline{\lambda})$, irrespective of initial talent.

4.5 Steady-State Payoffs

The model generates predictions about how value is shared between workers and firms. In steady state the flow wages $(\beta + \alpha)W^*(q)$ are given by³⁰

$$(\beta + \alpha)W_q^* = \frac{(\beta + \alpha)\lambda_q}{\beta + \alpha(1 - \lambda_p\psi_r)}. \quad (21)$$

Steady-state flow profits $\Pi^*(q)$ in turn are given by

$$\Pi_q^* = \rho_q - (\beta + \alpha)W_q^* = \frac{\beta\lambda_p\psi_r}{(1 - \lambda_p\psi_r)(\beta + \alpha(1 - \lambda_p\psi_r))}. \quad (22)$$

Two comparative statics help illustrate the inner workings of the model. First, we note that *any* degree of correlation between productivity and recruiting skills $\psi_r > 0$ gives rise to non-vanishing talent dispersion. In the limit, as the direct talent-skill relationship ψ_r goes to zero, equation (20) shows that the strategic effect gives rise to dispersion $R_x^* = \lambda_q Q_x^*$. Despite this talent dispersion, equation (22) shows that steady-state profits are zero since firms have weak incentives, as in Example 1. An increase in the importance of referrals in recruiting talent ψ_r then amplifies talent dispersion (20) and raises steady-state profits (22). This arises because (i) high-paying firms fish out more of the talented workers from the applicant pool, and (ii) talent accumulates at the top firms, further raising their screening ability and the talent of their recruits.

Second, a rise in turnover α raises wages and shifts steady-state rents from firms to workers. Indeed, a rise in turnover has no impact on the steady-state distribution of talent $R^*(x)$, but raises the rate at which the economy converges to steady state. Equations (21) and (22) then imply that flow wages $(\beta + \alpha)W^*(Q^*(x))$ rise, and flow profits $\Pi^*(Q^*(x))$ fall for all firms x . Intuitively, when turnover is high, a firm's stock of talent quickly depletes and a low-talent firm can achieve almost the same profits as a high-talent firm by mimicking its wage policy; this intensifies competition and drives up wages.

Our analysis assumes revenue is linear, and screening skills are weakly convex in talent r . Complementarities in production (in the form of convex revenue) reinforce equilibrium sorting and leaves steady-state dispersion unchanged. Our results are also robust to a little concavity in revenue or screening since incentives for positive assortative matching are strict. But sufficient concavity induces negative assortative matching: For example, if 10% of talented employees suffice to guarantee good screening outcomes, then firms below the 10% threshold would bid aggressively to improve their screening.

³⁰To see this, note that in steady state, the marginal value of talent (17) simplifies to $V_r^*(\rho(q)) = [\beta + \alpha(1 - \lambda_p\psi_r)]^{-1}$. Substituting this into the first-order condition (16) yields the result.

5 Conclusion

In their survey on personnel economics, Oyer and Schaefer (2011) write that “This literature has been very successful in generating models and empirical work about incentive systems [...] The literature has been less successful at explaining how firms can find the right employees in the first place.” By studying the equilibrium interaction of firms’ recruiting strategies, we hope this paper takes a step in this direction.

The paper has three major contributions. First, it provides a simple model to understand a competitive labor market where firms have independent information. Second, we show that compositional externalities mean that equilibrium sorting is inefficient and that the market is poor at aggregating firms’ private information. Third, we propose a new model of firm dynamics where similar firms diverge over time and the economy converges to a steady state with persistent dispersion in talent, wages, and productivity.

The tractable nature of the model means it can be extended in a number of directions. First, it would be natural to allow firms to differ in their technology as well as their talent, and study how they combine to define firms’ competitive advantages. For example, Waldinger (2016) argues the loss of human capital at German universities had a large, persistent effect on output, whereas the loss of physical capital had a small, temporary effect. Second, going beyond our one-dimensional model of talent, one might assume that firms also differ in the type of talent they seek to hire (e.g. generalists vs. specialists) which typically complements their broader organizational strategy. Third, we suppose that each worker is only on the job market once, but it would be natural to extend the model to allow for job-to-job transitions. Workers in low-wage jobs would then compete with new cohorts for high-wage jobs. Fourth, in many settings workers value peer quality or status as well as wages. For example, if all types of worker prefer to work at a firm with more talented colleagues, the allocation of talent is unchanged but wages are more compressed; moreover, firms have yet another reason to invest in talent, further transferring surplus from later generations to earlier ones.

The paper also raises questions further afield. On the theoretical side, our baseline model calls for a parsimonious foundation of firm-worker matching with fixed wages when firms are asymmetrically informed about worker talent. On the empirical side, we hope the paper stimulates work on the “production function of hiring”. What makes some managers (or firms) better at recruiting than others? How does this skill interact with the firms’ strategy (e.g. wage, advertising) in order to produce better recruits?

References

- ABOWD, J., F. KRAMARZ, AND D. MARGOLIS (1999): “High Wage Workers and High Wage Firms,” *Econometrica*, 67(2), 251–333.
- ALBRECHT, J., B. DECREUSE, AND S. VROMAN (2023): “Directed Search with Phantom Vacancies,” *International Economic Review*, 64(2), 837–869.
- ANDERSON, A. (2015): “A Dynamic Generalization of Becker’s Assortative Matching Result,” *Journal of Economic Theory*, 159, 290–310.
- ANDERSON, A., AND L. SMITH (2010): “Dynamic Matching and Evolving Reputations,” *Review of Economic Studies*, 77(1), 3–29.
- ATHEY, S., AND G. ELLISON (2011): “Position Auctions with Consumer Search,” *Quarterly Journal of Economics*, 126(3), 1213–1270.
- BANFI, S., AND B. VILLENA-ROLDÁN (2019): “Do High-Wage Jobs Attract more Applicants? Directed Search Evidence from the Online Labor Market,” *Journal of Labor Economics*, 37(3), 715–746.
- BARRON, J. M., J. BISHOP, AND W. C. DUNKELBERG (1985): “Employer Search: The Interviewing and Hiring of New Employees,” *Review of Economics and Statistics*, pp. 43–52.
- BARTH, E., A. BRYSON, J. C. DAVIS, AND R. FREEMAN (2016): “It’s Where you Work: Increases in the Dispersion of Earnings across Establishments and Individuals in the United States,” *Journal of Labor Economics*, 34(S2), S67–S97.
- BEAMAN, L., AND J. MAGRUDER (2012): “Who gets the Job Referral? Evidence from a Social Networks Experiment,” *American Economic Review*, 102(7), 3574–3593.
- BECKER, G. S. (1973): “A Theory of Marriage: Part I,” *Journal of Political Economy*, 81(4), 813–846.
- BEHRENTZ, L. (2001): “Who Gets the Job and Why? An Explorative Study of Employers’ Recruitment Behavior,” *Journal of Applied Economics*, 4(2), 255–278.
- BELOT, M., P. KIRCHER, AND P. MULLER (2022): “How Wage Announcements Affect Job Search - A Field Experiment,” *American Economic Journal: Macroeconomics*, 14(4), 1–67.
- BENDER, S., N. BLOOM, D. CARD, J. VAN REENEN, AND S. WOLTER (2018): “Management Practices, Workforce Selection, and Productivity,” *Journal of Labor Economics*, 36(S1), S371–S409.
- BENSON, A., D. LI, AND K. SHUE (2019): “Promotions and the Peter Principle,” *Quarterly Journal of Economics*, 134(4), 2085–2134.
- BERTRAND, M., AND A. SCHOAR (2003): “Managing With Style: The Effect Of Managers On Firm Policies,” *Quarterly Journal of Economics*, 118(4), 1169–1208.

- BLATTER, M., S. MUEHLEMAN, AND S. SCHENKER (2012): “The Costs of Hiring Skilled Workers,” *European Economic Review*, 56(1), 20–35.
- BOARD, S., M. MEYER-TER-VEHN, AND T. SADZIK (2017): “Recruiting Talent,” Working paper, UCLA.
- BOCK, L. (2015): *Work Rules! Insights from Inside Google that will Transform how you Live and Lead*. Twelve.
- BROECKER, T. (1990): “Credit-Worthiness Tests and Interbank Competition,” *Econometrica*, 58(1), 429–452.
- BURDETT, K., AND K. L. JUDD (1983): “Equilibrium Price Dispersion,” *Econometrica*, 51(4), 955–969.
- BURDETT, K., AND D. T. MORTENSEN (1998): “Wage Differentials, Employer Size, and Unemployment,” *International Economic Review*, 39(2), 257–273.
- CARD, D., A. R. CARDOSO, J. HEINING, AND P. KLINE (2018): “Firms and Labor Market Inequality: Evidence and Some Theory,” *Journal of Labor Economics*, 36(S1), S13–S70.
- CARD, D., J. HEINING, AND P. KLINE (2013): “Workplace Heterogeneity and the Rise of West German Wage Inequality,” *Quarterly Journal of Economics*, 128(3), 967–1015.
- CHAKRABORTY, A., A. CITANNA, AND M. OSTROVSKY (2010): “Two-Sided Matching with Interdependent Values,” *Journal of Economic Theory*, 145(1), 85–105.
- COLE, H. L., G. J. MAILATH, AND A. POSTLEWAITE (2001): “Efficient Non-contractible Investments in Large Economies,” *Journal of Economic Theory*, 101(2), 333–373.
- DAVIS, S. J., AND B. SAMANIEGO (2020): “Application Flows,” Working paper, Chicago Booth.
- FARBER, H. S., AND R. GIBBONS (1996): “Learning and Wage Dynamics,” *Quarterly Journal of Economics*, 111(4), 1007–1047.
- FENIZIA, A. (2022): “Managers and Productivity in the Public Sector,” *Econometrica*, 90(3), 1063–1084.
- FREDRIKSSON, P., L. HENSVIK, AND O. N. SKANS (2018): “Mismatch of Talent: Evidence on Match Quality, Entry Wages, and Job Mobility,” *American Economic Review*, 108(11), 3303–38.
- GIORCELLI, M. (2019): “The Long-Term Effects of Management and Technology Transfers,” *American Economic Review*, 109(1), 121–152.
- GUERRIERI, V., R. SHIMER, AND R. WRIGHT (2010): “Adverse Selection in Competitive Search Equilibrium,” *Econometrica*, 78(6), 1823–1862.
- GUPTA, S. (2018): “What Do Good Managers Do? Evidence from an Insurance Firm in India,” Working paper, Boston University.
- HARTMAN, P. (2002): *Ordinary Differential Equations*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA.

- HASTINGS, R., AND P. MCCORD (2009): “Netflix Culture: Freedom and Responsibility,” Presentation, <http://www.slideshare.net/reed2001/culture-1798664>.
- HOLZER, H. J. (1987): “Hiring Procedures in the Firm: Their Economic Determinants and Outcomes,” Working paper, NBER No. 2185.
- HOPENHAYN, H., AND R. ROGERSON (1993): “Job Turnover and Policy Evaluation: A General Equilibrium Analysis,” *Journal of Political Economy*, 101(5), 915–938.
- HOPENHAYN, H. A. (1992): “Entry, Exit, and Firm Dynamics in Long Run Equilibrium,” *Econometrica*, 60(5), 1127–1150.
- HUBER, K., V. LINDENTHAL, AND F. WALDINGER (2021): “Discrimination, Managers, and Firm Performance: Evidence from “Aryanizations” in Nazi Germany,” *Journal of Political Economy*, 129(9), 2455–2503.
- JOVANOVIĆ, B. (1982): “Selection and the Evolution of Industry,” *Econometrica*, 50(3), 649–670.
- KAYA, A., AND K. KIM (2018): “Changes in Relative Wages, 1963–1987: Trading Dynamics with Private Buyer Signals in the Market for Lemons,” *Review of Economic Studies*, 85, 2318–2352.
- KURLAT, P. (2016): “Asset Markets With Heterogeneous Information,” *Econometrica*, 84(1), 33–85.
- KURLAT, P., AND F. SCHEUER (2021): “Signalling to Experts,” *Review of Economic Studies*, 88(2), 800–850.
- LAUERMANN, S., AND A. WOLINSKY (2016): “Search with Adverse Selection,” *Econometrica*, 84(1), 243–315.
- LAZEAR, E. (2000): “Performance Pay and Productivity,” *American Economic Review*, 90(5), 1346–1361.
- LI, Q., AND R. SHIMER (2019): “Liquidity and Price Dispersion in Markets with Information Heterogeneity,” Working paper, 2019 Meeting Papers 1491, Society for Economic Dynamics.
- LIU, Q. (2020): “Stability and Bayesian Consistency in Two-Sided Markets,” *American Economic Review*, 110(8), 2625–2666.
- LIU, Q., G. J. MAILATH, A. POSTLEWAITE, AND L. SAMUELSON (2014): “Stable Matching with Incomplete Information,” *Econometrica*, 82(2), 541–587.
- LOCKWOOD, B. (1991): “Information Externalities in the Labour Market and the Duration of Unemployment,” *Review of Economic Studies*, 58(4), 733–753.
- LUCAS, R. E., AND E. C. PRESCOTT (1971): “Investment under Uncertainty,” *Econometrica*, 39(5), 659–681.
- MONTGOMERY, J. D. (1991): “Social Networks and Labor-Market Outcomes: Toward an Economic Analysis,” *American Economic Review*, 81(5), 1408–1418.

- OYER, P., AND S. SCHAEFER (2011): “Personnel Economics: Hiring and Incentives,” in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and D. Card, vol. 4b, pp. 1769–1823. North Holland.
- PALLAIS, A. (2014): “Inefficient Hiring in Entry-Level Labor Markets,” *American Economic Review*, 104(11), 3565–99.
- PALLAIS, A., AND E. G. SANDS (2016): “Why the Referential Treatment? Evidence from Field Experiments on Referrals,” *Journal of Political Economy*, 124(6), 1793–1828.
- PESENDORFER, W., AND J. M. SWINKELS (1997): “The Loser’s Curse and Information Aggregation in Common Value Auctions,” *Econometrica*, pp. 1247–1281.
- PHELPS, E. S. (1972): “The Statistical Theory of Racism and Sexism,” *American Economic Review*, 62(4), 659–661.
- POSTEL-VINAY, F., AND J.-M. ROBIN (2002a): “The Distribution of Earnings in an Equilibrium Search Model with State-Dependent Offers and Counter-Offer,” *International Economic Review*, 43(4), 989–1016.
- (2002b): “Equilibrium Wage Dispersion with Worker and Employer Heterogeneity,” *Econometrica*, 70(6), 2295–2350.
- SHAKED, M. (1982): “Dispersive Ordering of Distributions,” *Journal of Applied Probability*, 19(2), 310–320.
- SHAPLEY, L. S., AND M. SHUBIK (1971): “The Assignment Game I: The Core,” *International Journal of Game Theory*, 1(1), 111–130.
- SONG, J., D. J. PRICE, F. GUVENEN, N. BLOOM, AND T. VON WACHTER (2019): “Firming Up Inequality,” *Quarterly Journal of Economics*, pp. 1–50.
- STANTON, C. T., AND C. THOMAS (2015): “Landing the First Job: The Value of Intermediaries in Online Hiring,” *Review of Economic Studies*, 83(2), 810–854.
- VAN OURS, J., AND G. RIDDER (1992): “Vacancies and the Recruitment of New Employees,” *Journal of Labor Economics*, 10(2), 138–155.
- WALDINGER, F. (2012): “Peer Effects in Science: Evidence from the Dismissal of Scientists in Nazi Germany,” *Review of Economic Studies*, 79(2), 838–861.
- (2016): “Bombs, Brains, and Science: The Role of Human and Physical Capital for the Creation of Scientific Knowledge,” *Review of Economics and Statistics*, 98(5), 811–831.

Appendix

A Comparison with Kurlat (2016)

In this section we clarify the relationship between our model and that of Kurlat (2016) with “false positives”. As discussed in Section 1.1, the key differences are that, in the language of our model, Kurlat models workers’ labor as perfectly divisible and firms’ signals as perfectly nested.³¹

To appreciate these differences, we consider the effect of nested signals and divisible labor in our model. To reduce formalities, we try to keep these model variants as simple as possible. Indeed, we suppose that the outside option \underline{w} is close to 0 throughout.

Nested Signals. Assume there are two types of firms (skilled and unskilled). Signals are “nested” in that an untalented worker who passes the test of a skilled firm also passes the test of an unskilled firm. Workers thus fall into one of four categories: A) talented workers, who automatically pass both tests, B) untalented workers who pass both tests, C) untalented workers who pass the test of the unskilled firm but fail the test of the skilled firm, and D) untalented workers who fail both tests.³²

With such perfectly nested signals, adverse selection and outbidding incentives are one-sided. Unskilled firms, who hire proportionally from categories A, B, and C, do not impose adverse selection on other firms. Skilled firms, who hire proportionally from categories A and B, but screen out C workers, impose a negative externality on unskilled firms. Thus, unskilled firms have an incentive to outbid skilled firms but not vice versa (and neither type of firm has an incentive to outbid its own type).

The equilibrium wage distribution is thus degenerate, with all entering firms offering some wage w^* and workers endogenously breaking ties in favor of unskilled firms. The wage w^* is determined by the entry condition of unskilled firms. This equilibrium has negative assortative matching and efficient aggregate sorting, in contrast to the positive assortative matching and inefficient aggregate sorting of our model with independent signals. Note how the single-wage equilibrium relies on signals being perfectly nested: If

³¹Other differences between our model and Kurlat’s are less relevant for the analysis. For instance, Kurlat also allows for patient sellers of good assets, who never trade; in our model this would correspond to additional talented workers with reservation wage $\underline{w} = 1$. And Kurlat models buyers’ demand in terms of a monetary budget, while our firms have a single, physical job.

³²Category A corresponds to the “green assets” in Kurlat’s table I, category C to “red assets,” and category D to the “black assets.”

there are two firms who cannot perfectly screen out applicants who fail the other's test, both firms have an incentive to outbid each other, and equilibrium wages are dispersed.

Divisible Labor. Returning to independent signals, consider a version of our model with divisible labor, i.e. workers can rent themselves out by the second across many firms. As in Kurlat (2016) workers apply to all firms simultaneously, anticipating perfectly how much time each firm will buy. For simplicity, assume homogeneous firms with skill p .

In equilibrium, $1 - (1 - \bar{q})p$ firms enter and post the same wage $w^* = \lambda(\bar{q}, p)$; workers offer their entire time to each of these firms. At each entering firm, $1 - (1 - \bar{q})p$ applicants pass the test and each sells an infinitesimal share $1/(1 - (1 - \bar{q})p)$ of their time to the firm. High types pass all tests, and so sell all of their time. In contrast, low types pass each test with probability $1 - p$, and thus share p of their time remains unsold. Thus, adverse selection is avoided by firms hiring simultaneously and workers perfectly anticipating rationing outcomes to satisfy the budget constraint on their unit of labor.

This construction corresponds to the equilibrium in Kurlat (2016)'s online appendix with non-nested signals. The equilibrium is robust to any correlation structure between firms' signals. However, it relies on labor being perfectly divisible: If a worker can only work for finitely many firms, rationing is random, markets cannot clear simultaneously, and some firms face adversely selected applicants.

Closely related to the issue of divisibility is Kurlat's assumption of multiple markets with endogenous screening orders (e.g. top-to-bottom, or bottom-to-top), all of which clear simultaneously. Simultaneous market clearing requires workers to hit their budget constraint of selling one time unit across all markets; this requires perfectly divisible labor. With indivisible labor, markets cannot clear simultaneously: Workers that are not hired in market A apply in market B, inducing adverse selection. We must therefore specify a clearing order *across markets*, which means A and B are not really different markets, and renders moot the issue of endogenous clearing orders *within markets*.

Summary. The key differences in Kurlat's model are that labor is perfectly divisible and firms' signals are perfectly nested. Either one of these assumptions allows firms to avoid adverse selection, and equilibrium features a single wage and efficient aggregate sorting. But if labor is not perfectly divisible and signals are not perfectly nested, adverse selection emerges and equilibrium wages fan out. Our model assumes more strongly that labor is indivisible and signals are conditionally independent. Equilibrium sorting is then positive assortative and inefficient.

B Proofs from Section 2

B.1 Example 1: Comparative Statics of Wage Dispersion

Here we show that dispersion rises as screening improves and talent becomes more scarce. To see this, we use the demanding log-dispersive order (Shaked, 1982), so dispersion increases in p if the talent ratio $\lambda(Q(x'), p)/\lambda(Q(x), p)$ increases in p for all $x' > x$ (where applicant quality $Q(x)$ also depends on \bar{q}, p), and similarly for \bar{q} .

Proposition 1. *Assume talent is scarce, $\lambda(\bar{q}, p) \leq 1/2$:*

- (a) *The dispersion of wages and productivity increases in screening skill, p .*
- (b) *The dispersion of wages and productivity falls in average talent, \bar{q} .*

Part (a) means that as screening skills improve, the top-wage firms raise their productivity relative to the mean. Intuitively, these top firms extract more talented workers from the applicant pool, lowering the productivity and wages of lower-wage firms (even though their screening skill p increased by the same amount). Part (b) shows that a reduction in the number of talented workers, \bar{q} , also leads to an increase in the dispersion of productivity and wages. Intuitively, if the number of talented workers halves, the top firm's talent drops by less than half, as they are able to screen out many of the untalented workers, leaving proportionally less talent for lower firms, thereby raising inequality.

To prove Proposition 1, we first establish how applicant quality and employment depend on p and \bar{q} . Write the applicant quality for a firm with rank x as $Q(x, \bar{q}, p)$ to make explicit the effect of aggregate talent \bar{q} and screening skills p . Recall from (1) and (3) that

$$\lambda(q, p) := \frac{q}{1 - p(1 - q)} \quad \text{and} \quad \phi(x, q, p) := \frac{\lambda(q, p) - q}{x}.$$

Lemma 2. *Applicant quality $Q(x, \bar{q}, p)$ is:*

- (a) *Increasing in wage rank x with derivative $Q_x(x, \bar{q}, p) = \phi(x, Q(x, \bar{q}, p), p)$.*
- (b) *Increasing in aggregate talent \bar{q} with derivative*

$$Q_{\bar{q}}(x, \bar{q}, p) = \exp \left(- \int_x^1 \phi_q(\hat{x}, Q(\hat{x}, \bar{q}, p), p) d\hat{x} \right).$$

- (c) *Decreasing in screening skills p with derivative*

$$Q_p(x, \bar{q}, p) = - \int_x^1 \exp \left(- \int_x^{\hat{x}} \phi_q(\tilde{x}, Q(\tilde{x}, \bar{q}, p), p) d\tilde{x} \right) \phi_p(\hat{x}, Q(\hat{x}, \bar{q}, p), p) d\hat{x}.$$

- (d) Log-submodular in (x, \bar{q}) .
(e) Log-supermodular in (x, p) .

Proof. (a) is the sequential screening equation (3). Parts (b) and (c) follow from the theory of ordinary differential equations, e.g. Hartman (2002, Theorem 3.1), whereby the solution of the ODE $Q_x(x, \bar{q}, p) = \phi(x, Q(x, \bar{q}, p), p)$ with boundary condition $Q(1, \bar{q}, p) = \bar{q}$ satisfies $Q_{x\bar{q}} = \phi_q Q_{\bar{q}}$ and $Q_{xp} = \phi_q Q_p + \phi_p$ with boundary conditions $Q_{\bar{q}}(1, \bar{q}, p) = 1$ and $Q_p(1, \bar{q}, p) = 0$. Parts (d) and (e) follow because

$$(\log Q(x, \bar{q}, p))_x = \frac{\lambda(Q(x, \bar{q}, p), p) - Q(x, \bar{q}, p)}{Q(x, \bar{q}, p)x} = \frac{1}{x} \left(\frac{1}{1 - p(1 - Q(x, \bar{q}, p))} - 1 \right)$$

falls in \bar{q} by (b) and rises in p by (c). \square

Proof of Proposition 1. We wish to show that $\lambda(Q(x, \bar{q}, p), p) = \frac{Q(x, \bar{q}, p)}{1 - p + pQ(x, \bar{q}, p)}$ is (a) log-supermodular in (x, p) and (b) log-submodular in (x, \bar{q}) . For part (a) we compute

$$(\log \lambda(Q(x, \bar{q}, p), p))_p = \frac{\lambda_q(Q(x, \bar{q}, p), p)Q_p(x, \bar{q}, p) + \lambda_p(Q(x, \bar{q}, p), p)}{\lambda(Q(x, \bar{q}, p), p)}$$

and, omitting arguments for legibility,

$$(\log \lambda(Q(x, \bar{q}, p), p))_{px} = \frac{1}{\lambda^2} \left[\lambda \lambda_{qq} Q_x Q_p + \lambda \lambda_{qp} Q_x - \lambda_q Q_x \lambda_q Q_p + (\lambda \lambda_q Q_{px} - \lambda_q \lambda_p Q_x) \right].$$

To see that this is positive, recall $Q_x = \phi > 0$, $Q_p < 0$ from Lemma 2(a,c), and $\lambda_{qq} < 0$ and $\lambda_{qp} > 0$ from (5), using (4). Then the first three terms are positive. Using $Q_x = \phi$, the term in brackets then equals $\lambda_q(\lambda \phi_p - \lambda_p \phi) = \lambda_q(\lambda \lambda_p - \lambda_p(\lambda - q))/x > 0$.

For part (b) we compute

$$\begin{aligned} (\log \lambda(Q(x, \bar{q}, p), p))_{\bar{q}} &= (\log Q(x, \bar{q}, p))_{\bar{q}} - \frac{pQ_{\bar{q}}(x, \bar{q}, p)}{1 - p + pQ(x, \bar{q}, p)} \\ &= (\log Q(x, \bar{q}, p))_{\bar{q}} \left(\frac{1 - p}{1 - p + pQ(x, \bar{q}, p)} \right) \end{aligned}$$

which falls in x since $Q(x, \bar{q}, p)$ rises in x and is log-submodular in x and \bar{q} by Lemma 2(a,d). Intuitively, the larger proportional decline of applicant quality at lower-ranked firms is aggravated by the concavity of recruit quality $\lambda(q, p)$ in applicant quality q . \square

B.2 A Continuum Type Model

Our baseline model has two types, $\theta \in \{0, 1\}$. We now propose a version with continuous types $\theta \geq 0$. When they are exponentially distributed, we show that equilibrium matching is positive assortative.

Workers' types $\theta \geq 0$ are distributed with pdf $g(\theta)$. Firms' screening skills $p \in [0, 1]$ are distributed with cdf $F(p)$. We assume firms have linear hiring propensity, so the chance a type- θ worker is hired by firm p is proportional to

$$\mu(\theta, p) = 1 - (1 - \theta)p \geq 0. \quad (23)$$

Firm $p = 0$ hires indiscriminately; at firm $p = 1$, hiring chances scale with θ . For $\theta \in \{0, 1\}$, equation (23) reduces to our baseline model.

We first show how the distribution of worker types evolves as firms selectively recruit from the applicant pool. At wage rank x , write the pdf, expectation, and variance of types as $g(\theta; x)$, $\bar{\theta}(x) := \int \hat{\theta} g(\hat{\theta}; x) d\hat{\theta}$, and $V(x) := \bar{\theta}^2(x) - \bar{\theta}(x)^2$. Analogous to equation (1), the distribution of workers hired by firm p is then

$$\lambda(\theta; x, p) := \frac{\mu(\theta, p)g(\theta; x)}{\bar{\mu}(x, p)}$$

where $\bar{\mu}(x, p) := \int \mu(\hat{\theta}, p)g(\hat{\theta}, x)d\hat{\theta} = 1 - (1 - \bar{\theta}(x))p$. And the expected productivity is

$$\bar{\lambda}(x, p) := \frac{\bar{\mu}\bar{\theta}(x, p)}{\bar{\mu}(x, p)} := \frac{\int \mu(\hat{\theta}, p)\hat{\theta}g(\hat{\theta}, x)d\hat{\theta}}{\bar{\mu}(x, p)} = \int \hat{\theta}\lambda(\hat{\theta}; x, p)d\hat{\theta}. \quad (24)$$

The following result describes how the distribution of workers evolves. As in the baseline model, let $P(x)$ be the screening skill of firm x .

Lemma 3. *Assume linear hiring propensity (23). Relative applicant density decays exponentially:*

$$\frac{g(\theta'; x)}{g(\theta; x)} = \frac{g(\theta'; 1)}{g(\theta; 1)} \exp \left(-(\theta' - \theta) \int_x^1 \frac{P(\hat{x})}{\bar{\mu}(\hat{x}, P(\hat{x}))\hat{x}} d\hat{x} \right), \quad (25)$$

and the marginal benefit of screening skills is given by

$$\bar{\lambda}_p(x, p) = \frac{V(x)}{(1 - (1 - \bar{\theta}(x))p)^2}. \quad (26)$$

Proof. To see (25), let $\xi(\theta; x) := xg(\theta; x)$ be the non-normalized mass of type- θ applicants

at wage rank x ; this evolves according to

$$\xi_x(\theta; x) = \lambda(\theta; x, P(x)) = \frac{\mu(\theta, P(x))\xi(\theta; x)}{\bar{\mu}(x, P(x))x}.$$

Then

$$\frac{d}{dx} \log \frac{g(\theta'; x)}{g(\theta; x)} = \frac{d}{dx} \log \frac{\xi(\theta'; x)}{\xi(\theta; x)} = \frac{\xi_x(\theta'; x)}{\xi(\theta'; x)} - \frac{\xi_x(\theta; x)}{\xi(\theta; x)} = \frac{\mu(\theta', P(x)) - \mu(\theta, P(x))}{\bar{\mu}(x, P(x))x} = \frac{(\theta' - \theta)P(x)}{\bar{\mu}(x, P(x))x}.$$

Integrating yields (25). To see (26), differentiate (24) and multiply through with $\bar{\mu}^2$, noting that equation (23) implies $\mu_p = \theta - 1$,

$$\bar{\mu}^2 \bar{\lambda}_p = \bar{\mu}_p \bar{\theta} \bar{\mu} - \bar{\mu}_p \bar{\mu} \bar{\theta} = (\bar{\theta}^2 - \bar{\theta})(1 - (1 - \bar{\theta})p) + (1 - \bar{\theta})(\bar{\theta} - (\bar{\theta} - \bar{\theta}^2)p) = \bar{\theta}^2 - \bar{\theta}^2$$

yielding (26). Note that for binary types $\theta \in \{0, 1\}$, we have $\bar{\theta} = q$ and $V = q(1 - q)$, and so we recover the left expression in (5). \square

To prove that firms with better screening skills post higher wages, we wish to show the x -derivative of (26) is positive. This is tricky in general since the distribution of worker types evolves with the wage rank x . Fortunately, if worker types are initially exponentially distributed with $g(\theta) = \nu \exp(-\nu\theta)$, they remain exponentially distributed for all x .

Proposition 2. *Assume linear hiring propensity (23) and exponentially distributed types. Equilibrium exists and is unique. The equilibrium wage distribution is unique. Matching is positive assortative.*

Proof. Using (25), the wage-rank- x applicant distribution is exponential $g(\theta; x) = \nu(x) \exp(-\nu(x)\theta)$ with coefficient

$$\nu(x) = \nu + \int_x^1 \frac{P(\hat{x})}{\bar{\mu}(\hat{x}, P(\hat{x}))\hat{x}} d\hat{x}.$$

For the exponential, $V(x) = \bar{\theta}(x)^2$, so (26) simplifies to $\bar{\lambda}_p = ((1 - p)/\bar{\theta}(x) + p)^{-2}$ with positive cross-partial $\bar{\lambda}_{xp} > 0$, and matching is PAM. \square

C Proofs from Section 3

In this section we prove our main inefficiency result, Theorem 2, and extend it to two model variants with production complementarities and screening costs, introduced in Section 3.1. To accommodate these variants, we first formulate a more general model where the surplus when a firm with skills p hires from an applicant pool with quality q is given by a general function $\omega(q, p)$. The baseline model corresponds to $\omega(q, p) = \lambda(q, p) - \underline{w}$,

production complementarities correspond to $\omega(q, p) = p \cdot \lambda(q, p) - \underline{w}$, and screening costs to $\omega(q, p) = \lambda(q, p) - \kappa/(1 - p(1 - q)) - \underline{w}$.

We develop the apparatus to analyze aggregate surplus for general surplus functions $\omega(q, p)$ in Appendix C.1, specialize to the baseline model and prove Theorem 2 in Appendix C.2, prove some auxiliary claims from Section 3.1 in Appendix C.3, and then extend the inefficiency result to models with production complementarities and screening costs in Appendices C.4 and C.5.

C.1 Aggregate Surplus for General Surplus Functions $\omega(q, p)$

Fix the number of entering firms, and thereby the cutoff \underline{x} . Aggregate surplus under matching $P(\cdot)$ is given by

$$S(P(\cdot)) = \int_{\underline{x}}^1 \omega(Q(x), P(x)) dx \quad (27)$$

Consider adding a new firm p at rank x . This has three effects on surplus. First, firm p hires from a applicant pool with quality $Q(x)$ and so generates surplus $\omega(Q(x), p)$. Second, firm p pushes out the marginal firm, $P(\underline{x})$. Third, it lowers the ranking of intermediate firms, $\tilde{x} < x$, thereby lowering their applicant quality, $Q(\tilde{x})$. To quantify the latter externality, note that firm p 's hiring reduces applicant quality just below x by $dQ(x) = \phi(Q(x), p, x) dx$. For lower ranking firms \tilde{x} , this effect is mitigated since firm p pushes intermediate firms $\hat{x} \in [\tilde{x}, x]$ to lower wage ranks and thereby reduces their externality by $-\phi_x(Q(\hat{x}), p(\hat{x}), \hat{x})$. By standard results on ODEs in the proof of Lemma 2, firm p 's total effect on applicant quality $Q(\tilde{x})$ is given by

$$\chi(\tilde{x}; p, x, P(\cdot)) = -\phi(Q(x), p, x) \exp\left(-\int_{\tilde{x}}^x \phi_q\right) - \int_{\tilde{x}}^x \left[\exp\left(-\int_{\tilde{x}}^{\hat{x}} \phi_q\right) (-\phi_x(Q(\hat{x}), P(\hat{x}), \hat{x})) \right] d\hat{x} \quad (28)$$

where we dropped the arguments in the integrand $\phi_q = \phi_q(Q(\tilde{x}), P(\tilde{x}), \tilde{x})$ to enhance legibility. Putting these three effects together, firm p 's (infinitesimal) net-contribution to surplus when assigned to wage rank x in matching $P(\cdot)$ is thus given by

$$s(p, x, P(\cdot)) = \omega(Q(x), p) + \int_{\underline{x}}^x \chi(\tilde{x}; p, x, P(\cdot)) \omega_q(Q(\tilde{x}), P(\tilde{x})) d\tilde{x} - \omega(Q(\underline{x}), P(\underline{x})). \quad (29)$$

We wish to evaluate the effect of changing a matching function $P(\cdot)$ into a second matching function $P'(\cdot)$. To do this, suppose we move one firm p from rank $\underline{x}(p)$ to $\bar{x}(p)$

given matching $P(\cdot)$. The (infinitesimal) change in surplus equals

$$\int_{\underline{x}(p)}^{\bar{x}(p)} s_x(p, x, P(\cdot)) dx.$$

More generally, let us sequentially move all firms $p \in [\underline{p}, \bar{p}]$ in order of increasing p . The matching function changes over the course of this transformation, and we write $P^p(\cdot)$ for the matching function after firms $p' < p$ have been shifted; thus $P^{\underline{p}}(\cdot) = P(\cdot)$ and $P^{\bar{p}}(\cdot) = P'(\cdot)$.³³ The aggregate change in surplus is given by

$$S(P'(\cdot)) - S(P(\cdot)) = \int_{\underline{p}}^{\bar{p}} \left[\int_{\underline{x}(p)}^{\bar{x}(p)} s_x(p, x, P^p(\cdot)) dx \right] dF(p). \quad (30)$$

We first establish a general formula for the integrand in (30).

Lemma 4. *The marginal surplus of moving firm p past wage rank x given matching $P(\cdot)$ with $P(x) = \hat{p}$ equals*

$$s_x(p, x, P(\cdot)) = \left[\omega_q(Q(x), p) \phi(Q(x), \hat{p}, x) - \omega_q(Q(x), \hat{p}) \phi(Q(x), p, x) \right] \quad (31)$$

$$- \left[\phi_q(Q(x), p, x) \phi(Q(x), \hat{p}, x) + \phi_x(Q(x), p, x) - \phi_q(Q(x), \hat{p}, x) \phi(Q(x), p, x) - \phi_x(Q(x), \hat{p}, x) \right] \gamma$$

where

$$\gamma = \gamma(x, P(\cdot)) = \int_{\underline{x}}^x \exp \left(- \int_{\tilde{x}}^x \phi_q(Q(\hat{x}), P(\hat{x}), \hat{x}) d\hat{x} \right) \omega_q(Q(\tilde{x}), P(\tilde{x})) d\tilde{x}. \quad (32)$$

To understand these equations, consider swapping p and \hat{p} , and let us drop the arguments $Q(x)$ and x to enhance legibility. The first term in the first line of (31) $\omega_q(p) \phi(\hat{p})$ is the increased surplus contribution of firm p when selecting before \hat{p} from a pool with $dQ(x) = \phi(\hat{p}) dx$ higher quality. Conversely, the second term $\omega_q(\hat{p}) \phi(p)$ is \hat{p} 's decreased surplus when selecting after p . The term in the second line of (31) is the effect of switching p and \hat{p} 's screening order on residual pool quality $Q(x - dx)$. By screening applicants $q + \phi(\hat{p}) dx$ from a pool of size $x + dx$ (rather than applicants q from pool size x), firm p 's selection increases by $[\phi_q(p) \phi(\hat{p}) + \phi_x(p)] dx$; conversely firm \hat{p} 's selection de-

³³There are multiple ways to transform $P(\cdot)$ into $P'(\cdot)$. In a discrete analogue, if $p_1 < p_2 < p_3$, we can transform $\{p_1, p_3, p_2\}$ into $\{p_3, p_1, p_2\}$ by either moving p_1 to second position, or by moving p_1 to the top and then moving p_2 above it. Formally, a transformation is fully specified by the original matching $P(\cdot)$ and the intermediate target positions $\bar{x}(\cdot)$; the intermediate matchings $P^p(\cdot)$ as well as $\underline{x}(\cdot)$ are generated endogenously. In the two applications used in the proofs of Lemma 5 and Theorem 2 we move each firm immediately into its final position, $P'(\bar{x}(p)) = p$, but this need not be the case in general.

creases by $[\phi_q(\hat{p})\phi(p) + \phi_x(\hat{p})]dx$. This change in applicant quality $dQ(x)$ affects revenue $\omega(Q(\tilde{x}), P(\tilde{x}))$ of all lower-ranking firms $\tilde{x} \in [\underline{x}, x]$, where the exponential term in (32) equals $dQ(\tilde{x})/dQ(x)$ and captures differential selection by intermediate firms $\hat{x} \in [\tilde{x}, x]$.

Clearly, switching two identical firms $p = \hat{p} = P(x)$ makes no difference; formally

$$s_x(P(x), x, P(\cdot)) = 0. \quad (33)$$

Proof. Differentiating (29)

$$\begin{aligned} s_x(p, x, P(\cdot)) &= \omega_q(Q(x), p)Q_x(x) + \chi(x; p, x, P(\cdot))\omega_q(Q(x), P(x)) \\ &\quad + \int_{\underline{x}}^x \chi_x(\tilde{x}; p, x, P(\cdot))\omega_q(Q(x), P(\tilde{x}))d\tilde{x}. \end{aligned}$$

Since $Q_x(x) = \phi(Q(x), P(x), x)$ and $\chi(x; p, x, P(\cdot)) = -\phi(Q(x), p, x)$, and recalling $P(x) = \hat{p}$, the first two terms correspond to the first line of (31).

As for the last term, $\chi_x(\tilde{x}; p, x)$ equals

$$-\left[\phi_q(Q(x), p, x)Q_x(x) + \phi_x(Q(x), p, x) - \phi(Q(x), p, x)\phi_q(Q(x), P(x), x) - \phi_x(Q(x), P(x), x)\right] \exp\left(-\int_{\tilde{x}}^x \phi_q\right)$$

where the term in square-brackets equals the square-bracket term in the second line of (31), while integration over $\tilde{x} \in [\underline{x}, x]$ yields $\int_{\underline{x}}^x \exp\left(-\int_{\tilde{x}}^x \phi_q\right) \omega_q(Q(\tilde{x}), P(\tilde{x}))d\tilde{x} = \gamma$. \square

We cannot determine the sign of (31) in general. But at the lowest wage rank $x = \underline{x}$, the analysis simplifies because the integral domain in (32) collapses and so $\gamma = 0$, yielding a necessary condition for optimality of PAM that is easy to check. We will check that this condition is violated for the model variants in Propositions 3 and 4, below.

Lemma 5. *Assume that $1 - \underline{x}$ firms $p \in [\underline{p}, \bar{p}]$ enter. If*

$$\frac{\omega_{qp}(Q(\underline{x}), \underline{p})}{\omega_q(Q(\underline{x}), \underline{p})} < \frac{\phi_p(Q(\underline{x}), \underline{p}, \underline{x})}{\phi(Q(\underline{x}), \underline{p}, \underline{x})} \quad (34)$$

then PAM does not maximize surplus.

Proof. Let us transform $P(\cdot) = P^{PAM}(\cdot)$ into another matching $P'(\cdot)$ with NAM for $x \in [\underline{x}, \underline{x} + \epsilon]$ and PAM for $x \in (\underline{x} + \epsilon, 1]$. Intuitively, if there were 10 firms with skill $p_1 < \dots < p_{10}$ then this would mean swapping p_1 and p_2 , so firms are ranked $\{p_2, p_1, p_3, \dots, p_{10}\}$, from lowest to highest. Formally $P'(x) = F^{-1}(\underline{x} + \epsilon - (x - \underline{x}))$ for $x \in [\underline{x}, \underline{x} + \epsilon]$, and $P'(x) = F^{-1}(x)$ for $x > \underline{x} + \epsilon$, where $\epsilon > 0$ is small. We transform $P(\cdot)$ into $P'(\cdot)$ by shifting firms $p \in [\underline{p}, F^{-1}(\underline{x} + \epsilon)]$ (in rising order of p) to their $P'(\cdot)$ -wage

rank $\bar{x}(p) = \underline{x} + \epsilon - [F(p) - F(\underline{p})]$. Since lower firms $p' < p$ have already been shifted to $\bar{x}(p') > \bar{x}(p)$ at firm p 's "turn," firm p starts at rank $\underline{x}(p) = F(p) < \bar{x}(p)$ and is shifted exclusively past firms with higher screening skills $P^p(x) = F^{-1}[F(p) + x - \underline{x}] \geq p$, recalling the definition of the matching function $P^p(\cdot)$ at p 's "turn."

We now argue that given (34) the net value of this transformation (30) is positive. Using (33), the integrand in (30) equals

$$s_x(p, x, P^p(\cdot)) = s_x(P^p(x), x, P^p(\cdot)) - \int_p^{P^p(x)} s_{xp}(\check{p}, x, P^p(\cdot)) d\check{p} = - \int_p^{P^p(x)} s_{xp}(\check{p}, x, P^p(\cdot)) d\check{p}. \quad (35)$$

To see that this is positive, differentiate (31) wrt p , and evaluate for firm \underline{p} at wage rank \underline{x} . The derivative of the second line is zero because $\gamma = 0$ for $x = \underline{x}$. Thus,

$$s_{xp}(\underline{p}, \underline{x}, P^{PAM}(\cdot)) = \omega_{qp}(Q(\underline{x}), \underline{p}) \phi(Q(\underline{x}), \underline{p}, \underline{x}) - \omega_q(Q(\underline{x}), \underline{p}) \phi_p(Q(\underline{x}), \underline{p}, \underline{x})$$

which is negative by (34). Next, $(\check{p}, x, P^p(\cdot))$ converges to $(\underline{p}, \underline{x}, P^{PAM}(\cdot))$ for all $\check{p}, p \in [\underline{p}, F^{-1}(\underline{x} + \epsilon)]$ and $x \in [\underline{x}, \underline{x} + \epsilon]$ as $\epsilon \rightarrow 0$, using for instance the topology of uniform convergence on matching functions $P(\cdot)$. Thus, the integrand of (35) is negative, and so the integral (30) is positive, so the transformation from $P(\cdot)$ to $P'(\cdot)$ raises welfare. \square

C.2 Proof of Theorem 2

We now return to the baseline model where surplus is given by $\omega(q, p) = \lambda(q, p) - \underline{w}$. The comparison of the elasticities (10) implies (34), and so Lemma 5 already implies that PAM is inefficient. In particular, the proof of Lemma 5 shows that locally near the cutoff \underline{x} shifting low-skill firms ahead of high-skill firms increases surplus.

Theorem 2 claims more strongly that surplus is minimized by PAM and maximized by NAM. We show this by arguing that for the baseline surplus function $\omega(q, p) = \lambda(q, p) - \underline{w}$ our local argument at the bottom of the wage distribution in fact holds globally.

Indeed, (31) simplifies considerably:

Lemma 6. *When $\omega(q, p) = \lambda(q, p) - \underline{w}$, the marginal surplus of moving firm p past wage rank x given matching $P(\cdot)$ with $P(x) = \hat{p}$ equals*

$$s_x(p, x, P(\cdot)) = [\lambda_q(p) \phi(\hat{p}) - \lambda_q(\hat{p}) \phi(p)] (1 - \gamma/x) \quad (36)$$

where $1 - \gamma/x = \delta = \exp\left(-\int_{\underline{x}}^x \frac{\lambda_q(Q(\tilde{x}), P(\tilde{x}))}{\tilde{x}} d\tilde{x}\right) \in (0, 1)$.

The term in square-brackets corresponds to the incremental talent hired by firms p

and \hat{p} when the former is shifted ahead of the latter. The effect on aggregate surplus is scaled down by the factor δ from (9) since incremental talent hired by the marginal firms p and \hat{p} reduces the talent hired by lower-ranking firms.

Proof. We will show that the marginal surplus (31) equals

$$s_x(p, x, P(\cdot)) = \left[\lambda_q(p)\phi(\hat{p}) - \lambda_q(\hat{p})\phi(p) \right] - \left[\lambda_q(p)\phi(\hat{p}) - \lambda_q(\hat{p})\phi(p) \right] \frac{\gamma}{x} \quad (37)$$

and thus collapses to (36). Using the definition of $\omega(q, p)$, the first line in (31) equals the first square-bracket term in (37). Turning to the second line in (31) and recalling $\phi = \frac{\lambda-q}{x}$, elementary algebra implies

$$\phi_q(p)\phi(\hat{p}) + \phi_x(p) - \phi_q(\hat{p})\phi(p) - \phi_x(\hat{p}) = \frac{1}{x} [\lambda_q(p)\phi(\hat{p}) - \lambda_q(\hat{p})\phi(p)]. \quad (38)$$

Multiplying by γ , the second line in (31) equals the second square-bracket term in (37).

To evaluate γ observe that

$$\exp\left(-\int_{\tilde{x}}^x \phi_q\right) = \exp\left(-\int_{\tilde{x}}^x \frac{\lambda_q(Q(\hat{x}), P(\hat{x})) - 1}{\hat{x}} d\hat{x}\right) = \frac{x}{\tilde{x}} \exp\left(-\int_{\tilde{x}}^x \frac{\lambda_q(Q(\hat{x}), P(\hat{x}))}{\hat{x}} d\hat{x}\right).$$

Equation (32) thus simplifies to

$$\begin{aligned} \gamma &= x \int_{\underline{x}}^x \exp\left(-\int_{\tilde{x}}^x \frac{\lambda_q(Q(\hat{x}), P(\hat{x}))}{\hat{x}} d\hat{x}\right) \frac{\lambda_q(Q(\tilde{x}), P(\tilde{x}))}{\tilde{x}} d\tilde{x} \\ &= x \int_{\underline{x}}^x \frac{d}{d\tilde{x}} \exp\left(-\int_{\tilde{x}}^x \frac{\lambda_q(Q(\hat{x}), P(\hat{x}))}{\hat{x}} d\hat{x}\right) d\tilde{x} = x \left[1 - \exp\left(-\int_{\underline{x}}^x \frac{\lambda_q(Q(\tilde{x}), P(\tilde{x}))}{\tilde{x}} d\tilde{x}\right) \right], \end{aligned} \quad (39)$$

as required. \square

In the proof sketch in the body of the paper we argued that (36) is positive when $\hat{p} = p + dp$ by noting that $\lambda_q(p)\phi(p + dp) - \lambda_q(p + dp)\phi(p) = (\lambda_q\phi_p - \lambda_{qp}\phi)dp > 0$, as shown in (10). To prove Theorem 2, we now generalize this local argument to show that transforming an arbitrary matching $P(\cdot)$ into $P'(\cdot) = P^{NAM}(\cdot)$ raises surplus. Intuitively, if there were 10 firms with skill $p_1 < \dots < p_{10}$, we would shift firm p_1 to the highest position, then shift firm p_2 to the second-highest position, and so on. Formally, we shift type- p firms in rising order of p to their NAM-rank $\bar{x}(p) = 1 - [F(p) - F(\underline{p})]$. At firm p 's turn, i.e. in matching $P^p(\cdot)$, lower-skill firms $p' < p$ have already been shifted to their NAM rank in $\bar{x}(p') > \bar{x}(p)$, and so firm p starts at $\underline{x}(p) \leq \bar{x}(p)$ and is shifted past higher-skill firms $P^p(x) \geq p$ for all $x \in [\underline{x}(p), \bar{x}(p)]$.

Recalling $s_x(P^p(x), x, P^p(\cdot)) = 0$ from (33), the marginal surplus along this transformation is given by

$$s_x(p, x, P^p(\cdot)) = - \int_p^{P^p(x)} s_{xp}(\check{p}, x, P^p(\cdot)) d\check{p}. \quad (40)$$

To sign the integrand of this expression, we differentiate (35) with respect to p

$$s_{xp}(p, x, P^p(\cdot)) = [\lambda_{qp}(p)\phi(\hat{p}) - \phi_p(p)\lambda_q(\hat{p})](1 - \gamma/x) < 0.$$

Substituting back into (40) and recalling that $p < P^p(x)$, marginal surplus (40) is positive. Finally, integrating (40) over $p \in [\underline{p}, \bar{p}]$ and $x \in [\underline{x}(p), \bar{x}(p)]$, we conclude that the aggregate change of surplus (30) from this transformation is positive, and NAM maximizes aggregate surplus. The analogue argument implies that PAM minimizes aggregate surplus.

Finally we establish that the planner indeed wants the firms with the highest screening skills to enter the market. To see this, differentiate firm p 's contribution to surplus (29) with respect to p

$$\begin{aligned} s_p(p, x, P(\cdot)) &= \lambda_p(Q(x), p) + \int_{\underline{x}}^x \chi_p(\check{x}; p, x, P(\cdot)) \lambda_q(Q(\check{x}), P(\check{x})) d\check{x} \\ &= \lambda_p(Q(x), p) - \phi_p(Q(x), p, x) \int_{\underline{x}}^x \exp\left(-\int_{\check{x}}^x \phi_q\right) \lambda_q(Q(\check{x}), P(\check{x})) d\check{x} \\ &= \lambda_p(Q(x), p) - \left(\frac{\lambda_p(Q(x), p)}{x}\right) x \left[1 - \exp\left(-\int_{\underline{x}}^x \frac{\lambda_q(Q(\check{x}), P(\check{x}))}{\check{x}} d\check{x}\right)\right] \\ &= \lambda_p(Q(x), p) \exp\left(-\int_{\underline{x}}^x \frac{\lambda_q(Q(\check{x}), P(\check{x}))}{\check{x}} d\check{x}\right) > 0, \end{aligned} \quad (41)$$

where the second equality uses the definition of the externality χ in (28), and the third equality uses the definition of γ (32) and its evaluation in (39).

C.3 Wage Caps

Restricting firms to a single wage \check{w} implements a random screening order, which is constrained efficient by the proof of Theorem 2. The wage level \check{w} does not affect the screening order, but rather pins down the marginal entering firm $\check{p} = \check{p}(\check{w})$ via the indifference condition $\int_{F(\check{p})}^1 \lambda(Q(x), \check{p}) dx = \check{w}$. The optimal wage level \check{w} is the one that maximizes surplus $\int_{\check{p}}^{\bar{p}} [\int_{F(\check{p})}^1 \lambda(Q(x), \check{p}) dx - \underline{w}] dF(p)$. Since the marginal firm \check{p} exerts a negative externality on firms $p > \check{p}$, we know that firms pay workers more than their outside option, $\check{w} > \underline{w}$.

Here we argue that firms have no incentives to underbid, offering $w < \check{w}$; hence, making \check{w} a wage cap that firms are free to underbid also implements the second-best outcome. To see this, we need to check that the marginal firm \check{p} (which is most tempted to underbid) does not want to cut its wage to \underline{w} (which is the most profitable deviation). Indeed, note that at $w = \underline{w}$ firm \check{p} exerts no externality on other firms or workers, so captures its full contribution to social surplus. By definition of \check{p} , this social surplus is zero when the firm offers \check{w} and screens at a random rank, as instructed by the planner. When the firm disobeys the planner and posts $w = \underline{w}$, the contribution to social surplus is thus negative, meaning the firm's profits are also negative.

C.4 Production Complementarities

Theorem 2 is in stark contrast to the efficiency of equilibrium in the assignment model, where equilibrium also features PAM when the production function is supermodular. We now show that equilibrium continues to be inefficient even with exogenous complementarities.

Proposition 3. *Suppose firm p 's surplus equals $\omega(q, p) = h(p)\lambda(q, p) - \underline{w}$ with $h, h_p > 0$.³⁴*

(a) *Assume scarce talent (4). Equilibrium exists, and is unique. All firms above some threshold $p \geq \underline{p}$ enter and sort according to PAM.*

(b) *If $h(p) = p$, PAM is inefficient. For any $h(p)$ with bounded h_p/h , there exists $\bar{q}, \bar{p}, \underline{w}$ such that PAM is inefficient.*

Proof. (a) Surplus ω is increasing in p and supermodular, $\omega_{qp} = \lambda_{qp}h + \lambda_q h_p > 0$, so in equilibrium firms $p \geq \underline{p}$ enter and matching is PAM. Equilibrium existence and uniqueness then follow as in Theorem 1.

(b) To check the sufficient condition for inefficiency (34), note first that

$$\frac{\lambda_{qp}}{\lambda_q} = -\frac{1}{1-p} + 2\frac{1-q}{1-p(1-q)} = \frac{2(1-p)(1-q) - (1-p(1-q))}{(1-p)(1-p(1-q))}$$

and

$$\frac{\phi_p}{\phi} = \lambda_p \cdot \frac{1}{\lambda - q} = \frac{q(1-q)}{(1-p(1-q))^2} \cdot \frac{1-p(1-q)}{qp(1-q)} = \frac{1}{1-p(1-q)} \cdot \frac{1}{p}.$$

³⁴Given binary types $\theta \in \{L, H\}$, our functional form is essentially without loss. Indeed, consider any increasing, supermodular surplus function $\nu(\theta, p)$. Expected productivity of a worker with expected talent $\Pr(\theta = H) = \lambda(q, p)$ is then $\lambda\nu(H, p) + (1-\lambda)\nu(L, p) = \lambda[\nu(H, p) - \nu(L, p)] + \nu(L, p)$; the additive term $\nu(L, p)$ does not depend on θ , and so matters for neither equilibrium sorting nor efficient sorting.

Thus, equilibrium is inefficient (and could be improved by re-ordering low-wage firms) if

$$\frac{\phi_p}{\phi} - \frac{\omega_{qp}}{\omega_q} = \left(\frac{\phi_p}{\phi} - \frac{\lambda_{qp}}{\lambda_q} \right) - \frac{h'}{h} = \frac{q}{(1-p)(1-p(1-q))} + \frac{1}{p} - \frac{h'}{h} \quad (42)$$

is positive. This depends on the degree of supermodularity. For the standard specification with $h(p) = p$, it is positive. For general h , we choose \bar{p} such that $\frac{1}{2(1-\bar{p})} = \sup \frac{h'(p)}{h(p)}$, and \bar{q} such that $\lambda(\bar{q}, \bar{p}) = 1/2$, so that the positive first term in (42) outweighs the negative third term for \bar{p}, \bar{q} . For \underline{w} close enough to $1/2$ we have $\underline{p} \approx \bar{p}$ and $\underline{q} \approx \bar{q}$, so the positive second term ensures that (42) is positive also for $\underline{p}, \underline{q}$, as required by (34).

Thus, the compositional externality outweighs two sources of supermodularity: one in the recruiting function $\lambda(q, p)$, and another one in the production function $\omega = \lambda h - \underline{w}$. \square

C.5 Screening Costs

Here we show that Theorems 1 and 2 extend to a model with screening costs, where net surplus is given by $\omega(q, p) = \lambda(q, p) - \kappa/(1 - p(1 - q)) - \underline{w}$. Observe that screening costs $\kappa/(1 - p(1 - q))$ fall in applicant quality q but increase in screening skills p since more skillful firms interview more candidates.³⁵

Proposition 4. *Suppose there is a cost $\kappa \geq 0$ to screen each applicant.*

- (a) *Assume scarce talent (4). Equilibrium exists, and is unique. All firms above some threshold $p \geq \underline{p}$ enter and sort according to PAM.*
- (b) *PAM is inefficient.*

Proof. (a) As in Theorem 1, skilled firms post higher wages since

$$\omega_{qp} = \lambda_{qp} - \left(\frac{\kappa}{1 - p(1 - q)} \right)_{pq} = \frac{1 - p(1 - q) - 2q}{(1 - p(1 - q))^3} + \frac{1 + p(1 - q)}{(1 - p(1 - q))^3} \kappa > 0.$$

As before, the first term is positive since $\lambda = q/(1 - p(1 - q)) < 1/2$. Additionally, the second term is always positive. Thus $\omega(q, p)$ is supermodular on $q \in [0, \bar{q}]$, as required. As for entry, note that $\omega_p = \omega(1 - q)/(1 - p(1 - q)) \geq 0$. Equilibrium existence and uniqueness then follow as in Theorem 1.

³⁵By adopting this surplus function, we implicitly assume that firms prefer to screen candidates rather than hiring a random, unscreened candidate. This mild condition is satisfied if (i) the minimum wage \underline{w} is sufficiently high such that $\omega(q, p) > q$ for any firm p willing to hire a worker, $\lambda(q, p) \geq \underline{w}$, or (ii) screening additionally screens out unmodeled “terrible” types of workers.

(b) We apply Lemma 5 to show that PAM is inefficient. Indeed

$$\frac{\omega_{qp}}{\omega_q} = 2 \frac{1-q}{1-p(1-q)} - \frac{1-\kappa}{1-p(1-\kappa)} < \frac{1-q}{1-p(1-q)} = \frac{\lambda_p}{\lambda} < \frac{\lambda_p}{\lambda} \frac{\lambda}{\lambda-q} = \frac{\phi_p}{\phi}$$

where the first inequality follows because $\kappa < q$, which in turn follows from $\omega(q, p) = (q - \kappa)/(1 - p(1 - q)) - \underline{w} \geq 0$. \square

Proposition 4 shows that the main insights of our paper carry over to a model with screening costs. This extension also creates a role for different information structures because our “perfect bad news” screening is no longer without loss. We can show that Proposition 4 extends to “symmetric signals” where high (resp. low) types pass the screening tests with probability p (resp. $1-p$), and firms differ in their screening skill $p \in (0.5, 1)$. However, if firms receive “perfect good news” information whereby untalented applicants fail all tests, while firms differ in their probability p of identifying talented workers, then equilibrium matching is NAM and equilibrium is efficient. Intuitively, all firms are equally effective at screening and hire only talented workers, $\lambda = 1$, but skilled firms are more efficient and need not search as long to find a talented worker. We dislike this signal structure because of its counter-factual predictions that all firms hire workers of the same quality, and high-quality firms have lower search expenditure. Indeed, high-wage firms have more applications (Belot et al. (2022), Banfi and Villena-Roldán (2019)) and interview more applicants (Barron et al. (1985)).

D Proofs from Section 4

D.1 Derivation of Equation (17) and Proof that $V(r)$ is Convex

The envelope theorem applied to (14) implies

$$V_r(r(t), t) = \int_t^\infty e^{-\beta(s-t)} \frac{\partial r(s)}{\partial r(t)} ds.$$

To compute the integrand, we write the solution of the talent evolution (13) as a function of its initial condition $r_s(s, \underline{r}) = \zeta(r(s, \underline{r}), s)$ where $\zeta(r, s) = \alpha(\lambda(Q(s), \psi(r)) - r)$ and $r(t, \underline{r}) = \underline{r}$. Hartman (2002, Theorem 3.1) implies $r_{s\underline{r}} = \zeta_r r_{\underline{r}}$ with boundary condition $r_s(t, \underline{r}) = 1$. Hence, (17) follows from

$$\frac{\partial r(s)}{\partial r(t)} = r_{\underline{r}}(s, r(t)) = \exp \left(\int_s^t \zeta_r(r(u), u) du \right) = \exp \left(-\alpha \int_s^t [1 - \lambda_p(Q(u), \psi(r(u))) \psi_r(r(u))] du \right).$$

To see that $V(r, t)$ is convex in r , we differentiate again to obtain

$$\begin{aligned} V_{rr}(r(t), t) &= \int_t^\infty e^{-\beta(s-t)} \frac{\partial^2 r(s)}{\partial r(t)^2} ds \\ &= \int_t^\infty e^{-\beta(s-t)} \frac{\partial r(s)}{\partial r(t)} \alpha \int_s^t [\lambda_{pp}(Q(u), \psi(r(u))) \psi_r(r(u))^2 + \lambda_p(Q(u), \psi(r(u))) \psi_{rr}(r(u))] \frac{\partial r(u)}{\partial r(t)} du ds \end{aligned}$$

which is positive since $\lambda_p = \frac{q(1-q)}{(1-p(1-q))^2}$, $\lambda_{pp} = \frac{2q(1-q)^2}{(1-p(1-q))^3}$, ψ_r, ψ_{rr} are all positive. For an intuition consider the random recruiter example where $\psi(r) = p^L + r(p^H - p^L)$. Intermediate levels of recruiting skills r have the draw-back that the firm's wage must strike a compromise between the firm's low-skill and high-skill recruiters, while a firm with homogeneous recruiters, $r = 0$ or 1 , can choose the optimal wage for all.

D.2 Proof of Theorem 3

Here we complete the proof of Theorem 3 by arguing that if there is an atom of initially identical firms, these firms diverge immediately. Assume to the contrary, that at time $t > 0$ an atom of firms has the same worker quality $r(t)$ and write $[x^0, x^1]$ for the talent-ranks of these firms. Since optimal wages rise in talent and hence talent differences never vanish, firms in the atom must have identical talent $r(s)$ for all $s < t$. At any time $s < t$ the wage distribution must be smooth by the arguments in Section 2.2. If firms in the atom post different wages, they drift apart. Hence the firms must employ non-degenerate distributional strategies,³⁶ posting both high and low wages to attract good and bad applicants; they must thus be indifferent across a range of applicants $[q^0(s), q^1(s)]$ for all $s < t$. Thus, the first order condition (16) must hold with equality on $[q^0(s), q^1(s)]$ for all $s < t$ and the atom quality $r(s)$.

Such distributional strategies cannot be optimal because wages are dynamic complements: consider a firm that deviates by always attracting the best applicants in the atom $q^1(s)$, rather than mixing over good and bad applicants. At time $s = 0$, the choice $q^1(0)$ is optimal. Moreover, over time the firm's quality rises above $r(s)$ since it attracts better applicants. Since the marginal benefit of attracting better applicants, the RHS of (16), strictly increases in r , this deviation strictly improves on the posited distributional strategy. This proves that initially identical firms diverge immediately.

³⁶When using a distributional strategy, a firm posts an entire distribution of wages $\nu = \nu(w, t)$ of wages at any time t ; we then interpret $\psi(R(x, t))$ as the weighted-average skill of firms posting the x -ranked wage, and solve for the firm's evolution of talent by taking expectations over the RHS of (13).

D.3 Proof of Theorem 4

Here we show that firm x 's talent $R(x, t)$ and applicant quality $Q(x, t)$ converge to their steady state levels $R^*(x)$ and $Q^*(x)$. For constant applicants $Q(x, t) \equiv q$, talent drifts towards $\rho(q)$. The complication is that firm x 's applicant quality $Q(x, t)$ also changes over time, with $Q_x(x, t)$ given by (12) and $r_t(x, t)$ by (13). As discussed in Section 4.4, Figure 2 suggests a “proof by induction.” The formal proof is more complicated because x is continuous; it proceeds by showing that the steady state satisfies a contraction property, and then applies the contraction mapping theorem over a small interval, akin to the proof of the Picard-Lindelöf theorem.

First, we establish a contraction property. Define the limits $\underline{Q}(x) := \liminf_t Q(x, t)$, $\bar{Q}(x) := \limsup_t Q(x, t)$, $\underline{R}(x) := \liminf_t R(x, t)$, and $\bar{R}(x) := \limsup_t R(x, t)$ and interpret (12) as an operator \mathcal{Q} , mapping firm quality functions $R(\cdot, t)$ into applicant quality functions $Q(\cdot, t) = \mathcal{Q}[R(\cdot, t)](\cdot)$. We claim that:

$$\mathcal{Q}[\rho(\bar{Q}(\cdot))](x) \leq \underline{Q}(x) \leq \bar{Q}(x) \leq \mathcal{Q}[\rho(\underline{Q}(\cdot))](x). \quad (43)$$

To understand (43), first note that \mathcal{Q} is antitone: if $R(x) \geq \hat{R}(x)$ for all x then $Q(x) = \mathcal{Q}(R(\cdot))(x) \leq \mathcal{Q}(\hat{R}(\cdot))(x) = \hat{Q}(x)$, since $Q(1) = \hat{Q}(1) = \bar{q}$ and $\phi(q, \psi(r), x)$ rises in r . Intuitively, better recruiters introduce more adverse selection. Inequalities (43) then state that if applicant quality was equal to one of its limits, \underline{Q} and \bar{Q} , and talent r was in steady state, the induced difference in applicant pools is larger than the original difference.

We prove (43) in two steps. First, since $R(x, t)$ drifts towards $\rho(Q(x, t))$, which is asymptotically bounded by $\rho(\underline{Q}(x))$ and $\rho(\bar{Q}(x))$, we have

$$\rho(\underline{Q}(x)) \leq \underline{R}(x) \leq \bar{R}(x) \leq \rho(\bar{Q}(x)) \quad (44)$$

for all x . Second,

$$\underline{Q}(x) = \lim_{t \rightarrow \infty} \inf_{t' > t} Q(x, t') = \lim_{t \rightarrow \infty} \inf_{t' > t} \{ \mathcal{Q}[R(\cdot, t')](x) \} \geq \lim_{t \rightarrow \infty} \mathcal{Q}[\sup_{t' > t} \{ R(\cdot, t') \}](x) = \mathcal{Q}[\bar{R}(\cdot)](x).$$

The first equality is the definition of the \liminf , and the second the definition of the operator \mathcal{Q} . The inequality uses the antitonicity of \mathcal{Q} : since $R(\hat{x}, t') \leq \sup_{t' > t} \{ R(\hat{x}, t') \}$ for all $t' > t$ and \hat{x} , we know that $\mathcal{Q}[R(\cdot, t')](x)$ (weakly) exceeds $\mathcal{Q}[\sup_{t' > t} \{ R(\cdot, t') \}](x)$ for all $t' > t$ and x , and hence so does $\inf_{t' > t} \mathcal{Q}[R(\cdot, t')](x)$. The final equality uses the dominated convergence theorem to exchange the limit $t \rightarrow \infty$ and the operator \mathcal{Q} , as well as the definition of the \limsup , $\bar{R}(x) = \lim_{t \rightarrow \infty} \sup_{t' > t} R(x, t')$. Together with the analogue argument that $\bar{Q}(x) \leq \mathcal{Q}[\underline{R}(\cdot)](x)$, and applying the antitone operator \mathcal{Q} to (44)

we get

$$\mathcal{Q}[\rho(\bar{Q}(\cdot))](x) \leq \mathcal{Q}[\bar{R}(\cdot)](x) \leq \underline{Q}(x) \leq \bar{Q}(x) \leq \mathcal{Q}[\underline{R}(\cdot)](x) \leq \mathcal{Q}[\rho(\underline{Q}(\cdot))](x)$$

establishing (43).

To complete the proof of convergence, suppose “inductively” that applicant and firm quality converge above some $\hat{x} \in (0, 1]$, i.e. $\underline{Q}(x) = \bar{Q}(x)$, and hence $\underline{R}(x) = \bar{R}(x)$, for all $x \in (\hat{x}, 1]$. Fix ϵ , and let $\delta(\epsilon) := \max_{x \in [\hat{x}-\epsilon, \hat{x}]} |\bar{Q}(x) - \underline{Q}(x)|$ be the maximum distance between the liminf and limsup on $[\hat{x} - \epsilon, \hat{x}]$. Since $\rho(q)$ is locally Lipschitz in q with constant K ,³⁷ we have

$$\max_{x \in [\hat{x}-\epsilon, \hat{x}]} |\rho(\underline{Q}(\cdot))(x) - \rho(\bar{Q}(\cdot))(x)| \leq K\delta(\epsilon).$$

Next, since $\mathcal{Q}[\underline{R}(\cdot)](x)$ solves (12), the RHS of which is locally Lipschitz in q and r with constant K' , and choosing $\epsilon < 1/K'(1 + K)$ we get

$$\max_{x \in [\hat{x}-\epsilon, \hat{x}]} |\mathcal{Q}[\rho(\underline{Q}(\cdot))](x) - \mathcal{Q}[\rho(\bar{Q}(\cdot))](x)| \leq K'\epsilon(1 + K)\delta(\epsilon) < \max_{x \in [\hat{x}-\epsilon, \hat{x}]} |\bar{Q}(x) - \underline{Q}(x)|.$$

contradicting (43). Hence we must have $\underline{Q}(x) = \bar{Q}(x)$ and hence $\underline{R}(x) = \bar{R}(x)$, for all $x \in [\hat{x} - \epsilon, 1]$, and thus for all $x \in [0, 1]$.

³⁷Indeed, recall that $\rho(q) = r \in [0, 1]$ solves $\lambda(q, \psi(r)) - r = 0$. Thus, by convexity of the LHS we have $\lambda_p(q, \psi(\rho(q)))\psi_r(\rho(q)) - 1 < (\lambda(q, \psi(1)) - 1)/(1 - \rho(q)) < 0$ and so $R_q(q) = -\frac{\lambda_q(q, \psi(\rho(q)))}{\lambda_p(q, \psi(\rho(q)))\psi_r(\rho(q)) - 1} < \frac{\lambda_q(q, \psi(\rho(q)))(1 - \rho(q))}{1 - \lambda(q, \psi(1))} =: K$.