Economics 2102: Final

17 December, 2004

1. Random Pricing

Consider the pricing problem of a monopolist who has 300 units to sell and is only allowed to choose a price p per unit (i.e. no first degree price discrimination). There are 100 agents who are identical and have the following demand:

$$\begin{aligned} D(p) &= 0 \quad \text{if} \quad p > 2 \\ &= 1 \quad \text{if} \quad p \in (1,2] \\ &= 5 \quad \text{if} \quad p \in [0,1] \end{aligned}$$

(a) [5 points] Suppose the firm can charge a single price, p, per unit. What is the best they can do?

(b) [10 points] Suppose the firm can separate the agents into two groups. The first group of N are charged price p_1 per unit. The second are charged p_2 per unit. What is the best they can do?

(c) [5 points] Agents are identical so, intuitively, how can splitting them into two groups help? Does this relate to anything we covered in class? [100 words max.]

2. Nonlinear Pricing with Outside Options

Consider a second degree price discriminating firm facing customers with two possible types $\theta \in \{3, 4\}$ with equal probability. An agent with type θ gains utility $u(\theta) = \theta q - p$ from quality q supplied at price p. If the agent does not purchase they gain utility 0. The cost of quality q is $c(q) = q^2/2$.

(a)[5 points]. Suppose the firm could observe each agents type θ. What quantity would she choose for each type?

For the next two parts assume the firm cannot observe agents' types. She can choose two quantity–price bundles $\{q(\theta), p(\theta)\}$ for $\theta \in \{3, 4\}$.

(b)[10 points]. Suppose there is a single outside good of quality $q^* = 1$ and price $p^* = 1$. What quantity would the firm choose for each type?

(c)[10 points]. Now suppose the outside good has quality $q^* = 6$ and price $p^* = 18$. What quantity would the firm choose for each type?

3. Auctions with Hidden Quality

The economics department is trying to procure teaching services from one of N potential assistant professors. Candidate i has an outside option of wage $\theta_i \in [0, 1]$ with distribution function F. This wage is private information and can be thought of as the candidate's type. The department gets value $v(\theta_i)$ from type θ_i.

Consider a direct revelation mechanism consisting of an allocation function $P(\tilde{\theta}_1, \ldots, \tilde{\theta}_N)$ and a transfer function $t(\tilde{\theta}_1, \ldots, \tilde{\theta}_N)$. Suppose candidate i's utility is $u(\theta_i, \tilde{\theta}_i) = E_{-i}[t(\tilde{\theta}) - P(\tilde{\theta})\theta_i]$ and the department's profit is $\pi = E[P(\tilde{\theta})v(\theta_i) - t(\tilde{\theta})]$.

(a)[7 points] Characterise the agent's utility under incentive compatibility in terms of an integral equation and a monotonicity constraint.

(b)[8 points] Using (a), what is the department's profit?

For the rest of the question assume that

$$1 \geq \frac{d}{d\theta_i} \frac{F(\theta_i)}{f(\theta_i)} \geq 0$$

(c)[5 points] If $v'(\theta_i) \leq 1$ what is the department's optimal hiring policy (i.e. allocation function)? How can this be implemented?

(d)[5 points] Suppose $v'(\theta_i) \geq 2$ and $E[v(\theta_i)] \geq 1$. What is the department's optimal hiring policy (i.e. allocation function)? How can this be implemented?

4. Double Auction

A seller and buyer participate in a double auction. The seller's cost, $c \in [0,1]$, is distributed according to F_S. The buyer's value, $v \in [0,1]$, is distributed according to F_B. The seller names a price s and the buyer a price b. If $b \geq s$ the agents trade at price $p = (s+b)/2$, the seller gains $p - c$ and the buyer gains $v - p$. If $s < b$ there is no trade and both gain 0.

(a)[15 points] Write down the utilities of buyer and seller. Derive the FOCs for the optimal bidding strategies.

For the rest of the question assume $c \sim U[0,1]$ and $v \sim U[0,1]$.

(b)[10 points] Show that $S(c) = \frac{2}{3}c + \frac{1}{4}$ and $B(v) = \frac{2}{3}v + \frac{1}{12}$ satisfy the FOCs.

(c)[5 points] Under which conditions on (v,c) does trade occur?