# Contract Theory: Final

9:00am–12:00pm, 5th December, 2016

## 1. Affirmative Action

A continuum of students apply to be admitted to UCLA. There are two ethnicities of students, A and B. There is mass 20 of A students, with test scores $t \sim U[0, 20]$. There is mass 10 of B students, with test scores $t \sim U[0, 10]$. The university has capacity for mass 12.

The admissions office ("principal") wants students with high test scores and also wants a balanced racial mix. A mechanism describes an admissions probability for a student of ethnicity A,B with score $t$, i.e. $\{p_A(t), p_B(t)\}$. The admissions office wants to maximize

$$\int [tp_A(t)f_A(t) + tp_B(t)f_B(t)]dt - 4|N_A - N_B|$$

where $f_A(t)$ is the measure for type $A$ students, and $N_A$ describes the total mass of $A$ students admitted (and similarly for B).

(a) Suppose the principal chooses $p_A(t), p_B(t)$ to maximize their payoff subject to the capacity constraint. What is the optimal mechanism?

(b) Suppose we ban racial discrimination, so that $p_A(t) = p_B(t) =: p(t)$. What is the optimal mechanism?

(c) Suppose students can always deliberately fail test questions, meaning that $p(t)$ must be monotone. What is the optimal mechanism?

## 2. Transferring Knowledge

Time is continuous and infinite, $t \geq 0$, and the interest rate is $r$. An expert E has one unit of knowledge that she may pass to an apprentice A. Let $x_t \in [0,1]$ be the knowledge transferred by time $t$. This knowledge allows A to produce output $x_t$ whether he works for E or for himself. When A works for the expert, he is paid nothing. Rather, he is paid in future knowledge.

The principal commits to a weakly increasing knowledge trajectory, $\{x_t\}_{t\in[0,T]}$. At some endogenous time $T$, A quits and starts working for himself. Payoffs are as follows: The apprentice earns nothing when working for E, and a flow payoff of $rx_T$ thereafter. The expert E receives flow payoff $rx_t$ while A is employed, and nothing thereafter.

(a) Denote A's value function by $V(x)$. What is A's incentive compatibility constraint (to stop him quitting)?

(b) Suppose the IC constraint binds at each point in time. How does $x_t$ evolve?

(c) Argue that the IC constraint must bind at each point in time.

(d) Calculate the initial "gift" of knowledge, $x_0$, and the time apprenticeship lasts, $T$.

## 3. Reputation and Information Disclosure

Time is continuous and infinite, $t \geq 0$. There are a continuum of workers with type $\theta_t \in \{0, 1\}$. Workers all start off as high quality, $\theta_0 = 1$, but may become bad. A shock arrives at poisson rate $\lambda$; when this shock arrives a worker's type switches from good to bad, or from bad to good. That is, $\Pr(\theta_{t+dt} = \theta_t) = 1 - \lambda dt$. Let $x_t = \Pr(\theta_t = H)$ be the reputation of the worker.

A competitive market of short-lived employers offers spot contracts to workers. A worker is offered their productivity, $w_t = x_t - k$, where $k \in (1/2, 1)$ is the capital required to employ a worker. There is a minimum wage of 0, so a worker is unemployed if $w_t < 0$.

The market learns about workers quality via perfect bad news. That is, if $\theta_t = 1$, there is no news. If $\theta_t = 0$ then there are breakdowns at Poisson rate $\mu$.

(a) Show that an unemployed worker's reputation evolved according to

$$\dot{x} = \lambda(1 - 2x)$$

Show than an employed worker's reputation evolves according to

$$\dot{x} = \lambda(1 - 2x) + \mu x(1 - x)$$

in the absence of news, and $x_t = 0$ if there is a breakdown.

(b) Suppose that firms see a worker's entire history. What happens to his reputation after a breakdown? Argue that, as $t \to \infty$, all workers will be unemployed.

(c) Suppose that the economy runs until some exogenous time $T$, at which point the distribution of reputation is $f(x)$. At this time, the social planner chooses an information disclosure policy in order to maximize employment at time $T$. A disclosure policy maps reputations into signals $s$. Given a signal $s$, firms have correct beliefs, i.e. Bayes' rule must hold. Feasible signal policies thus include releasing workers' full reputation, releasing no information about their history, or anywhere in between. What is the planner's optimal disclosure policy?