Homework 0: Comparative Statics

September 30, 2007

1. Profits equal $\pi = k^\alpha l^\beta - wk - rl$, where $\alpha, \beta \in (0,1)$ and $\alpha + \beta < 1$. Suppose there is an increase in r. How does this affect the optimal choices of capital, k, and labour, l?

2. A consumer faces prices p_t over time $t \in \{1, 2, \ldots\}$. The agent chooses her optimal purchase time τ to maximise utility $u(\theta, \tau) = \theta\delta^\tau - p_\tau$, where $\delta < 1$ and $\theta > 0$. How does the optimal purchase time vary with θ?

3. Consider a sealed bid auction, where all agents simultaneously submit bids, and the highest bidder wins and pays her bid. Consider the problem of agent 1. Suppose the highest bid of her opponents has cumulative distribution function $F(\cdot)$ on $[0, \infty)$, and the agent has value v. If she bids b, her expected utility is thus

$$u(v, b) = (v - b)F(b)$$

How does the agent's optimal bid change with v? [Hint: the objective function is not supermodular, but this is not necessary.]

4. Suppose $f(x,t)$ and $g(x,t)$ are supermodular. Show that $f(x,t) + g(x,t)$ is supermodular.

5. Suppose $f(x,t)$ and $g(x,t)$ are quasi–supermodular. Show, by example, that $f(x,t) + g(x,t)$ may not be quasi–supermodular.

6. Suppose $f(x,t)$ is supermodular and $g : \mathbb{R} \to \mathbb{R}$ is increasing. Show, by example, that $g(f(x,t))$ may not be supermodular.

7. Suppose $f(x,t)$ is quasi–supermodular and $g : \mathbb{R} \to \mathbb{R}$ is increasing. Show that $g(f(x,t))$ is quasi–supermodular.