

Homework 1: Moral Hazard

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Question 1

The following normal-linear model is regularly used in applied models. Given action $a \in \mathfrak{R}$, output is $q = a + x$, where $x \sim N(0, \sigma^2)$. The cost of effort is $g(a)$ is increasing and convex. The agent's utility equals $u(w(q) - g(a))$, while the principal's is $q - w(q)$. Suppose the agent's outside option is $u(0)$.

We make two large assumptions. First, the principal uses a linear contract:

$$w(q) = \alpha + \beta q$$

Second, the agent's utility is CARA, i.e., $u(w) = -e^{-w}$.

(a) Suppose $w \sim N(\mu, \sigma^2)$. Denote the certainty equivalent of w by \bar{w} , where

$$u(\bar{w}) = E[u(w)]$$

Show that $\bar{w} = \mu - \sigma^2/2$.

(b) Suppose effort is unobservable. The principal's problem is

$$\begin{aligned} & \max_{w(q), a} E[q - w(q)] \\ & \text{s.t.} \quad E[u(w(q) - g(a)) | a] \geq u(0) \\ & \quad a \in \operatorname{argmax}_{a' \in \mathfrak{R}} E[u(w(q) - g(a')) | a'] \end{aligned}$$

Using the first order approach, characterise the optimal contract (α, β, a) . [Hint: write utilities in terms of their certainty equivalent.]

(c) How would the solution change if the agent knows x before choosing his action (but after signing the contract)?

Question 2

Consider the same normal-linear model as in Question 1. After a is chosen, the principal observes output q and a signal y that is correlated with x . For example, if the agent is selling cars, then y could be the sales of the dealer next door. Let

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix} \right)$$

Suppose the principal uses the linear wage function:

$$w = \alpha + \beta(q + \gamma y)$$

Using the same approach as above, solve for the optimal value of γ . How does γ vary with σ_{xy} ? Provide an intuition.

Question 3

Consider the same normal-linear model as in Question 1. This time the principal employs N agents. The performance of agent i is given by

$$q_i = e_i + x_i + x_c$$

where (x_1, \dots, x_N, x_c) are independent and normally distributed with variances $(\sigma_1^2, \dots, \sigma_N^2, \sigma_c^2)$. Assume the principal offers a linear contract

$$w_i = \alpha_i + \beta_i \left(q_i - \sum_{j \neq i} \gamma_j z_j \right)$$

The principal's profit is given by $E[\sum_i (q_i - w_i)]$.

Solve for the optimal contract γ_j 's. Interpret the coefficients $\{\gamma_j\}_{j \neq i}$. What implications does this have for the incentives in teams?

Question 4

An agent has increasing, concave utility $u(\cdot)$. They start with wealth W_0 and may have an accident costing x of their wealth. Assume x is publicly observable. The agent has access to a perfectly competitive market of risk-neutral insurers who offer payments $R(x)$ net of any insurance premium. The distribution of x is as follows

$$f(0, a) = 1 - p(a) \quad (1)$$

$$f(x, a) = p(a)g(x) \quad \text{for } x > 0 \quad (2)$$

where $\int g(x)dx = 1$. The agent can affect the probability of an accident through their choice of a . The cost is given by increasing convex function, $\psi(a)$. The function $p(a)$ is decreasing and convex.

- (a) Suppose there is no insurance market. What action \hat{a} does the agent take?
- (b) Suppose a is contractible. Describe the first-best payment schedule $R(x)$ and the effort choice, a^* .
- (c) Suppose a is not contractible. Describe the second-best payment schedule $R(x)$.
- (d) Interpret the second-best payment schedule. Would the agent ever have an incentive to hide an accident? (i.e. report $x = 0$ when $x > 0$).

Question 5

Consider Holstrom's model of moral hazard in teams. N agents work in a team with joint output $x(a_1, \dots, a_N)$, where a_i is the effort of agent i and $g(a_i)$ is the increasing, convex cost function.

- (a) Show that by introducing a principal (agent $N + 1$) who does not participate in the production process, we can sustain an efficient effort profile as a Nash equilibrium using a differentiable balanced-budget output-sharing rule, i.e. $\sum_i t_i(x) = x$ ($\forall x$).
- (b) Suppose the principal can collude with one agent (call her agent k). That is, the colluders secretly write a side contract based on x to increase their joint payoff (other agents are unaware of the side contract). Show the scheme in (a) is susceptible to collusion.

(c) Suppose we restricted ourselves to differentiable output-sharing schemes that are invulnerable to collusion. Show that it is impossible to sustain the efficient effort profile.

Question 6

Suppose a risk neutral principal employs a risk averse agent. The two parties both sign a contract stating wage profile $w(q)$. The agent then chooses action $a \in A$ at cost $g(a)$.

Payoffs are as follows. The agent gets

$$u(w - g(a))$$

where $g(a)$ is increasing and convex. Utility is strictly increasing and strictly concave. The principal gets

$$q - w$$

The principal has reservation profit 0; the agent has reservation utility $u(0)$.

First, suppose the principal makes a TIOLI offer to the agent.

- (a) Assume the effort a is observable. Set up and solve the principal's optimal contract.
- (b) Assume effort a is not observable. Set up the principal's problem.

Next, suppose the agent makes a TIOLI offer to the principal.

- (c) Assume the effort a is observable. Show that the optimal contract induces the same effort as when the principal proposes the contract.
- (d) Assume effort a is not observable. Set up the agent's problem. Next, suppose that utility is CARA, i.e. $u(w) = -\exp(-w)$, which implies that $u(w+x) = u(w)e^{-x}$. Show that the optimal contract induces the same effort as when the principal proposes the contract (part (b)).