Homework 1: Moral Hazard

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Question 1 (Normal–Linear Model)

The following normal–linear model is regularly used in applied models. Given action $a \in \Re$, output is $q = a + x$, where $x \sim N(0, \sigma^2)$. The cost of effort is $g(a)$ is increasing and convex. The agent's utility equals $u(w(q) - g(a))$, while the principals is $q - w(q)$. Suppose the agent's outside option is $u(0)$.

We make two large assumptions. First, the principal uses a linear contract:

$$w(q) = \alpha + \beta q$$

Second, the agent's utility is CARA, i.e., $u(w) = -e^{-w}$.

(a) Suppose $w \sim N(\mu, \sigma^2)$. Denote the certainty equivalent of w by $\bar{w}$, where

$$u(\bar{w}) = E[u(w)]$$

Show that $\bar{w} = \mu - \sigma^2/2$.

(b) Suppose effort is unobservable. The principal's problem is

$$\begin{aligned}
\max_{w(q),a}\ & E[q - w(q)] \\
\text{s.t.}\quad & E[u(w(q) - g(a))|a] \geq u(0) \\
& a \in \text{argmax}_{a' \in \Re} E[u(w(q) - g(a'))|a']
\end{aligned}$$

Using the first order approach, characterise the optimal contract (α, β, a). [Hint: write utilities in terms of their certainty equivalent.]

(c) How would the solution change if the agent knows x before choosing his action (but after signing the contract)?

Question 2 (Signal of Effort)

Consider the same normal–linear model as in Question 1. After a is chosen, the principal observes output q and a signal y that is correlated with x. For example, if the agent is selling cars, then y could be the sales of the dealer next door. Let

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix} \right)$$

Suppose the principal uses the linear wage function:

$$w = \alpha + \beta(q + \gamma y)$$

Using the same approach as above, solve for the optimal value of γ. How does γ vary with σ_{xy}? Provide an intuition.

Question 3 (Insurance)

An agent has increasing, concave utility $u(\cdot)$. They start with wealth W_0 and may have an accident costing x of their wealth. Assume x is publicly observable. The agent has access to a perfectly competitive market of risk–neutral insurers who offer payments $R(x)$ net of any insurance premium. The distribution of x is as follows

$$f(0,a) = 1 - p(a) \tag{1}$$
$$f(x,a) = p(a)g(x) \quad \text{for} \quad x > 0 \tag{2}$$

where $\int g(x)dx = 1$. The agent can affect the probability of an accident through their choice of a. The cost is given by increasing convex function, $\psi(a)$. The function $p(a)$ is decreasing and convex. Utility is then given by $u(W_0 - x + R(x)) - \psi(a)$.

(a) Suppose there is no insurance market. What action $\hat{a}$ does the agent take?

(b) Suppose a is contractible. Describe the first–best payment schedule $R(x)$ and the effort choice, a^*.

(c) Suppose a is not contractible. Describe the second–best payment schedule $R(x)$.

(d) Interpret the second–best payment schedule. Would the agent ever have an incentive to hide an accident? (i.e. report $x = 0$ when $x > 0$).

Question 4 (Private Evaluations with Limited Liability)

A principal employs an agent. The game is as follows.

1. The agent privately chooses an action $a \in \{L, H\}$. The cost of this action is $g(a)$.
2. The principal *privately* observes output $q \sim f(q|a)$ on $[\underline{q}, \overline{q}]$. Assume this distribution function satisfies strict MLRP. That is,
$$\frac{f(q|H)}{f(q|L)}$$
is strictly increasing in q.
3. Suppose the principal reports that output is $\tilde{q}$. The principal then pays out $t(\tilde{q})$, while the agent receives $w(\tilde{q})$, where $w(\tilde{q}) \leq t(\tilde{q})$. The difference is burned. The payments $\langle t, q \rangle$ are contractible.

Payoffs are as follows. The principal obtains
$$q - t$$
The agent obtains
$$u(w) - g(a)$$
where $u(\cdot)$ is strictly increasing and concave, and $g(\cdot)$ is increasing and convex. The agent has no (IR) constraint, but does have limited liability. That is, $w(q) \geq 0$ for all q.

First, assume the principal wishes to implement $a = L$.

(a) Characterise the optimal contract.

Second, assume the principal wishes to implement $a = H$.

(b) Write down the principal's problem as maximising expected profits subject to the agent's (IC) constraint, the principal's (IC) constraint, the limited liability constraint and the constraint that $w(q) \leq t(q)$.

(c) Argue that $t(q)$ is independent of q.

(d) Characterise the optimal contract. How does the wage vary with q?

Question 5 (Teams and Collusion)

Consider Holstrom's model of moral hazard in teams. N agents work in a team with joint output $x(a_i, \ldots, a_N)$, where a_i is the effort of agent i and $g(a_i)$ is is the increasing, convex cost function.

(a) Show that by introducing a principal (agent $N+1$) who does not participate in the production process, we can sustain an efficient effort profile as a Nash equilibrium using a differentiable balanced–budget output–sharing rule, i.e. $\sum_i t_i(x) = x$ $(\forall x)$.

(b) Suppose the principal can collude with one agent (call her agent k). That is, the colluders secretly write a side contract based on x to increase their joint payoff (other agents are unaware of the side contract). Show the scheme in (a) is susceptible to collusion.

(c) Suppose we restricted ourselves to differentiable output–sharing schemes that are invulnerable to collusion. Show that it is impossible to sustain the efficient effort profile.

Question 6 (Bargaining Power)

Suppose a risk neutral principal employs a risk averse agent. The two parties both sign a contract stating wage profile $w(q)$. The agent then chooses action $a \in A$ at cost $g(a)$.

Payoffs are as follows. The agent gets

$$u(w - g(a))$$

where $g(a)$ is increasing and convex. Utility is strictly increasing and strictly concave. The principal gets

$$q - w$$

The principal has reservation profit 0; the agent has reservation utility $u(0)$.

First, suppose the principal makes a TIOLI offer to the agent.

(a) Assume the effort a is observable. Set up and solve the principal's optimal contract.

(b) Assume effort a is not observable. Set up the principal's problem.

Next, suppose the agent makes a TIOLI offer to the principal.

(c) Assume the effort a is observable. Show that the optimal contract induces the same effort as when the principal proposes the contract.

(d) Assume effort a is not observable. Set up the agent's problem. Next, suppose that utility is CARA, i.e. $u(w) = -\exp(-w)$, which implies that $u(w+x) = u(w)e^{-x}$. Show that the optimal contract induces the same effort as when the principal proposes the contract (part (b)).