Practice Problems 1: Moral Hazard

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Question 1 (Comparative Performance Evaluation)

Consider the same normal–linear model as in Question 1 of Homework 1. This time the principal employs N agents. The performance of agent i is given by

$$q_i = e_i + x_i + x_c$$

where $(x_1, \ldots, x_N, x_c)$ are independent and normally distributed with variances $(\sigma_1^2, \ldots, \sigma_N^2, \sigma_c^2)$. Assume the principal offers a linear contract

$$w_i = \alpha_i + \beta_i(q_i - \sum_{j \neq i} \gamma_j^i z_j)$$

The principal's profit is given by $E[\sum_i (q_i - w_i)]$.

Solve for the optimal $\{\gamma_j^i\}_{j,i}$. Interpret these coefficients. What implications does this have for the incentives in teams?

Question 2 (Moral Hazard and Option Contracts)

A principal (P) and an agent (A) play the following game.

1. P announces an option contract (T, B).
2. A accepts or rejects the contract. Rejection yields utility $\overline{U}$
3. A chooses effort e^A. This action is observable but not verifiable. Effort costs the agent e^A and yields revenue $R(e^A)$, where $R(\cdot)$ is increasing and concave.
4. P chooses whether to keep the project or sell it to the agent. If he keeps it, he pays the agent T. Payoffs are then

$$U_P = R(e_A) - T \qquad U_A = T - e_A$$

Alternatively, P sells the project to the agent for price B. Payoffs are then

$$U_P = B \qquad U_A = R(e_A) - B - e_A$$

Let e_A^* maximise $R(e_A) - e_A$. A contract is first–best if it implements e_A^* and yields the agent utility $U_A = \overline{U}$.

Let $B = R(e_A^*) - T$ and $T - e_A^* = \overline{U}$. Show this contract implements the first–best. Provide an intuition

Question 3 (Team problem)

Two agents, $i \in \{1, 2\}$, simultaneously choose effort $e_i \in \{0, 1\}$ on a project. Exerting effort costs costs c_i, where $c_1 + c_2 < 1$ and $1 - x > c_i$. The output produced is given by

		Agent 2: 1	Agent 2: 0
Agent 1	1	1	x
	0	x	0

Fix (c_1, c_2). Suppose agent i gets share β_i of the output, where $\beta_1 + \beta_2 \leq 1$. We say the efficient outcome can be *implemented* if there exists an equilibrium where both agents exert high effort.

(a) For which values of x do there exist (β_1, β_2) such that the efficient outcome can be implemented?

(b) Show there exist sharing rules (β_1, β_2) which only depend on (c_1, c_2) and implement the efficient outcome whenever it is implementable.

We say the efficient outcome can be *implemented in dominant strategies* if the high effort choice is a dominant strategy.

(c) For which values of x do there exist (β_1, β_2) such that the efficient outcome can be implemented in dominant strategies?

(d) Show there exist sharing rules (β_1, β_2) which only depend on (c_1, c_2) and implement the efficient outcome in dominant strategies whenever it is implementable.

Question 4 (Debt Contracts)

An entrepreneur has access to a project requiring one unit of capital. If taken, the project succeeds with probability p and produces output $R(p)$, or fails with probability $1-p$ and produces 0. The entrepreneur can costlessly choose $p \in [0,1]$. This choice is unobservable to investors.

The entrepreneur is risk neutral and has initial wealth $w \in [0,1]$. The entrepreneur must raise the additional capital by issuing debt to perfectly competitive risk neutral investors.[1] This debt is secured only by the assets of the project. Both the investors and the entrepreneur have available a safe investment paying an interest rate 0 if they do not invest.

(a) For $w \in [0,1]$, determine the equation that defines the equilibrium relationship between w and p. (Assume an interior solution for p).

(b) Let $R(p) = 5 - 4p$. If $w = 1$, what value of p would the entrepreneur choose? If instead, $w \in (\frac{7}{32}, 1)$, show there are 2 possible equilibrium choices for p. Which of these solutions is more reasonable? What happens if $w < \frac{7}{32}$?

(c) Let $R(p) = 5 - 4p$. Plot the entrepreneur's expected final wealth as a function of initial wealth $w \in [0,1]$. Discuss the effect of agency costs on the return to wealth.

Question 5 (Private Evaluations)

Consider the private evaluations model. As in class, assume there are finite outputs, the distribution of outputs obeys MLRP and the principal wishes to implement the high action.

Now, consider the following game:
1. The principal designs a contract $\langle t(q), w(q) \rangle$.
2. The principal sells the excess, $t - w$, to a third party for $E[t(q) - q(q)|H]$.
3. The agent exerts effort.
4. Output is realised. The agent obtains $w(q)$, the principal obtains $q - t(q)$ and the third party obtains $t(q) - w(q)$.

What is the principal's optimal contract?

[1] A debt contract states that the first D dollars from the project goes to the investors.