Practice Problems 1: Moral Hazard

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Question 1 (Comparative Performance Evaluation)

Consider the same normal–linear model as in Question 1 of Homework 1. This time the principal employs \( N \) agents. The performance of agent \( i \) is given by

\[
q_i = \epsilon_i + x_i + x_c
\]

where \((x_1, \ldots, x_N, x_c)\) are independent and normally distributed with variances \((\sigma_1^2, \ldots, \sigma_N^2, \sigma_c^2)\).

Assume the principal offers a linear contract

\[
w_i = \alpha_i + \beta_i(q_i - \sum_{j \neq i} \gamma_{ij}z_j)
\]

The principal’s profit is given by \( E[\sum_i (q_i - w_i)] \).

Solve for the optimal \( \{\gamma_{ij}\}_{j,i} \). Interpret these coefficients. What implications does this have for the incentives in teams?

Question 2 (Moral Hazard and Option Contracts)

A principal (P) and an agent (A) play the following game.

1. P announces an option contract \((T, B)\).
2. A accepts or rejects the contract. Rejection yields utility \( U \)
3. A chooses effort \( e_A \). This action is observable but not verifiable. Effort costs the agent \( e_A \) and yields revenue \( R(e_A) \), where \( R(\cdot) \) is increasing and concave.
4. P chooses whether to keep the project or sell it to the agent. If he keeps the project, he pays the agent \( T \) and payoffs are

\[
U_P = R(e_A) - T \quad U_A = T - e_A
\]
If P sells the project to the agent, he receives $B$ and payoffs are

$$U_P = B \quad U_A = R(e_A) - B - e_A$$

Let $e_A^*$ maximise $R(e_A) - e_A$. A contract is first–best if it implements $e_A^*$ and yields the agent utility $U_A = U$.

Let $B = R(e_A^*) - T$ and $T - e_A^* = U$. Show this contract implements the first–best. Provide an intuition

**Question 3 (Debt Contracts)**

An entrepreneur has access to a project requiring one unit of capital. If taken, the project succeeds with probability $p$ and produces output $R(p)$, or fails with probability $1 - p$ and produces 0. The entrepreneur can costlessly choose $p \in [0, 1]$. This choice is unobservable to investors.

The entrepreneur is risk neutral and has initial wealth $w \in [0, 1]$. The entrepreneur must raise the additional capital by issuing debt to perfectly competitive risk neutral investors.\(^1\) This debt is secured only by the assets of the project. Both the investors and the entrepreneur have available a safe investment paying an interest rate 0 if they do not invest.

(a) For $w \in [0, 1]$, determine the equation that defines the equilibrium relationship between $w$ and $p$. (Assume an interior solution for $p$).

(b) Let $R(p) = 5 - 4p$. If $w = 1$, what value of $p$ would the entrepreneur choose? If instead, $w \in (\frac{7}{32}, 1)$, show there are 2 possible equilibrium choices for $p$. Which of these solutions is more reasonable? What happens if $w < \frac{7}{32}$?

(c) Let $R(p) = 5 - 4p$. Plot the entrepreneur’s expected final wealth as a function of initial wealth $w \in [0, 1]$. Discuss the effect of agency costs on the return to wealth.

\(^1\)A debt contract states that the first $D$ dollars from the project goes to the investors.