Practice Problems 1: Moral Hazard

October 13, 2010

Question 1 (Comparative Performance Evaluation)

Consider the same normal–linear model as in Question 1 of Homework 1. This time the principal employs N agents. The performance of agent i is given by

$$q_i = e_i + x_i + x_c$$

where $(x_1, \ldots, x_N, x_c)$ are independent and normally distributed with variances $(\sigma_1^2, \ldots, \sigma_N^2, \sigma_c^2)$. Assume the principal offers a linear contract

$$w_i = \alpha_i + \beta_i(q_i - \sum_{j \neq i} \gamma_j^i z_j)$$

The principal's profit is given by $E[\sum_i (q_i - w_i)]$.

Solve for the optimal $\{\gamma_j^i\}_{j,i}$. Interpret these coefficients. What implications does this have for the incentives in teams?

Question 2 (Moral Hazard and Option Contracts)

A principal (P) and an agent (A) play the following game.

1. P announces an option contract (T, B).
2. A accepts or rejects the contract. Rejection yields utility $\overline{U}$
3. A chooses effort e^A. This action is observable but not verifiable. Effort costs the agent e^A and yields revenue $R(e^A)$, where $R(\cdot)$ is increasing and concave.
4. P chooses whether to keep the project or sell it to the agent. If he keeps the project, he pays the agent T and payoffs are

$$U_P = R(e_A) - T \qquad U_A = T - e_A$$

If P sells the project to the agent, he receives B and payoffs are

$$U_P = B \qquad U_A = R(e_A) - B - e_A$$

Let e_A^* maximise $R(e_A) - e_A$. A contract is first–best if it implements e_A^* and yields the agent utility $U_A = \overline{U}$.

Let $B = R(e_A^*) - T$ and $T - e_A^* = \overline{U}$. Show this contract implements the first–best. Provide an intuition

Question 3 (Debt Contracts)

An entrepreneur has access to a project requiring one unit of capital. If taken, the project succeeds with probability p and produces output $R(p)$, or fails with probability $1 - p$ and produces 0. The entrepreneur can costlessly choose $p \in [0, 1]$. This choice is unobservable to investors.

The entrepreneur is risk neutral and has initial wealth $w \in [0, 1]$. The entrepreneur must raise the additional capital by issuing debt to perfectly competitive risk neutral investors.[1] This debt is secured only by the assets of the project. Both the investors and the entrepreneur have available a safe investment paying an interest rate 0 if they do not invest.

(a) For $w \in [0, 1]$, determine the equation that defines the equilibrium relationship between w and p. (Assume an interior solution for p).

(b) Let $R(p) = 5 - 4p$. If $w = 1$, what value of p would the entrepreneur choose? If instead, $w \in (\frac{7}{32}, 1)$, show there are 2 possible equilibrium choices for p. Which of these solutions is more reasonable? What happens if $w < \frac{7}{32}$?

(c) Let $R(p) = 5 - 4p$. Plot the entrepreneur's expected final wealth as a function of initial wealth $w \in [0, 1]$. Discuss the effect of agency costs on the return to wealth.

[1] A debt contract states that the first D dollars from the project goes to the investors.

Question 4 (Credible Wage Paths)

There are two periods, with no discounting. The firm proposes a contract (w_0, w_s) which the agent accepts if the sum of period 1 and period 2 utilities exceeds $\overline{u}$ in expectation. Their utility function is given by the increasing, strictly concave function $u(\cdot)$.

In the first period the worker gets paid w_0 (if they accept the contract). They then produce q for the firm.

In the second period, the state of the world $s \in S$ is the realised with probability f_s. The firm offers w_s, while there is an outside offer, $\overline{w}_s$. The worker accepts the larger. If they work for the firm, the worker produces $q > \max_s \overline{w}_s$.

(a) The firms problem is to maximise two–period profits subject to the first–period and second–period (IR) constraints. Write down this problem.

(b) Characterise the optimal wage path. If s is the state of the economy, how are wage affected by slumps and booms?

(c) Suppose the agent can commit to his period 2 behaviour in period 1. Describe the optimal contract.