Practice Problems 1: Moral Hazard

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Question 1 (Comparative Performance Evaluation)

Consider the same normal–linear model as in Question 1 of Homework 1. This time the principal employs N agents. The performance of agent i is given by

$$q_i = e_i + x_i + x_c$$

where $(x_1, \ldots, x_N, x_c)$ are independent and normally distributed with variances $(\sigma_1^2, \ldots, \sigma_N^2, \sigma_c^2)$. Assume the principal offers a linear contract

$$w_i = \alpha_i + \beta_i(q_i - \sum_{j \neq i} \gamma_j^i z_j)$$

The principal's profit is given by $E[\sum_i (q_i - w_i)]$.

Solve for the optimal $\{\gamma_j^i\}_{j,i}$. Interpret these coefficients. What implications does this have for the incentives in teams?

Question 2 (Moral Hazard and Option Contracts)

A principal (P) and an agent (A) play the following game.

1. P announces an option contract (T, B).
2. A accepts or rejects the contract. Rejection yields utility $\overline{U}$
3. A chooses effort e^A. This action is observable but not verifiable. Effort costs the agent e^A and yields revenue $R(e^A)$, where $R(\cdot)$ is increasing and concave.
4. P chooses whether to keep the project or sell it to the agent. If he keeps the project, he pays the agent T and payoffs are

$$U_P = R(e_A) - T \qquad U_A = T - e_A$$

If P sells the project to the agent, he receives B and payoffs are

$$U_P = B \qquad U_A = R(e_A) - B - e_A$$

Let e_A^* maximise $R(e_A) - e_A$. A contract is first–best if it implements e_A^* and yields the agent utility $U_A = \overline{U}$.

Let $B = R(e_A^*) - T$ and $T - e_A^* = \overline{U}$. Show this contract implements the first–best. Provide an intuition

Question 3 (Debt Contracts)

An entrepreneur has access to a project requiring one unit of capital. If taken, the project succeeds with probability p and produces output $R(p)$, or fails with probability $1 - p$ and produces 0. The entrepreneur can costlessly choose $p \in [0, 1]$. This choice is unobservable to investors.

The entrepreneur is risk neutral and has initial wealth $w \in [0, 1]$. The entrepreneur must raise the additional capital by issuing debt to perfectly competitive risk neutral investors.[1] This debt is secured only by the assets of the project. Both the investors and the entrepreneur have available a safe investment paying an interest rate 0 if they do not invest.

(a) For $w \in [0, 1]$, determine the equation that defines the equilibrium relationship between w and p. (Assume an interior solution for p).

(b) Let $R(p) = 5 - 4p$. If $w = 1$, what value of p would the entrepreneur choose? If instead, $w \in (\frac{7}{32}, 1)$, show there are 2 possible equilibrium choices for p. Which of these solutions is more reasonable? What happens if $w < \frac{7}{32}$?

(c) Let $R(p) = 5 - 4p$. Plot the entrepreneur's expected final wealth as a function of initial wealth $w \in [0, 1]$. Discuss the effect of agency costs on the return to wealth.

[1] A debt contract states that the first D dollars from the project goes to the investors.

Question 4 (Credible Wage Paths)

There are two periods, with no discounting. The firm proposes a contract (w_0, w_s) which the agent accepts if the sum of period 1 and period 2 utilities exceeds $\overline{u}$ in expectation. Their utility function is given by the increasing, strictly concave function $u(\cdot)$.

In the first period the worker gets paid w_0 (if they accept the contract). They then produce q for the firm.

In the second period, the state of the world $s \in S$ is the realised with probability f_s. The firm offers w_s, while there is an outside offer, $\overline{w}_s$. The worker accepts the larger. If they work for the firm, the worker produces $q > \max_s \overline{w}_s$.

(a) The firms problem is to maximise two–period profits subject to the first–period and second–period (IR) constraints. Write down this problem.

(b) Characterise the optimal wage path. If s is the state of the economy, how are wage affected by slumps and booms?

(c) Suppose the agent can commit to his period 2 behaviour in period 1. Describe the optimal contract.

5. Motivating Information Acquisition

A potential house buyer (principal) hires a real estate broker (agent) to collect information about a house. The house has quality $q \in \{L, H\}$. A high quality house delivers utility 1 to the principal, a low quality house delivers utility -1 (this is net of the price paid). The prior is $\Pr(q = H) = \gamma$. Both the agent's and principal's utility are quasi-linear.

The agent invests effort e at cost $c(e)$ into observing a signal $s \in \{G, B\}$. The signal is informative with probability

$$\Pr(s = G|q = H) = \Pr(s = B|q = L) = \frac{1}{2} + e =: \eta(e)$$

The signal provides 'hard' information, so the agent cannot lie about the value of the signal. The cost function $c(e)$ is increasing and convex, and obeys $c'''(e) > 0$. To obtain internal optima assume that $c'(0) = 0$, $c''(0) = 0$ and $\lim_{e \to 1/2} c(e) = \infty$.

After observing the signal, the principal can choose to buy the house or not. If her decision to buy is independent of the signal, there is no reason to have the agent exert effort. Hence we assume the principal buys if $s = G$ and does not buy if $s = B$.

(a) Suppose the principal can observe the agent's effort choice. Show the welfare maximising effort satisfies the first order condition $c'(e) = 1$. [Here, welfare is the sum of the agents and principal's utility].

Now, consider the second best contract, where e is not observed by the principal. A contract consists of a wage $w_G \geq 0$ when the good signal is observed, and a wage $w_B \geq 0$ when the bad signal is observed [Note the limited liability constraint; there is no other (IR) constraint].

(b) Write down the agent's utility. Show the agent's optimal level of effort satisfies the first order condition $(w_G - w_B)(2\gamma - 1) = c'(e)$.

(c) Using the FOC in (b), what can we say about how the optimal wages change in the prior, γ? Is it always possible to motivate positive effort? Provide an intuition.

(d) Suppose $\gamma > 1/2$. Replacing the agent's (IC) constraint with their first-order condition, show the principal's optimal effort satisfies the first order condition

$$1 = c''(e)\left[e + \frac{1}{2(2\gamma - 1)}\right] + c'(e)$$

so that effort is increasing in γ.