Lecture Notes - Reputation Applications

Simon Board and Moritz Meyer-ter-Vehn

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1 Career Concerns

- Holmstrom (1999 - really early '80s)
- Background
  - Chicago: Why do we need contract theory? Reputational/Career Concerns will discipline manager
  - Holmstrom: Will it?
- How to model reputation?
  - Pure moral hazard: Equilibrium of a repeated game
  - Pure adverse selection: Learn about type
  - Moral hazard & Adverse Selection
    - Action affects learning about type
      - Crazy/Inept types
      - Signal jamming
    - Actions control inert type

1.1 Model

- 1 long-run manager & passive market
- Time $t = 1, 2, 3, \cdots$, discount rate β
- Realized output $y_t = \theta + a_t + \varepsilon_t$
  - $\theta \sim N(m_\theta, 1/h_0)$ manager's type (unknown to market)
  - a_t manager's effort (unobserved)

- $\varepsilon_t \sim N\left(0, 1/h_\varepsilon\right)$ error (unobserved)

- Competitive wage $w_t = \mathbb{E}\left[\theta|y^{t-1}\right] + a_t^*\left(y^{t-1}\right)$, where $a_t^*\left(y^{t-1}\right)$ expected effort in equilibrium
- Learning via $z_t = y_t - a_t^*\left(y^{t-1}\right) = \theta + \varepsilon_t + \left(a_t\left(y^{t-1}\right) - a_t^*\left(y^{t-1}\right)\right)$
  - Hamster analogy: Manager tries to pretend θ is high by choosing a_t high, but market takes account of this in equilibrium and subtracts $a_t^*\left(y^{t-1}\right)$ from evaluation
- Bayesian updating -> $\theta|y^t = \theta|z^t$ normal with
  - mean $\frac{h_0 m_\theta + h_\varepsilon \sum_{s=1}^t z_s}{h_0 + t h_\varepsilon}$
  - precision $h_0 + t h_\varepsilon$ ($\rightarrow \infty$, so θ is perfectly learnt asymptotically)
- Manager maximizes

$$\max_a \sum \beta^t \mathbb{E}\left[w_t - g\left(a_t\right)\right]$$

1.2 Two period model

1.2.1 Period 2

- $a_2 = 0$
- $w_2 = \mathbb{E}[\theta|y_1] = \frac{h_0 m_\theta + h_\varepsilon z_1}{h_0 + h_\varepsilon}$

1.2.2 Period 1

- Choose a_1 to maximize

$$w_1 - g(a_1) + \delta w_2 = w_1 - g(a_1) + \frac{\delta}{h_0 + h_\varepsilon}\left(h_0 m_\theta + h_\varepsilon z_1\right)$$

- FOC

$$g'(a_1) = \frac{\delta h_\varepsilon}{h_0 + h_\varepsilon}$$

1.3 Infinite period model

- FOC for one-step deviation after history y^{t-1}

$$\begin{aligned} MC\left(a_t\right) &= g'\left(a_t\right) \\ MB\left(a_t\right) &= \sum_{s=1}^{\infty} \beta^s \frac{\partial w_{t+s}}{\partial a_t} = \sum_{s=1}^{\infty} \beta^s \frac{h_\varepsilon}{h_0 + \left(t+s\right) h_\varepsilon} \end{aligned}$$

because

$$
\begin{aligned}
w_{t+s} &= \frac{h_0 m_\theta + h_\varepsilon \sum_{r=1}^{t+s} z_s}{h_0 + (t+s)h_\varepsilon} + a_{t+s}^* \\
z_t &= \theta + a_t + \varepsilon_t - a_t^* \left(y^{t-1}\right) - m_\theta \\
\frac{\partial w_{t+s}}{\partial a_t} &= \frac{h_\varepsilon}{h_0 + (t+s)\, h_\varepsilon}
\end{aligned}
$$

- One-step deviation justified because optimal a_t is independent of
  - past outcomes y^{t-1}
  - own type θ
- Note
  - can "overwork" initially if $\beta \approx 1$
  - eventually $MB\,(a_t) \to 0$ as θ becomes known, i.e. $h_0 + th_\varepsilon \to \infty$

1.4 Extensions

- Let type θ evolve with $\theta_t = \theta_{t-1} + \delta_t$ where $\delta_t \sim N\,(0, 1/h_\delta)$
- Market is learning moving target θ_t - never perfectly
- Steady state effort a_∞ inefficiently low $MB\,(a_\infty) < 1$
- Convergence from
  - above if uncertainty decreasing over time, i.e. $h_0 < h_\infty = f\,(h_\varepsilon, h_\delta)$
  - below if uncertainty increasing over time, i.e. $h_0 > h_\infty$

2 Reputation and Exit

- Bar-Isaac (2003)

2.1 Model

- Firm has fixed type $\theta \in \{H, L\}$ where $H = 1$ and $L = 0$
- Reputation of firm $x_t = \mathbb{E}_t[\theta]$; initially x_0
- Time continuous, interest rate r
- Market learns about θ via Brownian motion

$$dY_t = \theta dt + dW_t$$

- Bayes' rule
  - given θ
$$dx = x(1-x)[(\theta - x)dt + dW]$$
  - in expectation
$$dx = x(1-x) + dW$$
- Payoffs
  - Consumer's gross utility θ, expectation x_t
  - Firm charges x_t and has cost c
  - Firm value
$$V = \mathbb{E}\left[\int_0^T e^{-rt}(x_t - c)dt\right]$$
    if firm exits at time T
- When does firm exit?

2.2 Firm does not know own type

- Take-away: Some good firms exit; market may never learn type
- Evolution of value function

$$V(x) = (x-c)dt + (1 - rdt)\mathbb{E}_x\left[V(x+dx)\right]$$

- Ito's Lemma: If
$$dx = \gamma(x)dt + \sigma(x)dW$$
and $V(x)$ smooth, then
$$\mathbb{E}\left[dV(x)\right] = \gamma(x)V'(x)dt + \frac{1}{2}\sigma(x)^2V''(x)dt$$
- Hence
$$\mathbb{E}_x\left[V(x+dx)\right] = V(x) + \frac{1}{2}x^2(1-x)^2V''(x)dt$$
- Thus
$$rV(x) = (x-c) + \frac{1}{2}x^2(1-x)^2V''(x)$$
- Boundary conditions at exit threshold x^*
  - Value matching $V(x^*) = 0$
  - Smooth pasting $V'(x^*) = 0$; Idea
    * If $V'(x^*) > 0$, want to stay at x^*
    * Brownian motion goes $\pm\sigma\sqrt{\Delta t}$ over Δt
    * Then value from staying another Δt

$$V(stay) = \underbrace{\Delta t(x^* - c)}_{\text{current loss; order } -\Delta t} + (1 - rdt)(\underbrace{V(\sigma\sqrt{\Delta t})/2}_{\text{order } +\sqrt{\Delta t}} + \underbrace{V(-\sigma\sqrt{\Delta t})/2}_{=0})$$

2.3 Firm knows own type

- Staying in market is signal of quality
- Show that
  - Only low firms exit
  - Cannot have deterministic exit cutoff, where low firm exits with probability 1. Then belief $\to 1$, so low firms would deviate
  - Market eventually learns quality
- Claim: $V_H(x) = V_L(x)$ for all x
  - Idea: $x_t^H > x_t^L$ for all t and realizations of W_t
- Form of equilibrium

- At x^*, low firm quits with probability qdt
- High firm never quits
- q chosen such that reputation does not fall below x^* and low firm stays indifferent

3 Reputation for Competence

- Mailath, Samuelson (2001) "Who wants a Good Reputation"
- Standard reputational motive: Strategic type "mimics good type", i.e. Stackelberg, high productivity, ethical...
- Alternative model: Strategic type "distinguishes from bad type":
  - Strategic type θ_0 can exert effort $\eta \in \{L, H\}$ with $L \in (0, 1/2)$ and $H = 1 - L$
  - Incompetent type θ_L exerts effort $\eta = L$
  - Market posterior $q_t = \Pr\left(\theta_0 | h^{t-1}\right)$ that firm is strategic
- Imperfect monitoring through consumer utility
  - Success $\Pr(u = 1) = \eta$
  - Failure $\Pr(u = 0) = 1 - \eta$
- Players:
  - Long-lived firm with type $\theta \in \{\theta_0, \theta_L\}$
  - Short-lived consumers, 2 per period
- Strategies
  - Firm θ_0 exerts effort $\eta_t = \eta\left(h^{t-1}\right)$
  - Consumers "bid" $p\left(h^t\right) = \mathbb{E}\left[u | h^{t-1}, \widetilde{\eta}_t\right] = \mathbb{E}\left[\widetilde{\eta}_t | h^{t-1}\right]$
- Public history $h^t \in \{0, 1\}^{t-1}$, e.g. $(u_1 = 1, u_2 = 0, \cdots, u_{t-1} = 0)$
- Equilibrium $(\eta, \widetilde{\eta})$
  - Firm plays optimally: η maximizes $\mathbb{E}\left[\sum_{t=1}^{\infty} \beta^t \left(p\left(h^t\right) - c\left(\eta\left(h^t\right)\right)\right)\right]$
  - Consumers play optimally: $\widetilde{\eta} = \eta$
- Assume effort is
  - costly: $c(H) > c(L) = 0$
  - but 1st best: $c(H) - c(L) < H - L$

Proposition 1 *If $c(H)$ small, then "grim-trigger in beliefs" is an equilibrium*

$$\widetilde{\eta}\left(h^t\right)=\eta\left(h^t\right)=\begin{cases} H & \textit{if } h^t=(1,1,\cdots,1) \\ L & \textit{else} \end{cases}$$

Proof.

- Punishment phase $\widetilde{\eta}\left(h^t\right)=\eta\left(h^t\right)=L$ is an equilibrium with payoffs $(0,0)$
- Effort phase is equilibrium as long as

$$\begin{aligned} -c(H)+\frac{\beta}{1-\beta}\Pr(u=1|H)\,v &\geq \frac{\beta}{1-\beta}\Pr(u=1|L)\,v \\ c(H) &\leq \frac{\beta}{1-\beta}(H-L)\,v \end{aligned}$$

  where the firm equilibrium value v is given by $v=\frac{1}{1-\beta H}(H-c(H))$

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- But one idea of reputation models was to get away from "boot-strapped" equilibria
- Markovian equilibria $\eta(q),\widetilde{\eta}(q)$ where $q=\Pr(\theta_0)$
- In pure-strategy equilibrium posterior hops on a grid:

$$\frac{q\left(h^t\right)}{1-q\left(h^t\right)}=\frac{q\left(h^{t-1}\right)}{1-q\left(h^{t-1}\right)}*\begin{cases} 1 & \text{if } \widetilde{\eta}\left(h^{t-1}\right)=L \\ H/L & \text{if } \widetilde{\eta}\left(h^{t-1}\right)=H \text{ and } u_t=1 \\ L/H & \text{if } \widetilde{\eta}\left(h^{t-1}\right)=H \text{ and } u_t=0 \end{cases}$$

- Denote posteriors on grid by $q_t=q(z)$ for $z\in\mathbb{Z}$, where

$$z=\#\left(s\leq t:\widetilde{\eta}_s=H,u=1\right)-\#\left(s\leq t:\widetilde{\eta}_s=H,u=0\right)$$

- Note: Every posterior q with $\widetilde{\eta}(q)=L$ is absorbing and $v(q)=0$

Proposition 2 *There is a unique Markov-perfect equilibrium in pure strategies: $\eta\equiv L$*

Proof.

- Clearly, $\eta\equiv L$ is an equilibrium
- This is the only pure strategy equilibrium

1. Induction Base: $\eta(q(z)) = H$ for all $z \geq z^*$ is not an equilibrium: For $z \geq z^{**} \gg z^*$, the posterior will stay above z^* for too long to incentivize effort $\eta = H$; more formally $\Pr(\widetilde{\eta}_T = H | u_0 = \cdots = u_{T-1} = 0) \approx 1$ for T finite but large
2. Induction Step: If $\eta(q_z) = L$ then $\eta(q_{z-1}) = L$:

$$\begin{aligned} v(H, q_{z-1}) &= -c(H) + \beta(Hv(q_z) + Lv(q_{z-2})) \\ v(L, q_{z-1}) &= \beta(Lv(q_z) + Hv(q_{z-2})) \end{aligned}$$

so $v(H, q_{z-1}) < v(L, q_{z-1})$ because $v(q_z) = 0$ and $v(q_{z-2}) \geq 0$.

- Idea:
  - Reputation for competence q is only valuable in conjunction with $\widetilde{\eta}(q) = 1$
  - Reputation for commitment on contrary is more directly valuable

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- Paper then goes on to firms selling their name/reputation; this re-introduces uncertainty about θ and bounds q away from 0 and 1

4 Reputation Acquisition in Debt Markets

- Diamond (1989)
- T rounds
- Manager can invest \$1 in one of two projects
  - Good with certain payoff $G > 1 + r$
  - Bad with payoff
$$\begin{cases} B & \text{with prob. } \pi \\ 0 & \text{else} \end{cases}$$
    where $B > G$ but $\pi B < 1 + r$
- Three types of debtors
  - BG: can choose between B and G
  - B: always picks B
  - G: always picks G

- Combination of KW 82 and MS 01
  - separate yourself from B
  - mimic G
- In round t, debtor
  - gets loan at rate $r_t \in (r, G-1)$ (lender breaks even in eq.)
  - picks project B or G
  - privately observes payoff $0, G$, or B
    * repays $1 + r_t$ after G or B
    * defaults after 0 (game over)

Proposition 3 *For adequate parameter values, there is a reputation acquisition equilibrium, characterized by $\underline{t} < \bar{t} < T$ such that manager BG chooses project*

$$\begin{array}{ll} B & \text{at } t \leq \underline{t} \\ G & \text{at } t \in \left(\underline{t}, \bar{t}\right) \\ B, G & \text{at } t \geq \bar{t} \end{array}$$

- Initially
  - interest rate r_t is high because of B types in population
  - this reduces margin $G - r_t$ of good projects and gambling on bad projects with payoff $\pi (B - r_t)$ is profitable
- Then, if manager survives
  - as B types are sorted out interest rate falls
  - margin $G - r_t$ large enough to protect reputation by choosing G
- Eventually, in the endgame as $T - t$ becomes small effects like in Kreps, Wilson

5 Regulating a Firm with Reputational Concerns

- Atkeson, Hellwig, Ordonez (2010)
- Firm with moral hazard to produce experience good of high quality
- Reputational concerns provide incentives, but not perfectly
- How can regulation improve incentives?

5.1 Model

- Time $t \in [0, \infty)$
- Continuum of firms (but they don't really interact)
- Strategies: At time t
  - Enter and choose type θ
  - Exit (with rate bounded below)
- Type = Quality $\theta \in \{L, H\}$ chosen once and for all; constant; $L < 0, H > 0$
- Reputation $x_t \in \mathbb{E}\left[\theta | h^t\right]$
- Reputational Evolution
  - Three cases of learning:
    - Good news: Learn $\theta = H$ at rate μ
    - Bad news: Learn $\theta = L$ at rate μ
    - Brownian news: Learn from $dZ = \mu\theta dt + dW$
  - If $\theta = L$ exits at x^*, then $x_t \geq x^*$

5.1.1 Payoffs

- Price $p_t = x_t Y^{-\eta}$
  - Expected quality x_t
  - Marginal utility of quality $Y^{-\eta}$ (where Y total production and $-\eta$ funky macro stuff)

- Value function

$$V_\theta(x_0)\, Y^{-\eta}$$
$$V_\theta(x_0) \;=\; \mathbb{E}_{\theta,x_0}\left[\int_0^{Exit} e^{-rt} x_t dt\right]$$

- Free entry, but type $\theta = H$ costs $C > 0$

5.2 Equilibrium

- Value functions $V_\theta(x)$ pinned down by ODE
- Entry and exit at x^* with
$$V_L(x^*) = 0$$
- Indifference condition for investment
$$V_H(x^*)\, Y^{-\eta} = C$$
pins down $Y^{-\eta}$
- (Very weird: With fixed Y there is no equilibrium.... Why?)

5.2.1 Good News

- Low firm stuck at x^* -> $x^* = 0$; might as well exit
- High firm waits to jump to $x = H$
- $(V_H - V_L)(x)$ decreasing to 0
- (draw it)

5.2.2 Bad News

- Firms drift up from x^* -> $x^* < 0$
- Low firm fails eventually
- High firm drifts to $x = H$
- $(V_H - V_L)(x)$ increasing
- (draw it)

5.2.3 Brownian News

- $(V_H - V_L)(x)$ hump-shaped

5.3 Regulation

- If government can tax/subsidize based on reputation it can almost achieve first best
  - Choose $\widehat{x}$ close to H
  - Heavily subsidize firms with $x > \widehat{x}$
  - Heavily tax firms with $x < \widehat{x}$
  - Then get $x^* = \widehat{x} - \varepsilon$
  - (draw it)
- But what if it can only impose an entry fee F?
- Entry conditions

$$\begin{aligned} V_L(x^e) Y^{-\eta} &= F \\ V_H(x^e) Y^{-\eta} &= C + F \end{aligned}$$

- Twin goals
  - Quality: $x^e(F)$ increasing in F because

$$\frac{F}{F+C} = \frac{V_L(x^e)}{V_H(x^e)}$$

    which is increasing in all three cases -> so maximize F
  - Quantity:

$$Y = \sqrt[\eta]{(V_H(x_e) - V_L(x_e))/C}$$

    so maximize $V_H(x_e) - V_L(x_e)$

5.3.1 Bad news

- Two goals coincide
- Set $F = rC/\mu$ to achieve first-best quality $x^e = 1$ and first-best quantity

5.3.2 Good news

- Goals opposed
- May be optimal to set $F = 0$ so as not to decrease Y