Contract Theory: Final

10:00am–1:00pm, 6th December, 2011

1. Debt Contracts and Information Acquisition

An Entrepreneur seeks financing I from a competitive market of Investors. E has a project that pays off random return q that is observed by both players ex-post, but not before the contract $\langle r(q)\rangle$ is signed.

Unlike previous models, the project will take place whether or not the investor makes the investment. Rather, the funding is used to pay E, who gains utility αI. We assume $\alpha \geq 1$, so that investment is efficient.

If the investor makes the investment, she is repayed according to $r(q) \in [0, q]$. Hence her payoff is $U_I = r(q) - I$, while E's utility is $U_E = q - r(q) + \alpha I$. In the first-best contract, any feasible $r(q)$ such that $E[r(q)] = I$ maximises E's utility.

(a) Suppose that, just before they sign the contract, the investor can pay to observe the success of the project, q. Suppose we wish to choose $r(q)$ to minimize the incentive for the investor to acquire information (i.e. minimize the increase in their payoffs obtained by acquiring). Argue that the optimal contract has $r(q) \geq \min\{q, I\}$ and therefore that a debt contract is optimal (although not necessarily uniquely optimal).

(b) Now suppose that, just before they sign the contract, the Entrepreneur can pay to observe the success of the project, q. Suppose we wish to choose $r(q)$ to minimize the incentive for the entrepreneur to acquire information. Again, argue that a debt contract is optimal.

(c) Suppose the cost of information acquisition are c_E and c_I for the entrepreneur and investor. What is the highest level of investment I that is sustainable if we do not wish either party to acquire information?

[Note: If you get stuck, you might find it easier to work with $\alpha = 1$.]

2. Revenue Management

A firm has one unit of a good to sell over T periods. One agent enters each period and has a value drawn IID from $F(\cdot)$ on [0,1]. Agents values are privately known. The discount rate is $\delta \in (0,1)$.

First, assume there is no recall, so agent t leaves at the end of period t if they do not buy. A mechanism $\langle P_t, Y_t \rangle$ gives the probability of allocating the object in period t as a function of the reports until time t, and the corresponding payment. Agent t's utility is then given by

$$u_t = v_t \delta^t P_t - Y_t$$

and the firm's profits equal $\Pi = \sum_t Y_t$. Assume $\mathrm{MR}(v) = v - (1 - F(v))/f(v)$ is increasing.

(a) Argue that the firm's profit is given by

$$\Pi = E_0 \left[\sum_t \delta^t P_t \mathrm{MR}(v_t) \right]$$

where E_0 is the expectation at time 0, over all sequences of values. [Note: you don't have to be formal]

(b) What is the optimal allocation in period T? [Hint: it is easy to think in terms of cutoffs v_T^*, where the firm is indifferent between allocating the object and not]

(c) Using backwards induction, characterize the optimal cutoff in period $t < T$? What happens to the cutoffs over time?

Next, suppose there is recall. Hence agent t can buy in any period $\tau \geq t$. A mechanism $\langle P_{t,\tau}, Y_t \rangle$ gives the probability of allocating the object to agent t in period τ as a function of the reports until time τ, and the corresponding payment. Agent t's utility is then given by

$$u_t = \sum_{\tau \geq t} v_t \delta^\tau P_{t,\tau} - Y_t$$

and the firm's profits equal $\Pi = \sum_t Y_t$.

(d) Argue that the firm's profit is given by

$$\Pi = E_0 \left[\sum_t \sum_{\tau \geq t} \delta^\tau P_{t,\tau} \mathrm{MR}(v_t) \right]$$

(e) When there is recall, what is the optimal allocation in period T?

(f) Using backwards induction, what is the optimal allocation in period $t < T$? What happens to the cutoffs over time? [Hint: try $t = T - 1$ and $t = T - 2$, and the general case only if you have time]

3. Moral Hazard with Persistent Effort

An agent chooses effort $e \in \{e_L, e_H\}$ at time 0 at cost $c(e) \in \{0, c\}$. At time $t \in \{1, 2\}$, output $y_t \in \{y_1, \ldots, y_N\}$ is realized according to the IID distribution $\Pr(y_t = y_n | e) = f(y_n | e)$.

A contract is a pair of wages $\langle w_1(y_1), w_2(y_1, y_2) \rangle$. The agent's utility is then

$$u(w_1(y_1)) + u(w_2(y_1, y_2)) - c(e)$$

where $u(\cdot)$ is increasing and concave, while the firm's profits are

$$y_1 + y_2 - w_1(y_1) - w_2(y_1, y_2)$$

where we ignore discounting. The agent has outside option $2u_0$. Also, assume the principal wishes to implement effort e_H.

(a) What is the first best contract, assuming effort is observable? [Note: A formal derivation is not necessary].

(b) Suppose the firm cannot observe the agent's effort. Set up the firm's problem.

(c) Characterise the optimal first-period and second-period wages.

(d) Suppose output is binomial, $y_t \in \{y_L, y_H\}$. Let $f(y_H | e_L) = \pi_L$ and $f(y_H | e_H) = \pi_H$. How do wages vary over time? In particular, can you provide a full ranking of wages across the different states and time periods?