Economics 211A: Final

11:30am, Thursday, 13th December, 2007
Time limit: 3 hours

This test is closed book. There are four questions. Each is worth 25 points. Good luck.

Question 1 (Moral Hazard and Option Contracts)

A principal (P) and an agent (A) play the following game.

1. P announces an option contract (T, B).

2. A accepts or rejects the contract. Rejection yields utility $\overline{\overline{U}}$

3. A chooses effort e^A. This action is observable but not verifiable. Effort costs the agent e^A and yields revenue $R(e^A)$, where $R(\cdot)$ is increasing and concave.

4. P chooses whether to keep the project or sell it to the agent. If he keeps it, he pays the agent T. Payoffs are then

$$U_P = R(e_A) - T \qquad U_A = T - e_A$$

Alternatively, P sells the project to the agent for price B. Payoffs are then

$$U_P = B \qquad U_A = R(e_A) - B - e_A$$

Let e_A^* maximise $R(e_A) - e_A$. A contract is first–best if it implements e_A^* and yields the agent utility $U_A = \overline{U}$.

Let $B = R(e_A^*) - T$ and $T - e_A^* = \overline{U}$. Show this contract implements the first–best. Provide an intuition

Question 2 (Bilateral Trade)

Suppose two agents wish to trade a single good. The seller has privately known cost $c \sim g(\cdot)$ on $[0,1]$. The buyer has privately known value $v \sim f(\cdot)$ on $[0,1]$. These random variables are independent of each other. The agents' payoffs are

$$U_S = t - cp$$
$$U_B = vp - t$$

where $t \in \Re$ is a transfer and $p \in [0,1]$ is the probability of trade. If an agent abstains from trade, they receive 0.

In class, we showed that it is impossible to implement the ex–post efficient allocation. We now wish to find the welfare maximising mechanism.

(a) Consider the problem of a middleman who runs mechanism $\langle p(\tilde{v}, \tilde{c}), t_B(\tilde{v}, \tilde{c}), t_S(\tilde{v}, \tilde{c}) \rangle$ where t_B and t_S are the transfers from the buyer and to the seller respectively. Show that a middleman can make profit

$$\Pi = E\Big[[MR(v) - MC(c)]p(v,c)\Big] - U_B(\underline{v}) - U_S(\overline{c})$$

where

$$MR(v) = v - \frac{1 - F(v)}{f(v)} \quad \text{and} \quad MC(c) = c + \frac{G(c)}{g(c)}$$

(b) Maximise expected welfare subject to $\Pi = 0$. [Note: We have not shown that $\Pi = 0$ implies one can find a common transfer function $t(v, c)$. We leave this for another day.]

Question 3 (Private Evaluations with Limited Liability)

A principal employs an agent. The game is as follows.

1. The agent privately chooses an action $a \in \{L, H\}$. The cost of this action is $g(a)$.

2. The principal *privately* observes output $q \sim f(q|a)$ on $[\underline{q}, \bar{q}]$. Assume this distribution function satisfies strict MLRP. That is,

$$\frac{f(q|H)}{f(q|L)}$$

   is strictly increasing in q.

3. Suppose the principal reports that output is $\tilde{q}$. The principal then pays out $t(\tilde{q})$, while the agent receives $w(\tilde{q})$, where $w(\tilde{q}) \leq t(\tilde{q})$. The difference is burned. The payments $\langle t, q \rangle$ are contractible.

Payoffs are as follows. The principal obtains

$$q - t$$

The agent obtains

$$u(w) - g(a)$$

where $u(\cdot)$ is strictly increasing and concave, and $g(\cdot)$ is increasing and convex. The agent has no (IR) constraint, but does have limited liability. That is, $w(q) \geq 0$ for all q.

First, assume the principal wishes to implement $a = L$.

(a) Characterise the optimal contract.

Second, assume the principal wishes to implement $a = H$.

(b) Write down the principal's problem as maximising expected profits subject to the agent's (IC) constraint, the principal's (IC) constraint, the limited liability constraint and the constraint that $w(q) \leq t(q)$.

(c) Argue that $t(q)$ is independent of q.

(d) Characterise the optimal contract. How does the wage vary with q?

Question 4 (Sequential Screening with Different Priors)

At time I, a principal signs a contract $\langle q_1, t_1, q_2, t_2 \rangle$ with an agent for trade conducted at time II. At the time of contracting, the principal and agent are both uninformed of the agent's period II utility.

At time II, the state $s \in \{1, 2\}$ is revealed. The agent's utility in state s is $u_s(q) - t$. The cost to the principal of providing quantity q is $c(q)$ in both states. A contract $\langle q_1, t_1, q_2, t_2 \rangle$ then specifies the quantity $q \in \Re_+$ and transfer $t \in \Re$ in both states of the world. Assume that $u_1'(q) > u_2'(q)$ ($\forall q$). For technical simplicity, also assume that utility functions are increasing and concave, while the cost function is increasing and convex.

The agent and principal have different priors over the state. The principal is experienced and knows that state 1 will occur with probability p. The agent is mistaken, and believes that state 1 will occur with probability θ. Assume that $\theta > p$, so the agent is more confident than the principal.

(a) Suppose that the state s is publicly observable. The principal thus maximises her profit

$$\Pi = p[t_1 - c(q_1)] + (1 - p)[t_2 - c(q_2)]$$

subject to the individual rationality constraint of the agent,

$$\theta[u_1(q_1) - t_1] + (1 - \theta)[u_2(q_2) - t_2] \geq 0$$

Describe the principal's profit–maximising contract.

For the rest of this question, suppose the state s is only observed by the agent.

(b) Show that your optimal contract from (a) is not incentive compatible after the state has been revealed.

(c) Suppose the principal maximises her profit subject to individual rationality and incentive compatibility. Derive the optimal contract. [Hint: you can ignore one of the (IC) constraints and later show that it does not bind at the optimal solution].