# Economics 211A: Final

9:00am–12:00pm, 10th December, 2008

## 1. Durable Goods with New Entrants

A firm is selling a durable good over 3 periods. Each period mass 2 of agents enter the market: of these, mass 1 has valuation $L$ and mass 1 has valuation $H$, where $H > L$. An agent with valuation $v$ who consumes in period $t$ and pays price $p_t$ has utility

$$u_t = \delta^t (v - p_t)$$

The firm has zero costs and therefore makes profits

$$\pi_t = \delta^t p_t$$

for each mass of agents.

We make some parametric assumptions. First, suppose that $3L > H > 2L$. Second, suppose that $\delta \in (1/2, 1)$.

(a) Suppose the firm can commit to a sequence of prices. What is the optimal sequence? [Note: If you know the result, you can simply state it without derivation.]

The rest of the question assumes the firm has no commitment power: each period it names a price $p_t$, taking it's future behaviour as given. We wish to find the subgame perfect equilibria of this game.

(b) Suppose we are in period 3. Characterise the firm's pricing as a function of the number of high and low agents she faces.

(c) Suppose we are in period 2 and the firm faces mass (1,1) of (H,L) agents. What is the firm's optimal price? [Hint: calculate the highest price at which the firm can sell to the high types and the highest price at which the firm can sell to both types, and compare the profits from each.]

(d) Suppose we are in period 2 and the firm faces mass (1,2) of (H,L) agents. What is the firm's optimal price?

(e) Suppose we are in period 1 and the firm faces mass (1,1) of (H,L) agents. What is the firm's optimal price?

(f) Putting this together, what is the firm's optimal strategy? Can you provide an intuition behind this pricing policy?

## 2. Nonlinear Pricing and Risk Aversion

A firm sells to an agent of unknown type. The agent has valuation $\theta$ distributed according to $f(\theta)$ on $[\underline{\theta}, \overline{\theta}]$. A selling mechanism consists of a quantity and a price $\langle q(\theta), t(\theta)\rangle$. The agent's utility is

$$u(\theta, \hat{\theta}) = \theta q(\hat{\theta}) - t(\hat{\theta})$$

The firm is infinitely risk averse and wishes to maximise its utility in the worst possible state. Hence it chooses $\langle q(\theta), t(\theta)\rangle$ to maximise

$$\min_{\theta} \pi(\theta) = [t(\theta) - c(q(\theta))]$$

where the cost function is convex.

What is the firm's optimal mechanism?

[Aside: If you cannot solve the continuous type model, you may wish to try the two type model either for general convex costs or for $c(q) = q^2/2$.]

## 3. Debt Contracts

A risk neutral agent seeks funding from a risk neutral principal. The game is as follows:

1. The project requires investment $I$ from the principal.
2. The agent chooses effort $a \in \{L, H\}$ at cost $c(a)$. Assume $c(H) > c(L)$.
3. Output $q$ is realised. Assume $q$ takes values $\{q_1, \ldots, q_N\}$, where $q_{i+1} > q_i$. Output is distributed according to $f(q_i|a)$.

4. If $q_i$ is realised, the principal obtains payment $B_i$ and the agent obtains $q_i - B_i$. The agent's utility is $u = q_i - B_i - c(a)$; the principal's profit is $\pi = B_i - I$.

A contract specifies the payment to the principal as a function of the output $\langle B_i \rangle$. Assume the principal has outside option 0 and the agent makes a TIOLI offer to the principal. We also assume the contract satisfies feasibility (FE):

$$0 \leq B_i \leq q_i$$

and monotonicity (MON):

$$B_i \text{ is increasing in } i$$

Finally assume that $f(q_i|a)$ satisfies the monotone hazard rate principle (MHRP):

$$\frac{f(q_i|L)}{1 - F(q_i|L)} \geq \frac{f(q_i|H)}{1 - F(q_i|H)} \qquad \text{for each } q_i.$$

Assume the agent wishes to implement the high action. Show that a debt contract is optimal.

[Aside: In class we showed that MLRP implies debt contracts are optimal. The key insight is that, if we use the (MON) condition, we can use the weaker MHRP assumption.]