Economics 2102: Final

13 December, 2005

Question 1

[25 points] A principal employs an agent who privately observes the state of the world $\theta \in [\underline{\theta}, \overline{\theta}]$ which is distributed with density $f(\theta)$. The principal first makes a report to the principal who chooses an action $q \in \{1, 2\}$. Consider the following direct–revelation mechanism:

1. The principal commits to a mechanism $q(\hat{\theta}) \in \{1, 2\}$.
2. The agent observes the state θ.
3. The agent then sends a message to the principal $\hat{\theta}$.
4. The principal receives payoff $v(\theta, q)$ and the agent receive payoff $u(\theta, q)$.

(a) Suppose $u(\theta, q)$ is supermodular in that

$$u(\theta_H, q_H) + u(\theta_L, q_L) > u(\theta_H, q_L) + u(\theta_L, q_H)$$

for $\theta_H > \theta_L$ and $q_H > q_L$. Show incentive compatibility implies that $q(\theta)$ is increasing.

(b) Characterise the mechanism, $q(\cdot)$, that maximises the principal's expected payoff.

(c) Intuitively, what happens to the optimal mechanism as the principal's preferences converge to those of the agent's? That is, $v(\theta, q) \to u(\theta, q)$ in L^1.

Question 2

[25 points] Consider the following holdup game where the quantity traded is $q \in \{0, 1\}$. Suppose the agents sign a contract that gives the seller the option to sell $q = 0$ at price p_0 or $q = 1$ at price $p_1 = p_0 + k$. The game works as follows.

1. Investments are made simultaneously. The buyer invests $b \in \mathbb{R}$ and the seller invests $s \in \mathbb{R}$.
2. The state of nature θ is revealed.
3. The seller has the option to supply $q = 0$ at price p_0 or $q = 1$ at price $p_1 = p_0 + k$. The buyer makes a TIOLI renegotiation offer to the seller.
4. Payoffs are $v(b, \theta)q - p - b$ for the buyer and $p - c(s, \theta)q - s$ for the seller, where p is the traded price.

Assumptions:

- $v(b, \theta)$ is concave in b. $c(s, \theta)$ is convex in s.
- v, u, s, b, θ are observable but not verifiable.
- There exists states θ such that $c(s, \theta) > v(b, \theta)$ and $c(s, \theta) < v(b, \theta)$.

(a) Define the first–best investment for the buyer and seller.

(b) What are the seller's payoffs after renegotiation? [Note, this will depend on whether or not $c(s, \theta) > k$].

(c) Write down the seller's investment problem.

(d) Show that there exists a choice of k such that the seller chooses the first best investment.

(e) Show that under the optimal k the buyer also chooses the first–best investment level.

Question 3

[25 points] Suppose a buyer invests b at cost $c(b)$, where $c(\cdot)$ is increasing and convex. Investment b induces a stochastic valuation v for one unit of a good. The valuation is observed by the buyer and is distributed according to $f(v|b)$.

The seller then makes a TIOLI offer to the buyer of a price p. The buyer accepts or rejects.

(a) First suppose the seller observes v. How much will the buyer invest?

For the rest of the question, suppose that the seller observes neither b nor v. Assume that buyer's and seller's optimisation problems are concave.

(b) Assume $f(v|b)$ satisfies the hazard rate order in that

$$\frac{f(v|b)}{1-F(v|b)} \quad \text{decreases in } b \tag{HR}$$

Derive the seller's optimal price. How does the optimal price vary with b?

(c) Derive the buyer's optimal investment choice. Notice that (HR) implies that $F(v|b)$ decreases in b. How does the optimal investment vary with the expected price, p?

(d) Argue that there will be a unique Nash equilibrium in (b,p) space.

(e) How does the level of investment differ from part (a)? Why?

Question 4

[25 points] A firm employs an agent who is risk–neutral, but has limited liability (i.e. they cannot be paid a negative wage). There is no individual rationality constraint. The agent can choose action $a \in \{L, H\}$ at cost $\{0, c\}$. There are two possible outputs $\{q_L, q_H\}$. The high output occurs with probability p_L or p_H if the agent takes action L or H, respectively. The agent's payoff is

$$w - c(a)$$

where w is the wage and $c(a)$ the cost of the action. The principal's payoff is

$$q - w$$

where q is the output and w is the wage.

(a) Characterise the optimal wages and action.

Suppose there are two types of agents, $i \in \{1, 2\}$. The principal cannot observe an agent's type but believes the probability of either type is 1/2. The agents are identical except for their cost of taking the action: for agent $i \in \{1, 2\}$ the cost of $a \in \{L, H\}$ is $\{0, c^i\}$, where $c^2 > c^1$.

(b) What are the optimal wages if the principal wishes to implement $\{a^1, a^2\} = \{L, L\}$?

(c) What are the optimal wages if the principal wishes to implement $\{a^1, a^2\} = \{H, H\}$?

(d) What are the optimal wages if the principal wishes to implement $\{a^1, a^2\} = \{L, H\}$?

(e) What are the optimal wages if the principal wishes to implement $\{a^1, a^2\} = \{H, L\}$?