# Practice Problems 1: Moral Hazard 

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## 1. Credible Wage Paths

There are two periods, with no discounting. The firm proposes a contract ( $w_{0}, w_{s}$ ) which the agent accepts if the sum of period 1 and period 2 utilities exceeds $\bar{u}$ in expectation. Their utility function is given by the increasing, strictly concave function $u(\cdot)$.

In the first period the worker gets paid $w_{0}$ (if they accept the contract). They then produce $q$ for the firm.

In the second period, the state of the world $s \in S$ is the realised with probability $f_{s}$. The firm offers $w_{s}$, while there is an outside offer, $\bar{w}_{s}$. The worker accepts the larger. If they work for the firm, the worker produces $q>\max _{s} \bar{w}_{s}$.
(a) The firms problem is to maximise two-period profits subject to the first-period and secondperiod (IR) constraints. Write down this problem.
(b) Characterise the optimal wage path. If $s$ is the state of the economy, how are wage affected by slumps and booms?
(c) Suppose the agent can commit to his period 2 behaviour in period 1. Describe the optimal contract.

## 2. Short-term and long-term contracts

Suppose there are three periods, $t \in\{1,2,3\}$. Each period a principal and an agent must share a good; let $x_{t} \in \mathbb{R}$ be the share obtained by the agent. The principal gets $\sum_{t} \pi_{t}\left(x_{t}\right)$ and the agent gets $\sum_{t} u_{t}\left(x_{t}\right)$, where $\pi_{t}\left(x_{t}\right)$ is decreasing in $x_{t}$ and $u_{t}\left(x_{t}\right)$ is increasing in $x_{t}$. The agent's outside option is a share of the assets $\left(\underline{x}_{1}, \underline{x}_{2}, \underline{x}_{3}\right)$.
(a) Suppose the principal can write a long term contract. Write down the program of maximising profit subject to individual rationality.
(b) Now suppose the principal offered a spot contract each period. Using backwards induction derive the optimal sequence of spot contracts. Explain why this may differ from the long-term contract.
(c) Suppose the principal offers two-period contracts. In the first period they offer $\left({ }_{1} x_{1},{ }_{1} x_{2}\right)$. If it is rejected the agent gets $\underline{x}_{1}$. At the start of the second period a new contract $\left({ }_{2} x_{2},{ }_{2} x_{3}\right)$ may be proposed by the principal. If this is rejected the agent gets ${ }_{1} x_{2}$ if they accepted the first contract or $\underline{x}_{2}$ otherwise. In the third period a spot contract is offered to the agent. If this is rejected, the agent gets ${ }_{2} x_{3}$ if they accepted the second contract, or $\underline{x}_{3}$ otherwise. Show that if $\lim _{x \rightarrow-\infty} u_{t}(x)=-\infty$ and $\lim _{x \rightarrow \infty} u_{t}(x)=\infty$ then this can implement the optimal long term contract.
(d) Provide an example (outside options, utility functions, profit function) where the two-period contracts cannot implement the long-term contract.

