Practice Problems 1: Moral Hazard

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1. Credible Wage Paths

There are two periods, with no discounting. The firm proposes a contract (w_0, w_s) which the agent accepts if the sum of period 1 and period 2 utilities exceeds \overline{u} in expectation. Their utility function is given by the increasing, strictly concave function $u(\cdot)$.

In the first period the worker gets paid w_0 (if they accept the contract). They then produce q for the firm.

In the second period, the state of the world $s \in S$ is the realised with probability f_s . The firm offers w_s , while there is an outside offer, \overline{w}_s . The worker accepts the larger. If they work for the firm, the worker produces $q > \max_s \overline{w}_s$.

(a) The firms problem is to maximise two-period profits subject to the first-period and second-period (IR) constraints. Write down this problem.

(b) Characterise the optimal wage path. If s is the state of the economy, how are wage affected by slumps and booms?

(c) Suppose the agent can commit to his period 2 behaviour in period 1. Describe the optimal contract.

2. Short–term and long–term contracts

Suppose there are three periods, $t \in \{1, 2, 3\}$. Each period a principal and an agent must share a good; let $x_t \in \mathbb{R}$ be the share obtained by the agent. The principal gets $\sum_t \pi_t(x_t)$ and the agent gets $\sum_t u_t(x_t)$, where $\pi_t(x_t)$ is decreasing in x_t and $u_t(x_t)$ is increasing in x_t . The agent's outside option is a share of the assets $(\underline{x}_1, \underline{x}_2, \underline{x}_3)$.

(a) Suppose the principal can write a long term contract. Write down the program of maximising profit subject to individual rationality.

(b) Now suppose the principal offered a spot contract each period. Using backwards induction derive the optimal sequence of spot contracts. Explain why this may differ from the long-term contract.

(c) Suppose the principal offers two-period contracts. In the first period they offer $(_1x_1, _1x_2)$. If it is rejected the agent gets \underline{x}_1 . At the start of the second period a new contract $(_2x_2, _2x_3)$ may be proposed by the principal. If this is rejected the agent gets $_1x_2$ if they accepted the first contract or \underline{x}_2 otherwise. In the third period a spot contract is offered to the agent. If this is rejected, the agent gets $_2x_3$ if they accepted the second contract, or \underline{x}_3 otherwise. Show that if $\lim_{x\to-\infty} u_t(x) = -\infty$ and $\lim_{x\to\infty} u_t(x) = \infty$ then this can implement the optimal long term contract.

(d) Provide an example (outside options, utility functions, profit function) where the two–period contracts cannot implement the long–term contract.