PARTIAL EQUILIBRIUM
Welfare Analysis

[See Chap 12]

Welfare Analysis

- We would like welfare measure.
- Normative properties of competitive markets.
  - First welfare theorem.
- Use to analyze policies
  - What is effect of tax?
  - What is effect of price control?
  - What is effect of banning imports?

Pareto Efficiency

- An allocation is Pareto efficient if there is no other allocation that makes everyone else better off.
- Weak notion of efficiency.
  - Necessary condition for desirable allocation.
  - May not be sufficient: If one agent has everything, this is Pareto efficient.

Welfare Measures

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Consumer Surplus

- Suppose utility is quasi-linear

$$u_j(x_1,x_2) = v_j(x_1) + x_2$$

  - We are interested in good 1
  - Think of good 2 as "money on everything else".
- Individual consumer surplus is area under Marshallian demand function.
  - See consumer surplus notes.

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Consumer Surplus

- Suppose there is amount X_1 of good 1.
  - Divide X_1 between J agents.
  - Allocations $\{x_1^1,x_1^2,\ldots,x_1^J\}$.
- In any Pareto efficient allocation, the social planner wishes to maximize $\sum_j v_j(x_1^j)$.
  - Suppose good is given to agent 1 (value \$10) and not agent 2 (value \$20).
  - Everyone better off if give good to agent 2, and transfer \$15 from agent 2 to agent 1.
- Hence aggregate consumer surplus is area under aggregate demand function.

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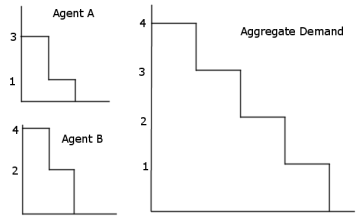
Consumer Surplus

- Agent A has values {3,1}, B has values {4,2}.
- CS is sum of values minus price.



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Producer Surplus

- The producer surplus of a single firm equals its profit.
  - Profit equals [AC(q)-p]q
  - Profit equals area over MC curve (net of fixed costs).
  - Ignore fixed costs since don't affect welfare comparisons.
- Aggregate producer surplus
  - In any Pareto Efficient allocation, social planner wishes to minimize $\sum_k c_k(q^k)$.
  - Hence aggregate producer surplus equals area over the market supply function.

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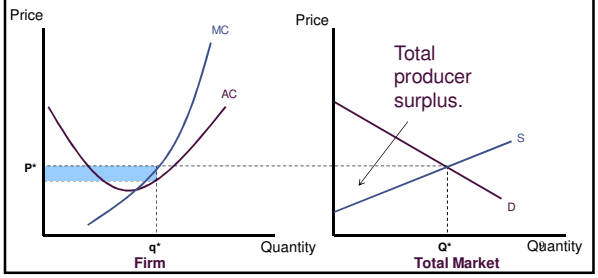
Producer Surplus

Producer surplus of a typical firm and market.



Total Welfare

- The area between the demand and supply curve equals the sum of CS and PS.
  - Measures the value of agents/firms from being able to make market transactions.
- First Welfare Theorem: Any competitive equilibrium is Pareto efficient.
  - PS+CS maximized in a competitive equilibrium.
  - Trade occurs if and only if the marginal utility exceeds the marginal cost.

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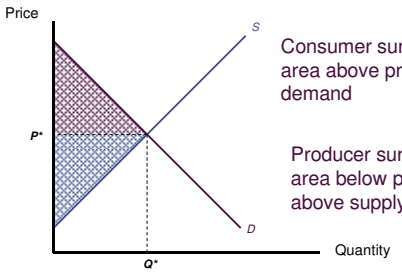
First Welfare Theorem



Consumer surplus is the area above price and below demand

Producer surplus is the area below price and above supply

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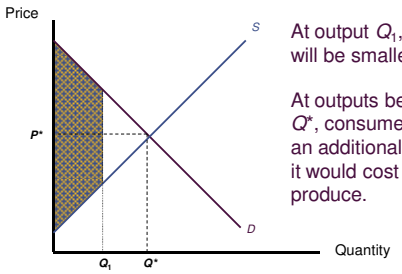
First Welfare Theorem



At output Q_1, total surplus will be smaller

At outputs between Q_1 and Q^*, consumers would value an additional unit more than it would cost suppliers to produce.

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Interpretation

- In any Pareto efficient allocation, social planner maximizes CS plus PS.
- If Q<Q*, then everyone can be made better off by increasing Q.
  - Does not mean everyone is necessarily better off.
  - Need transfers to redistribute money.
- Interpret CS + PS as gains from trade than can be distributed between agents and firms.

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Welfare Loss Computations

- We can use CS and PS to explicitly calculate the welfare losses caused by restrictions on voluntary transactions
  - In general, we have to integrate the area between demand and supply.
  - With linear demand and supply, the calculation is simple because the areas are triangular.

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Welfare Loss: Example

- Suppose that the demand is given by

$$Q_D = 10 - P$$

and supply is given by

$$Q_S = P - 2$$

- Market equilibrium occurs where $P^* = 6$ and $Q^* = 4$

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Welfare Loss: Example

- Restriction of output to $Q_0 = 3$ would create a gap between what demanders are willing to pay (P_D) and what suppliers require (P_S)

$$P_D = 10 - 3 = 7$$

$$P_S = 2 + 3 = 5$$

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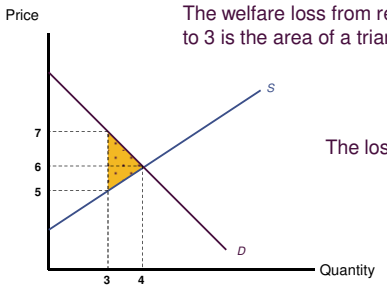
Welfare Loss: Example

The welfare loss from restricting output to 3 is the area of a triangle



The loss = (0.5)(2)(1) = 1

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Welfare Loss Computations

- The welfare loss is shared by producers and consumers
- The elasticity of demand and elasticity of supply to determine who bears the larger portion of the loss
  - the side of the market with the smallest price elasticity (in absolute value)

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Application: Price Controls

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Price Controls and Shortages

- Sometimes governments seek to control prices at below equilibrium levels.
  - This will lead to a shortage
- We can analyze impact on welfare
  - Price floor will lead to forgone transactions.
  - Welfare loss since these transactions would benefit consumers and producers.

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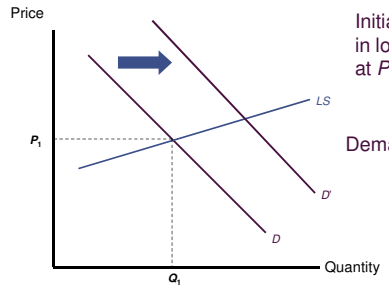
Price Controls and Shortages



Initially, the market is in long-run equilibrium at P_1, Q_1

Demand increases to D'

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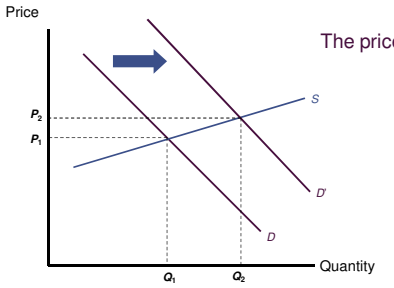
Price Controls and Shortages



The price rises to P_2

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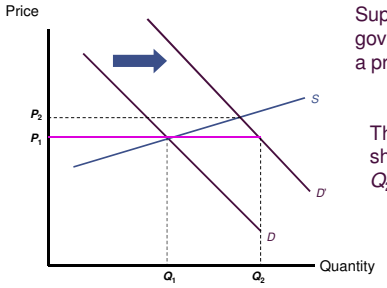
Price Controls and Shortages



Suppose that the government imposes a price ceiling at P_1

There will be a shortage equal to Q_2 - Q_1

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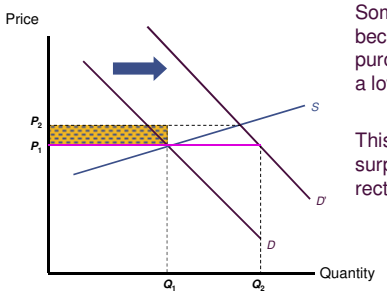
Price Controls and Shortages



Some buyers will gain because they can purchase the good for a lower price

This gain in consumer surplus is the shaded rectangle

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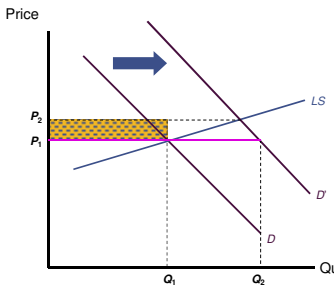
Price Controls and Shortages



The gain to consumers is also a loss to producers who now receive a lower price

The shaded rectangle therefore represents a pure transfer from producers to consumers

No welfare loss there

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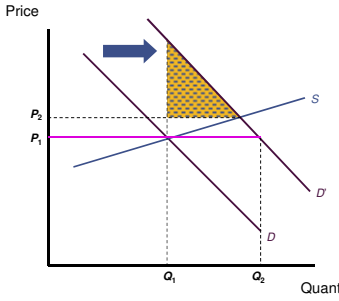
Price Controls and Shortages



Assume: (a) the Q_1 goods go to the agents with the highest values, (b) no resources wasted in competing for goods. This gives lower bound on welfare loss.

This shaded triangle represents the value of additional consumer surplus that would have been attained without the price control

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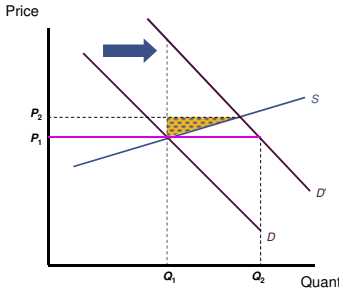
Price Controls and Shortages



This shaded triangle represents the value of additional producer surplus that would have been attained without the price control.

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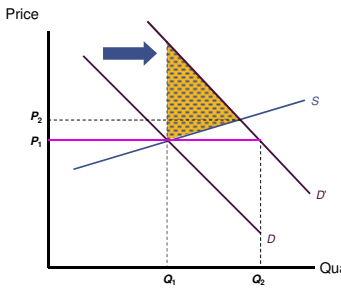
Price Controls and Shortages



This shaded area represents the value of mutually beneficial transactions that are prevented by the government

This is a measure of the welfare costs of this policy

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Bigger Picture

- Static model
  - Price floor causes welfare loss since firms do not supply enough.
- Argentina's agriculture
  - Government tries to force firms to raise Q.
  - Firms make loss and exit
- Rent control
  - Reduces investment in housing stock.
- Drug control (like price floor of ∞)
  - Black markets

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APPLICATION: TAXES

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Tax Incidence

- To discuss the effects of a per-unit tax (t), we need to make a distinction between the price paid by buyers (P_D) and the price received by sellers (P_S)

$$P_D - P_S = t$$

- Who pays the taxes is irrelevant. E.g.,
  - Income tax of 10% (paid by workers)
  - Payroll tax of 10% (paid by firms)

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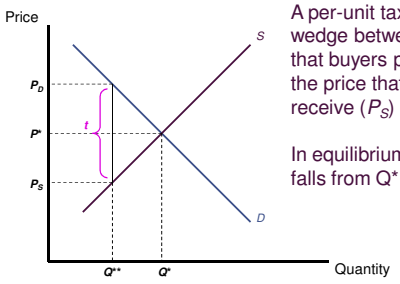
Tax Incidence



A per-unit tax creates a wedge between the price that buyers pay (P_D) and the price that sellers receive (P_S)

In equilibrium, quantity falls from Q* to Q**.

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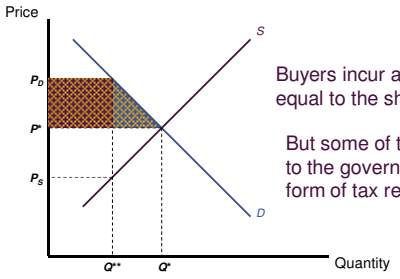
Tax Incidence



Buyers incur a welfare loss equal to the shaded area

But some of this loss goes to the government in the form of tax revenue

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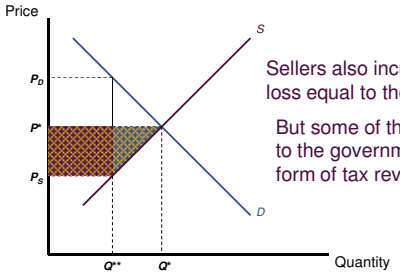
Tax Incidence



Sellers also incur a welfare loss equal to the shaded area

But some of this loss goes to the government in the form of tax revenue

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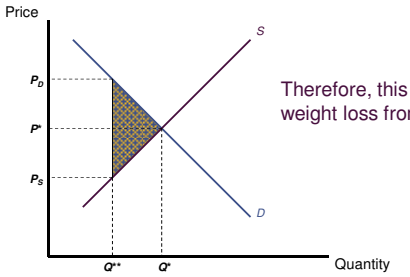
Tax Incidence



Therefore, this is the dead-weight loss from the tax

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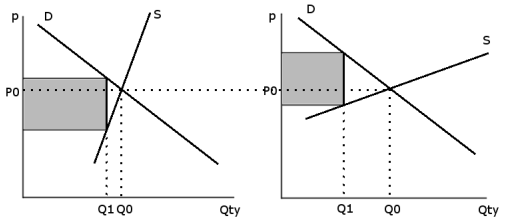
Tax Incidence

- Do consumers or producers lose more?



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Tax Incidence

- Suppose there is small change in tax, dt.
- Prices change so that

$$dP_D - dP_S = dt$$

- In equilibrium, supply equals demand. Hence

$$dD = dS$$

- Differentiating,

$$D'(P)dP_D = S'(P)dP_S$$

- Substituting, for dP_S we get

$$D'(P)dP_D = S'(P)(dP_D - dt)$$

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Tax Incidence

- We can now solve for the effect of the tax on P_D:

$$\frac{dP_D}{dt} = \frac{S'(P)}{S'(P) - D'(P)} = \frac{e_S}{e_S - e_D}$$

where e_S is the price elasticity of supply and e_D is the price elasticity of demand,

- Similarly, if we solve for dP_S,

$$\frac{dP_S}{dt} = \frac{D'(P)}{S'(P) - D'(P)} = \frac{e_D}{e_S - e_D}$$

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Tax Incidence

- Since $e_D \leq 0$ and $e_S \geq 0$, $dP_D/dt \geq 0$ and $dP_S/dt \leq 0$
- If demand is perfectly inelastic ($e_D = 0$), the tax is completely paid by consumers.
- If demand is perfectly elastic ($e_D = \infty$), the tax is completely paid by suppliers.
- In general, the side with the more elastic responses will experience less of the price change

$$-\frac{dP_S / dt}{dP_D / dt} = -\frac{e_D}{e_S}$$

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Deadweight Loss

- We showed taxes induce deadweight losses
  - the size of the losses will depend on the elasticities of supply and demand
- Start from tax t=0.
- The deadweight loss is given by the triangle. This area equals

$$DW = 0.5(dt)(dQ)$$

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Deadweight Loss

- Suppose tax is small, so use local approx.
- From the definition of elasticity, we know that

$$dQ = e_D dP_D \cdot Q^*/P^*$$

where Q* is qty before tax, and P* is price.

- Tax incidence equation says $dP_D = e_S/(e_S - e_D)dt$.
- Substituting, $dQ = e_D[e_S/(e_S - e_D)]\, t\, Q^*/P^*$
- Substituting, we get

$$DW = -0.5\left(\frac{dt}{P_0}\right)^2 [e_D e_S/(e_S - e_D)]P^* Q^*$$

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Deadweight Loss

- Deadweight losses are smaller in situations where e_D or e_S are small
  - Deadweight losses are zero if either e_D or e_S are zero
  - The tax does not alter the quantity of the good that is traded
- Deadweight loss is proportional to dt^2.
  - Loss small when tax small, since lose low value transactions.
  - Loss large when tax big, since lose high value transactions.

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Transactions Costs

- Transactions costs create a wedge between the price the buyer pays and the price the seller receives
  - real estate agent fees
  - broker fees for the sale of stocks
- These can be modeled as taxes
  - Middleman gains area labeled "government revenue"
- Costs are shared by the buyer and seller
  - Who pays depends on elasticities

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Application: International Trade

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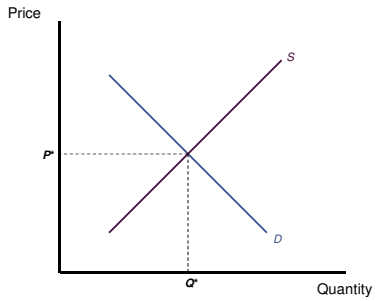
Gains from International Trade



Consider a small country.

In the absence of international trade, the domestic equilibrium price is *P** and the domestic equilibrium quantity is *Q**

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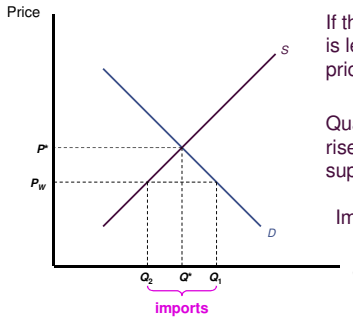
Gains from International Trade



If the world price (P_W) is less than the domestic price, the price falls to P_W

Quantity demanded rises to Q_1 and quantity supplied falls to Q_2

Imports = Q_1 - Q_2

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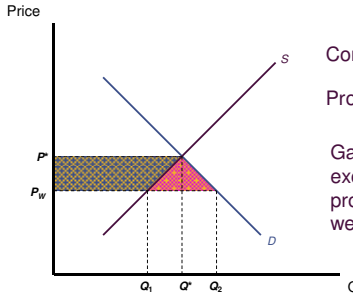
Gains from International Trade



Consumer surplus rises

Producer surplus falls

Gain to consumers exceeds loss to producers so overall welfare rises.

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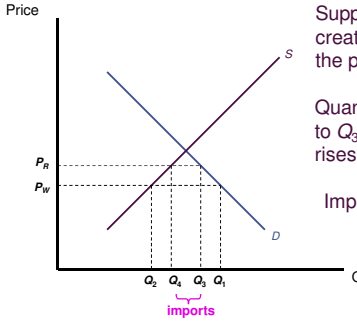
Effects of a Tariff



Suppose the government creates a tariff that raises the price to P_R

Quantity demanded falls to Q_3 and quantity supplied rises to Q_4

Imports are now Q_3 - Q_4

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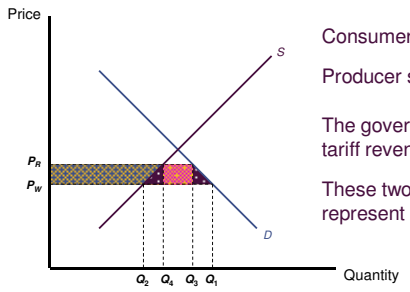
Effects of a Tariff



Consumer surplus falls

Producer surplus rises

The government gets tariff revenue

These two triangles represent deadweight loss

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Estimating Deadweight Loss

- We can estimates of the size of the welfare loss triangles.
- Suppose tariff is a percentage, so $P_R = (1+t)P_W$.
- Elasticity of demand: $e_D = (P/Q)(\Delta Q/\Delta P)$.
- Letting $\Delta Q = Q_3 - Q_1$ and $\Delta P = P_R - P_W$,

$$Q_3 - Q_1 = \frac{P_R - P_W}{P_W} \cdot e_D \cdot Q_1 = te_D \cdot Q_1$$

where we use $\Delta P = tP_W$.

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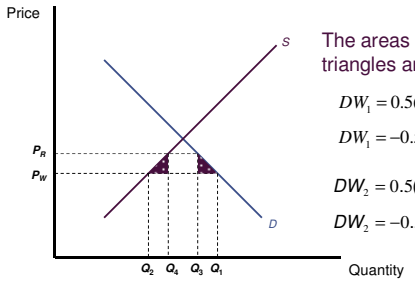
Estimating Deadweight Loss



The areas of these two triangles are

$$DW_1 = 0.5(P_R - P_W)(Q_1 - Q_3)$$

$$DW_1 = -0.5t^2 e_D P_W Q_1$$

$$DW_2 = 0.5(P_R - P_W)(Q_4 - Q_2)$$

$$DW_2 = -0.5t^2 e_S P_W Q_2$$

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Example

- Market demand is

$$D = 200/P$$

- Market supply curve is

$$S = 2P,$$

- Domestic equilibrium is $P^* = 10$ and $Q^* = 20$
- World price is $P_W = 8$,
  - Demand is D = 200/8 = 25
  - Supply is S = 2 x 8 = 16
  - Imports equal D-S = 9

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Example

- Suppose government places tariff of 1 on each unit sold,
  - Restricted price is $P_R = 9$
  - Imports fall to 200/9 – 2x9 = 22.2 – 18 = 4.2
- Welfare effect of the tariff can be calculated

$$DW_1 = (0.5)(P_R\text{-}P_W)(Q_1\text{-}Q_3) = 0.5(1)(25 - 22.2) = 1.4$$

$$DW_2 = (0.5)(P_R\text{-}P_W)(Q_4\text{-}Q_2) = 0.5(1)(18 - 16) = 1$$

- The total deadweight loss from the tariff is 2.4.

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Other Trade Restrictions

- A quota that limits imports to Q_3 - Q_4 would have effects that are similar to those for the tariff
  - same decline in consumer surplus
  - same increase in producer surplus
- Revenue rectangle
  - Goes to government if sell quota.
  - Goes to foreign firms if give away quota.

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Big Picture

- Trade restrictions such as tariffs or quotas create
  - Transfers between consumers and producers
  - Deadweight loss of economic welfare
- To justify trade restrictions
  - Care about producers more than consumers (and transfers not possible).
  - Externality when firms exit.
  - Imperfect competition among firms.
  - Irreversible exit and poor financial markets.

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