PARTIAL EQUILIBRIUM
Welfare Analysis

[See Chap 12]

Welfare Analysis

• We would like welfare measure.
• Normative properties of competitive markets.
  – First welfare theorem.
• Use to analyze policies
  – What is effect of tax?
  – What is effect of price control?
  – What is effect of banning imports?

Pareto Efficiency

• An allocation is Pareto efficient if there is no other allocation that makes everyone else better off.

• Weak notion of efficiency.
  – Necessary condition for desirable allocation.
  – May not be sufficient: If one agent has everything, this is Pareto efficient.
WELFARE MEASURES

Consumer Surplus

• Suppose utility is quasi-linear
  \[ u_j(x_1, x_2) = v_j(x_1) + x_2 \]
  – We are interested in good 1
  – Think of good 2 as “money on everything else”.

• Individual consumer surplus is area under Marshallian demand function.
  – See consumer surplus notes.

Consumer Surplus

• Suppose there is amount \( X_1 \) of good 1.
  – Divide \( X_1 \) between \( J \) agents.
  – Allocations \( \{x_{11}, x_{12}, \ldots, x_{1J}\} \).

• In any Pareto efficient allocation, the social planner wishes to maximize \( \sum_j v_j(x_j) \).
  – Suppose good is given to agent 1 (value $10) and not agent 2 (value $20).
  – Everyone better off if give good to agent 2, and transfer $15 from agent 2 to agent 1.

• Hence aggregate consumer surplus is area under aggregate demand function.
### Consumer Surplus
- Agent A has values \(\{3, 1\}\), B has values \(\{4, 2\}\).
- CS is sum of values minus price.

![Graph showing consumer surplus with demand curve and agent values](image1)

### Producer Surplus
- The producer surplus of a single firm equals its profit.
  - Profit equals \([AC(q) - p]q\)
  - Profit equals area over MC curve (net of fixed costs).
  - Ignore fixed costs since don’t affect welfare comparisons.
- Aggregate producer surplus
  - In any Pareto Efficient allocation, social planner wishes to minimize \(\sum c_i(q^*)\).
  - Hence aggregate producer surplus equals area over the market supply function.

![Graph showing producer surplus and firm supply](image2)

### Producer Surplus
Producer surplus of a typical firm and market.

![Graph showing total producer surplus and market supply](image3)
Total Welfare

- The area between the demand and supply curve equals the sum of CS and PS.
  - Measures the value of agents/firms from being able to make market transactions.

- First Welfare Theorem: Any competitive equilibrium is Pareto efficient.
  - PS+CS maximized in a competitive equilibrium.
  - Trade occurs if and only if the marginal utility exceeds the marginal cost.

First Welfare Theorem

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Price</th>
</tr>
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<tbody>
<tr>
<td>Q*</td>
<td>P*</td>
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Consumer surplus is the area above price and below demand
Producer surplus is the area below price and above supply

At output Q₁, total surplus will be smaller
At outputs between Q₁ and Q*, consumers would value an additional unit more than it would cost suppliers to produce.
Interpretation

- In any Pareto efficient allocation, social planner maximizes CS plus PS.
- If Q<Q*, then everyone can be made better off by increasing Q.
  - Does not mean everyone is necessarily better off.
  - Need transfers to redistribute money.
- Interpret CS + PS as gains from trade than can be distributed between agents and firms.

Welfare Loss Computations

- We can use CS and PS to explicitly calculate the welfare losses caused by restrictions on voluntary transactions
  - In general, we have to integrate the area between demand and supply.
  - With linear demand and supply, the calculation is simple because the areas are triangular.

Welfare Loss: Example

- Suppose that the demand is given by
  \[ Q_D = 10 - P \]
  and supply is given by
  \[ Q_S = P - 2 \]
- Market equilibrium occurs where \( P^* = 6 \) and \( Q^* = 4 \)
Welfare Loss: Example

- Restriction of output to $Q_0 = 3$ would create a gap between what demanders are willing to pay ($P_D$) and what suppliers require ($P_S$)

  \[ P_D = 10 - 3 = 7 \]
  \[ P_S = 2 + 3 = 5 \]

The welfare loss from restricting output to 3 is the area of a triangle

\[ \text{The loss} = (0.5)(2)(1) = 1 \]

Welfare Loss Computations

- The welfare loss is shared by producers and consumers
- The elasticity of demand and elasticity of supply to determine who bears the larger portion of the loss
  - the side of the market with the smallest price elasticity (in absolute value)
APPLICATION: PRICE CONTROLS

Price Controls and Shortages
- Sometimes governments seek to control prices at below equilibrium levels.
  - This will lead to a shortage
- We can analyze impact on welfare
  - Price floor will lead to forgone transactions.
  - Welfare loss since these transactions would benefit consumers and producers.

Price Controls and Shortages

Initially, the market is in long-run equilibrium at \( P_1, Q_1 \)
Demand increases to \( D' \)
The price rises to $P_2$

There will be a shortage equal to $Q_2 - Q_1$

Some buyers will gain because they can purchase the good for a lower price

This gain in consumer surplus is the shaded rectangle.
The shaded rectangle therefore represents a pure transfer from producers to consumers. No welfare loss there.

Assume: (a) the $Q_1$ goods go to the agents with the highest values, (b) no resources wasted in competing for goods. This gives lower bound on welfare loss.

This shaded triangle represents the value of additional consumer surplus that would have been attained without the price control.

This shaded triangle represents the value of additional producer surplus that would have been attained without the price control.
This shaded area represents the value of mutually beneficial transactions that are prevented by the government.

This is a measure of the welfare costs of this policy.

**Bigger Picture**

- **Static model**
  - Price floor causes welfare loss since firms do not supply enough.
- **Argentina’s agriculture**
  - Government tries to force firms to raise Q.
  - Firms make loss and exit
- **Rent control**
  - Reduces investment in housing stock.
- **Drug control (like price floor of ∞)**
  - Black markets

**APPLICATION:**

**TAXES**
To discuss the effects of a per-unit tax \((t)\), we need to make a distinction between the price paid by buyers \((P_D)\) and the price received by sellers \((P_S)\):

\[ P_D - P_S = t \]

Who pays the taxes is irrelevant. E.g.,
- Income tax of 10% (paid by workers)
- Payroll tax of 10% (paid by firms)

A per-unit tax creates a wedge between the price that buyers pay \((P_D)\) and the price that sellers receive \((P_S)\).

In equilibrium, quantity falls from \(Q^*\) to \(Q^{**}\).

Buyers incur a welfare loss equal to the shaded area, but some of this loss goes to the government in the form of tax revenue.
Sellers also incur a welfare loss equal to the shaded area. But some of this loss goes to the government in the form of tax revenue.

Therefore, this is the deadweight loss from the tax.

• Do consumers or producers lose more?
Tax Incidence

• Suppose there is small change in tax, $dt$.
• Prices change so that
  \[dP_D - dP_S = dt\]
• In equilibrium, supply equals demand. Hence
  \[dD = dS\]
• Differentiating,
  \[D'(P)dP_D = S'(P)dP_S\]
• Substituting, for $dP_S$ we get
  \[D'(P)dP_D = S'(P)(dP_D - dt)\]

We can now solve for the effect of the tax on $P_D$:

\[
\frac{dP_D}{dt} = \frac{S'(P)}{S'(P) - D'(P)} e_S - e_D
\]

where $e_S$ is the price elasticity of supply and $e_D$ is the price elasticity of demand,

• Similarly, if we solve for $dP_S$,
  \[
  \frac{dP_S}{dt} = \frac{D'(P)}{S'(P) - D'(P)} e_D - e_S
  \]

Since $e_D \leq 0$ and $e_S \geq 0$, $dP_D/dt \geq 0$ and $dP_S/dt \leq 0$
• If demand is perfectly inelastic ($e_D = 0$), the tax is completely paid by consumers.
• If demand is perfectly elastic ($e_D = \infty$), the tax is completely paid by suppliers.
• In general, the side with the more elastic responses will experience less of the price change

\[
\frac{dP_S}{dt} / \frac{dP_D}{dt} = \frac{e_D}{e_S}
\]
Deadweight Loss

• We showed taxes induce deadweight losses
  – the size of the losses will depend on the elasticities
    of supply and demand

• Start from tax $t=0$.
• The deadweight loss is given by the triangle.
  This area equals
  \[ DW = 0.5(dt)(dQ) \]

Deadweight Loss

• Suppose tax is small, so use local approx.
• From the definition of elasticity, we know that
  \[ dQ = e_D dP_D \cdot Q^*/P^* \]
  where $Q^*$ is qty before tax, and $P^*$ is price.
• Tax incidence equation says $dP_D = e_S/(e_S - e_D)dt$.
• Substituting, $dQ = e_D \cdot \frac{e_S}{(e_S - e_D)} t Q^*/P^*$
• Substituting, we get
  \[ DW = 0.5 \left( \frac{dt}{P^*} \right) \left( e_S/(e_S - e_D) \right) P^* Q^* \]

Deadweight Loss

• Deadweight losses are smaller in situations
  where $e_D$ or $e_S$ are small
  – Deadweight losses are zero if either $e_D$ or $e_S$ are
    zero
  – The tax does not alter the quantity of the good that
    is traded
• Deadweight loss is proportional to $dt^2$.
  – Loss small when tax small, since lose low value
    transactions.
  – Loss large when tax big, since lose high value
    transactions.
Transactions Costs

- Transactions costs create a wedge between the price the buyer pays and the price the seller receives
  - real estate agent fees
  - broker fees for the sale of stocks
- These can be modeled as taxes
  - Middleman gains area labeled "government revenue"
- Costs are shared by the buyer and seller
  - Who pays depends on elasticities

APPLICATION: INTERNATIONAL TRADE

Gains from International Trade

Consider a small country.

In the absence of international trade, the domestic equilibrium price is \( P^* \) and the domestic equilibrium quantity is \( Q^* \)
Gains from International Trade

If the world price \( P_W \) is less than the domestic price, the price falls to \( P_W \).

Quantity demanded rises to \( Q_1 \) and quantity supplied falls to \( Q_2 \).

Imports = \( Q_1 - Q_2 \)

Consumer surplus rises

Producer surplus falls

Gain to consumers exceeds loss to producers so overall welfare rises.

Effects of a Tariff

Suppose the government creates a tariff that raises the price to \( P_R \).

Quantity demanded falls to \( Q_3 \) and quantity supplied rises to \( Q_4 \).

Imports are now \( Q_3 - Q_4 \).
Effects of a Tariff

Consumer surplus falls
Producer surplus rises
The government gets
tariff revenue
These two triangles
represent deadweight loss

Estimating Deadweight Loss

- We can estimate the size of the welfare loss
  triangles.
- Suppose tariff is a percentage, so $P_R = (1+t)P_W$.
- Elasticity of demand: $e_D = (P/Q)(\Delta Q/\Delta P)$.
- Letting $\Delta Q = Q_3 - Q_1$ and $\Delta P = P_R - P_W$, 
  $$Q_3 - Q_1 = \frac{P_R - P_W}{P_W}, e_D = \frac{\Delta Q}{\Delta P}, Q_1$$
  where we use $\Delta P = tP_W$.

Estimating Deadweight Loss

The areas of these two
triangles are

$$DW_1 = 0.5(P_R - P_w)(Q_2 - Q_1)$$
$$DW_2 = -0.5t e_D P_w Q_1$$

$$DW_3 = 0.5(P_R - P_w)(Q_3 - Q_2)$$
$$DW_4 = -0.5t e_D P_w Q_2$$
Example

• Market demand is 
\[ D = \frac{200}{P} \]

• Market supply curve is 
\[ S = 2P \]

• Domestic equilibrium is \( P^* = 10 \) and \( Q^* = 20 \)

• World price is \( P_W = 8 \).
  – Demand is \( D = \frac{200}{8} = 25 \)
  – Supply is \( S = 2 \times 8 = 16 \)
  – Imports equal \( D - S = 9 \)

Example

• Suppose government places tariff of 1 on each unit sold,
  – Restricted price is \( P_R = 9 \)
  – Imports fall to \( \frac{200}{9} - 2 \times 9 = 22.2 - 18 = 4.2 \)

• Welfare effect of the tariff can be calculated
  \[ DW_1 = 0.5(P_R - P_W)(Q_1 - Q_3) = 0.5(1)(25 - 22.2) = 1.4 \]
  \[ DW_2 = 0.5(P_R - P_W)(Q_4 - Q_2) = 0.5(1)(18 - 16) = 1 \]

• The total deadweight loss from the tariff is 2.4.

Other Trade Restrictions

• A quota that limits imports to \( Q_3 - Q_4 \) would have effects that are similar to those for the tariff
  – same decline in consumer surplus
  – same increase in producer surplus

• Revenue rectangle
  – Goes to government if sell quota.
  – Goes to foreign firms if give away quota.
Big Picture

• Trade restrictions such as tariffs or quotas create
  – Transfers between consumers and producers
  – Deadweight loss of economic welfare

• To justify trade restrictions
  – Care about producers more than consumers (and transfers not possible).
  – Externality when firms exit.
  – Imperfect competition among firms.
  – Irreversible exit and poor financial markets.