Preferences and Utilities

- Consumer is central figure in microeconomics.
- Need model of consumer behavior.
- Consumer chooses from set of feasible options.
- How does consumer choose?

Preferences

- Suppose that an individual must choose between goods x, y, and z
- For example, x = IPhone app, y = MP3, z = Movie.
- To decide, agent forms ranking
- For example, x is be preferred to z; and z is be preferred to y.
Preferences

Preferences
- Write \( x \succeq y \) if \( x \) is weakly preferred to \( y \).
- Write \( x \succ y \) if \( x \) is strictly preferred to \( y \).
- Write \( x \sim y \) if \( x \) is indifferent to \( y \).

Choice
- We can only observe agents’ choices.
- Hence preferences come from choice.
- Thus, \( x \succeq y \) if \( x \) is chosen over \( y \).

Axioms of Rational Choice

Axiom 1: Completeness
- Given good \( x \) and \( y \), then either
  - \( x \) is preferred to \( y \), or
  - \( y \) is preferred to \( x \), or
  - both.

  - Is this reasonable?

2. Transitivity
- If \( x \) is preferred to \( y \), and \( y \) is preferred to \( z \), then \( x \) is preferred to \( z \)
- Assumes that the individual’s choices are internally consistent

  - Is this reasonable?
Utility Functions

• We say the utility function \( u(.) \) represents preferences if
  \( u(x) \geq u(y) \) if and only if \( x \succeq y \)

Hence we can use utility function to see if agent prefers \( x \) or \( y \).

Theorem: Suppose there are a finite number of goods. If preferences satisfy completeness and transitivity then there exists a utility function that represents them.

Utility Functions

Idea behind theorem:
• Suppose there are three goods \{x, y, z\}.
• Suppose \( x \succ y \) and \( y \succ z \).
• Then let \( u(x) = 3 \), \( u(y) = 2 \), and \( u(z) = 1 \).

Utility is not happiness
• Utilities represent choices, not “happiness”.
• E.g. An addict has high utility for heroin.
• Numbers are meaningless. Only rankings matter.
• Cannot compare utilities across people.

Utility is Ordinal

• Ordinal – only rankings matter
• Cardinal – magnitudes have meaning

• Suppose agent has utility \( u(x) \)
• Suppose \( f(.) \) is strictly increasing function.
• Then agent’s preferences are represented by \( v(x) = f(u(x)) \).

• Example: Ranking movies on 1-10 scale.
Choosing from Budget Sets

- Suppose agent chooses from subset of \( \mathbb{R}^N \)
  - Mathematically useful: Calculus.

- Axiom 3: Continuity
  - if \( x \) is strictly preferred to \( y \), then goods "close to" \( x \) are also preferred to \( y \)

Utility Functions

- Suppose agent chooses from subset of \( \mathbb{R}^N \)
- Suppose preferences satisfy completeness, transitivity and continuity.
- Then there exists a utility function \( u(\cdot) \) that represents preferences.

- If \( N=2 \), then write utility as \( u(x_1, x_2) \)

What is a Good?

- My utility from an umbrella depends on the whether.
- We can think of (umbrella, rain) and (umbrella, sunny) as different goods.
- There are many dimensions to goods:
  - State of the world
  - Time
  - Psychological state
  - With whom good is consumed.
Properties of Preferences

Monotonicity

- Suppose agent chooses from $\mathbb{R}^N$
- Preferences are monotone if $x \succeq y$ when
  1. $x_i \geq y_i$ for $i=1..N$, and
  2. $x \neq y$
- Utility is increasing if $u(x) > u(y)$ when
  1. $x_i \geq y_i$ for $i=1..N$, and
  2. $x \neq y$
- Preferences are monotone if and only if the corresponding utility function is increasing.

Monotonicity

- More is better:
Convexity

• Suppose consumer chooses from $\mathbb{R}^N$
• Preferences are convex if $x \succ y$ and $1 \geq \alpha \geq 0$, imply $\alpha x + (1-\alpha)y \succ y$.
• Motivation: Agent prefers averages to extremes. To see this suppose $x \sim y$.
• Utility is quasi-concave if $u(x) \geq t$ and $u(y) \geq t$ implies $u(\alpha x + (1-\alpha)y) \geq t$.
• Preferences are convex if and only if the corresponding utility function is quasi-concave.

INDIFFERENCE CURVES

Utility and Indifference Curves

• Sometimes, instead of using the utility function we will use the indifference curves.
• There is one set of indifference curves that corresponds to a particular utility function.
• The indifference curves and the utility function describe the same preferences.
Indifference Curves

- An indifference curve shows a set of consumption bundles among which the individual is indifferent.

\[ (x_1, y_1) \] and \( (x_2, y_2) \) provide the same level of utility.

Indifference Curve Map

- Each point has an IC through it.
- Hence preferences completely determined by ICs.

Increasing utility

\[ U_1 < U_2 < U_3 \]

Indifference Curves

- For example, if you are indifferent between:
  - A = eating in your favorite restaurant once a week and going to a movie 3 times a week
  - B = eating in your favorite restaurant twice a week and going to a movie once a week
- Then the two consumption bundles A and B are on the same indifference curve.
Properties of Indifference Curves

Assume preferences satisfy completeness, transitivity, continuity and monotonicity.

1. ICs are thin.
2. ICs never cross.
3. ICs are strictly downward sloping.
4. ICs are continuous and have no gaps.
5. If preferences convex, ICs are convex to origin.

ICs Do Not Cross

• Can any two of an individual’s indifference curves intersect?

The individual is indifferent between A and C. The individual is indifferent between B and C. Transitivity suggests that the individual should be indifferent between A and B but B is preferred to A because B contains more x and y than A.

Convexity

• A set of points is convex if any two points can be joined by a straight line that is contained completely within the set.

The assumption of a diminishing MRS is equivalent to the assumption that all combinations of x₁ and x₂ which are preferred to x₁* and x₂* form a convex set.
Convexity

• If the indifference curve is convex, then the combination \((x_1 + y_1)/2, (x_2 + y_2)/2\) will be preferred to either \((x_1, x_2)\) or \((y_1, y_2)\).

This implies that "well-balanced" bundles are preferred to bundles that are heavily weighted toward one commodity.

Marginal Rate of Substitution

• The negative of the slope of the indifference curve at any point is called the marginal rate of substitution \((MRS)\).

\[ MRS = -\frac{dx_2}{dx_1} \]

At \((x_1, x_2)\), the indifference curve is steeper. The person would be willing to give up more of \(x_2\) to gain additional units of \(x_1\).

At \((y_1, y_2)\), the indifference curve is flatter. The person would be willing to give up less \(x_2\) to gain additional units of \(x_1\).
Marginal Rate of Substitution

- The MRS is the number of $x_2$ the agent is willing to give up to get 1 unit of $x_1$.
- Example:
  - How many MP3s would you give up to get one movie?
  - Tradeoff depends on how many MP3s you own.
- If preferences convex then MRS decreases in $x_1$.

MRS and Utility

- Totally differentiating the utility function $dU = U_1 dx_1 + U_2 dx_2$ where $U_1$ and $U_2$ are the partial derivatives.
- Along an indifference curve $dU=0$. Hence
  \[ MRS = -\frac{dx_2}{dx_1} \bigg|_{U=\text{const}} = \frac{U_1}{U_2} \]
  - Suppose $U_1=3U_2$, so $x_1$ is 3 times as valuable.
  - Then if lose 3 $x_2$ and gain 1 $x_1$ utility is constant.

Examples
Examples of Utility Functions

- Cobb-Douglas Utility
  \[ U(x_1, x_2) = x_1^\alpha x_2^\beta \]
  where \( \alpha \) and \( \beta \) are positive constants
  - The relative sizes of \( \alpha \) and \( \beta \) indicate the relative importance of the goods

MRS and Convexity

- Differentiating
  \[ MRS = \frac{\partial U}{\partial x_1} \frac{\partial U}{\partial x_2} = \frac{\alpha x_2^\beta}{\beta x_1^{\alpha-1}} \]
  - Along an indifference curve \( x_1^\alpha x_2^\beta = k \)
  - That is, \( y = k^{\alpha/\beta} x_1^{\alpha/\beta} \). Substituting,
    \[ MRS = \frac{\alpha}{\beta} k^{\alpha/\beta} x_1^{-(\alpha/\beta) - 1} \]
  - MRS is decreasing in \( x_1 \), so prefs are convex

Perfect Substitutes

- \( U(x_1, x_2) = \alpha x_1 + \beta x_2 \)
- \( MRS = \alpha/\beta \), independent of \((x_1, x_2)\)
Perfect Complements

- \( U(x_1, x_2) = \min(\alpha x_1, \beta x_2) \)
- \( MRS = \infty \) if \( x_2/x_1 > \alpha/\beta \), undefined if \( x_2/x_1 = \alpha/\beta \), and \( MRS = 0 \) if \( x_2/x_1 < \alpha/\beta \)

The indifference curves will be L-shaped. Only by choosing more of the two goods together can utility be increased.

Constant Elasticity of Substitution Utility

- The Constant Elasticity of Substitution Utility (CES) can take two forms.
- If \( \delta \neq 0 \) and \( \delta < 1 \), utility = \( U(x_1, x_2) = x_1^{\delta} + x_2^{\delta} \)
- If \( \delta = 0 \), utility = \( U(x_1, x_2) = \ln x_1 + \ln x_2 \)

CES Utility

- The following parameter is called the substitution parameter:
  \[ \sigma = 1/(1 - \delta) \]
- It measures how much the consumer is willing to substitute between the two goods:
  - Perfect substitutes \( \Rightarrow \delta = 1 \)
  - Cobb-Douglas \( \Rightarrow \delta = 0 \)
  - Perfect complements \( \Rightarrow \delta = -\infty \)
Qualilinear Preferences

- Suppose utility is linear in $x_2$,
  \[ U(x_1, x_2) = v(x_1) + x_2 \]
- Differentiating,
  \[ MRS = \frac{\partial M}{\partial U} = v'(x_1) \]
- This only depends on $x_1$ and not $x_2$
- Hence ICs parallel shifts of each other

Homothetic Preferences

- Preferences are homothetic if the MRS depends only on the ratio of the amount consumed of two goods.
- With homothetic preferences all indifference curves have the same shape.

- Homothetic: Cobb-Douglas, perfect substitutes, perfect complements, CES.
- Not homothetic: Quasilinear.

The Many-Good Case

- Suppose utility is a function of $n$ goods given by
  \[ \text{utility} = U(x_1, x_2, \ldots, x_n) \]
The Many-Good Case

- We can find the MRS between any two goods by setting $dU = 0$
  \[ dU = 0 = \frac{\partial U}{\partial x_i} dx_i + \frac{\partial U}{\partial x_j} dx_j \]
- Rearranging, we get
  \[ MRS(x_i \text{ for } x_j) = -\frac{dx_j}{dx_i} = -\frac{\frac{\partial U}{\partial x_i}}{\frac{\partial U}{\partial x_j}} \]

Multigood Indifference Surfaces

- We will define an indifference surface as being the set of points in $n$ dimensions that satisfy the equation
  \[ U(x_1, x_2, \ldots, x_n) = k \]
  where $k$ is a constant level of utility