# PROFIT MAXIMIZATION

[See Chap 11]

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## Profit Maximization

- A profit-maximizing firm chooses both its inputs and its outputs with the goal of achieving maximum economic profits

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## Model

- Firm has inputs $(z_1,z_2)$. Prices $(r_1,r_2)$.
  - Price taker on input market.
- Firm has output $q=f(z_1,z_2)$. Price p.
  - Price taker in output market.
- Firm's problem:
  - Choose output q and inputs $(z_1,z_2)$ to maximise profits. Where:

$$\pi = pq - r_1 z_1 - r_2 z_2$$

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## One-Step Solution

- Choose $(z_1,z_2)$ to maximise

$$\pi = pf(z_1,z_2) - r_1z_1 - r_2z_2$$

- This is unconstrained maximization problem.
- FOCs are

$$p\frac{\partial f(z_1,z_2)}{z_1} = r_1 \quad \text{and} \quad p\frac{\partial f(z_1,z_2)}{z_2} = r_2$$

- Together these yield optimal inputs $z_i^*(p,r_1,r_2)$.
- Output is $q^*(p,r_1,r_2) = f(z_1^*, z_2^*)$. This is usually called the supply function.
- Profit is $\pi(p,r_1,r_2) = pq^* - r_1z_1^* - r_2z_2^*$



## Example: $f(z_1,z_2)=z_1^{1/3}z_2^{1/3}$

- Profit is $\pi = pz_1^{1/3}z_2^{1/3} - r_1z_1 - r_2z_2$
- FOCs are

$$\frac{1}{3}pz_1^{-2/3}z_2^{1/3} = r_1 \quad \text{and} \quad \frac{1}{3}pz_1^{1/3}z_2^{-2/3} = r_2$$

- Solving these two eqns, optimal inputs are

$$z_1^*(p,r_1,r_2) = \frac{1}{27}\frac{p^3}{r_1^2r_2} \quad \text{and} \quad z_2^*(p,r_1,r_2) = \frac{1}{27}\frac{p^3}{r_1r_2^2}$$

- Optimal output

$$q^*(p,r_1,r_2) = (z_1^*)^{1/3}(z_2^*)^{1/3} = \frac{1}{9}\frac{p^2}{r_1r_2}$$

- Profits

$$\pi^*(p,r_1,r_2) = pq^* - r_1z_1^* - r_2z_2^* = \frac{1}{27}\frac{p^3}{r_1r_2}$$



## Two-Step Solution

**Step 1:** Find cheapest way to obtain output q.

$$c(r_1,r_2,q) = \min_{z1,z2} r_1z_1+r_2z_2 \;\; \text{s.t} \;\; f(z_1,z_2) \geq q$$

**Step 2:** Find profit maximizing output.

$$\pi(p,r_1,r_2) = \max_q pq - c(r_1,r_2,q)$$

This is unconstrained maximization problem.

- Solving yields optimal output $q^*(r_1,r_2,p)$.
- Profit is $\pi(p,r_1,r_2) = pq^* - c(r_1,r_2,q^*)$.

## Step 2: Output Choice

- We wish to maximize

$$pq - c(r_1,r_2,q)$$

- The FOC is

$$p = dc(r_1,r_2,q)/dq$$

- That is,

$$p = MC(q)$$

- Intuition: produce more if revenue from unit exceeds the cost from the unit.
- SOC: MC'(q)≥0, so MC curve must be upward sloping at optimum.

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## Example: $f(z_1,z_2)=z_1^{1/3}z_2^{1/3}$

- From cost slides (p18),

$$c(r_1,r_2,q) = 2(r_1r_2)^{1/2} q^{3/2}$$

- We wish to maximize

$$\pi = pq - 2(r_1r_2)^{1/2} q^{3/2}$$

- FOC is

$$p = 3(r_1r_2)^{1/2} q^{1/2}$$

- Rearranging, optimal output is

$$q^*(p,r_1,r_2) = \frac{1}{9}\frac{p^2}{r_1r_2}$$

- Profits are $\pi^*(p,r_1,r_2) = pq^* - c(r_1,r_2,q^*) = \frac{1}{27}\frac{p^3}{r_1r_2}$

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## Profit Function

- Profits are given by

$$\pi = pq^* - c(q^*)$$

- We can write this as

$$\pi = pq^* - AC(q^*)q^* = [p-AC(q^*)]q^*$$

- We can also write this as

$$\pi = pq^* - \int_0^q MC(x)dx - F = \int_0^q [p - MC(x)]dx - F$$

where F is fixed cost

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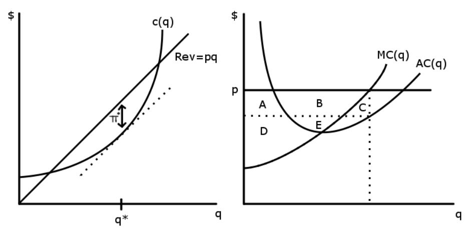
## Profit Function




Left: Profit is distance between two lines.

Right: Max profit equals A+B+C. If no fixed cost, this equals A+B+D+E.

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## Supply Functions

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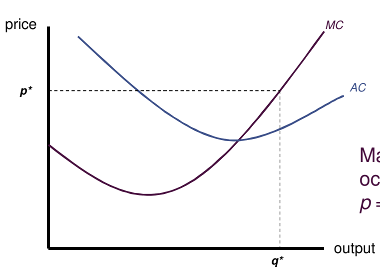
## Supply with Fixed Cost




Maximum profit occurs where *p = MC*

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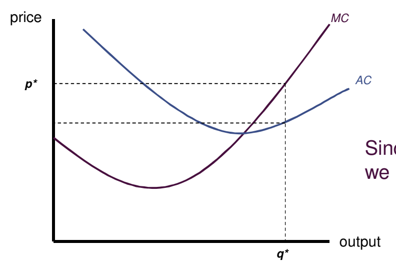
## Supply with Fixed Cost




Since $p > AC$, we have $\pi > 0$.

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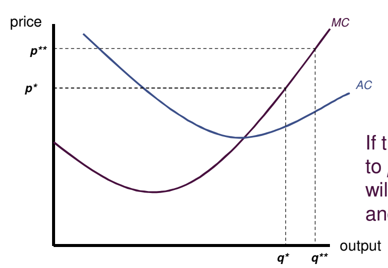
## Supply with Fixed Cost




If the price rises to $p^{**}$, the firm will produce $q^{**}$ and $\pi > 0$

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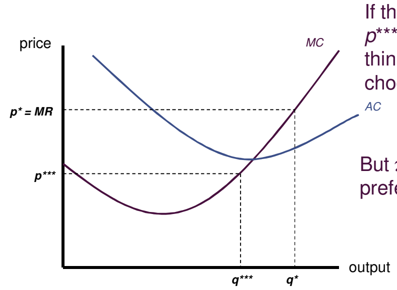
## Supply with Fixed Cost

If the price falls to $p^{***}$, we might think the firm chooses $q^{***}$.




But $\pi < 0$ so firm prefers q=0.

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## Supply Curve

- We can use the marginal cost curve to show how much the firm will produce at every possible market price.
- The firm can always choose q=0
  - Firm only operates if revenue covers costs.
  - Firm chooses q=0 if $pq<c$ or $p < AC$.

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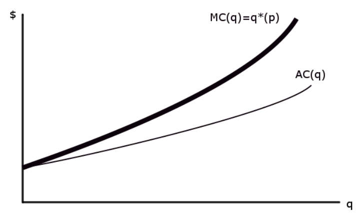
## Supply Function #1




- Increasing marginal costs.
- No fixed cost

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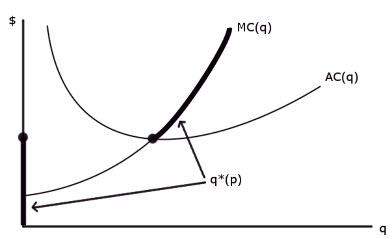
## Supply Function #2




- Increasing marginal costs.
- Fixed cost

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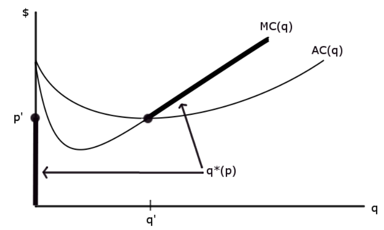
## Supply Function #3




- U-shaped marginal costs.
- No fixed cost.

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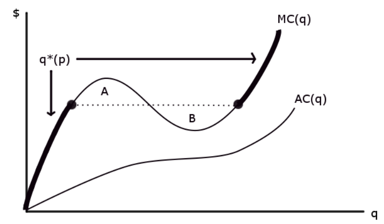
## Supply Function #4




- Wiggly marginal costs.
- No fixed cost

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## Sunk Costs

- In the short run, there may be sunk costs (i.e. unavoidable costs).
- Let AVC = average variable cost (excluding sunk cost).
- Then supply function is given by MC curve if p>AVC.
- Firm shuts down if p<AVC.

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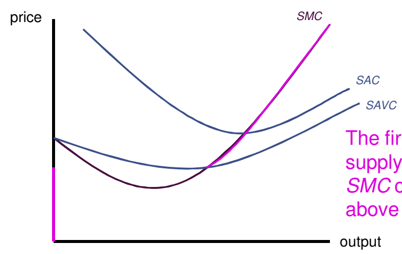
## Short-Run Supply by a Price-Taking Firm




The firm's short-run supply curve is the *SMC* curve that is above *SAVC*



## Cost Function: Properties

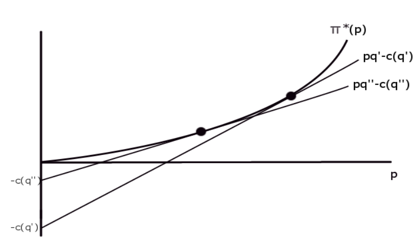
1. $\pi(p,r_1,r_2)$ is homogenous of degree 1 in $(p,r_1,r_2)$
   – If prices double, profit equation scales up so optimal choices unaffected and profit doubles.
2. $\pi(p,r_1,r_2)$ increases in p and decreases in $(r_1,r_2)$
3. Hotelling's Lemma:

$$\frac{\partial}{\partial p}\pi(p,r_1,r_2) = q^*(p,r_1,r_2)$$

   – If p rises by $\Delta p$, then $\pi(.)$ rises by $\Delta p \times q^*(.)$
   – Optimal output also changes, but effect second order.
4. $\pi(p,r_1,r_2)$ is convex in p.

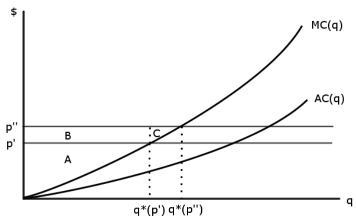


## Convexity and Hotelling's Lemma




- This shows the pseudo-profit functions, when output is fixed, and real profit function.

## Convexity and Hotelling's Lemma




When p' → p'', profit rises by B+C.

- Hotelling Lemma: B+C ≈ B when Δp small.
- Convexity: C means profit increases more than linearly.

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## Supply Functions: Properties

1. $q^*(p,r_1,r_2)$ is homogenous of degree 0 in $(p,r_1,r_2)$
   - If prices double profit equation scales up, so optimal output unaffected.

2. Law of supply

$$\frac{\partial}{\partial p} q^* = \frac{\partial}{\partial p}\left[\frac{\partial}{\partial p}\pi\right] \leq 0$$

   - Uses Hotelling's Lemma and convexity of π(.)
   - Hence supply curve is upward sloping!

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