Economics 11: Homework 1

1. Optimisation (4 points)

We wish to maximise \( f(x) = 5 - 2x - x^2 \).

(a) Find the optimal value of \( x \) and the resulting value of the objective. Verify the SOC.

(b) Suppose we introduce the constraint that \( x \geq 0 \). Find the optimal value of \( x \) and the resulting value of the objective. [Hint: Plot the function]

2. Optimisation with 2 variables (4 points)

Find the values of \((x_1, x_2)\) that maximise \( f(x) = 20x_1^{1/4}x_2^{1/4} - x_1 - x_2 \).

3. Constrained Optimisation (4 points)

Suppose that \( f(x_1, x_2) = x_1x_2 \). Suppose the constraint is \( x_1 + x_2 = 1 \).

(a) Draw the constraint in \((x_1, x_2)\) space.

(b) Draw the level curves of the objective in \((x_1, x_2)\) space. A level curve are the values of \((x_1, x_2)\) such that \( f(x_1, x_2) \) is constant.\(^1\)

We now wish to find the maximum value of \( f \) given the constraint.

(c) Solve the problem by substituting the constraint into the objective.

(d) Solve the problem by using the Lagrange method.

4. Constrained Optimisation (4 points)

The dual problem to the last question is to minimise \( g(x_1, x_2) = x_1 + x_2 \) subject to \( x_1x_2 = 1/4 \).

\(^1\)We will later think of these as indifference curves.
(a) Solve this problem using the Lagrangian technique.

(b) How does the solution compare to part (d) of the last question?

5. Partial Derivatives and Total Differential (4 points)

Suppose $u = f(x_1, x_2) = 4x_1^2 + 3x_2^2$.

(a) Calculate $\partial u / \partial x_1$ and $\partial u / \partial x_2$.

(b) Write the total differential for $u$.

(c) Calculate $dx_2 / dx_1$ when $du = 0$. That is, what is the implied trade-off between $x_1$ and $x_2$ holding $u$ constant.