1. Profit Maximisation: Perfect Substitutes

A firm has production function \( f(z_1, z_2) = (z_1 + z_2)^{1/2} \) where the prices of the inputs are \( r_1 < r_2 \). The output price is \( p \).

(a) The firm wishes to attain target output \( q \). Derive the cost minimising inputs \( z_i^*(r_1, r_2, q) \).

(b) Derive the cost function \( c(r_1, r_2, q) \).

(c) Using the cost function, derive the profit-maximising output and the firm’s optimal profits.

(d) Using the one-step method, directly solve for the firm’s optimal inputs, the resulting output and the optimal profit. [When using the one-step method the firm chooses \((z_1, z_2)\) to maximise \( pf(z_1, z_2) - r_1 z_1 - r_2 z_2 \) subject to \( z_i \geq 0 \).]

2. Profit Maximisation: Perfect Complements

A firm has production function \( f(z_1, z_2) = (\min\{z_1, z_2\})^{1/2} \) where the prices of the inputs are \( r_1 < r_2 \). The output price is \( p \).

(a) The firm wishes to attain target output \( q \). Derive the cost minimising inputs \( z_i^*(r_1, r_2, q) \).

(b) Derive the cost function \( c(r_1, r_2, q) \).

(c) Using the cost function, derive the profit-maximising output and the firm’s optimal profits.

(d) Using the one-step method, directly solve for the firm’s optimal inputs, the resulting output and the optimal profit. [When using the one-step method the firm chooses \((z_1, z_2)\) to maximise \( pf(z_1, z_2) - r_1 z_1 - r_2 z_2 \).]
3. Profit Maximisation

(a) A firm has cost function \( c(q) = 20q - 10q^2 + q^3 \). The output price is \( p = 8 \). Solve for the profit maximising output.

(b) A firm has cost function \( c(q) = 40q - 10q^2 + q^3 \). The output price is \( p = 8 \). Solve for the profit maximising output.

[Hint: you may wish to plot these functions]

4. Equilibrium and Inputs Markets

A firm has a Cobb–Douglas production function \( f(z_1) = 10z_1^{1/2} \). The price of the input is \( r_1 \).

(a) Find the supply function of the firm and its input demand (as a function of \( p \) and \( r_1 \)).

Suppose there are 4,000 identical firms in the market, and the supply of \( z_1 \) is given by \( z_1^S = 40r_1^2 \).

b) Assume that the output price is \( p = 1 \). Calculate the equilibrium input price \( r_1 \), the amount of \( z_1 \) each firm demands, the amount of \( z_1 \) the entire market demands and the output of each firm.

c) Suppose the output price rises to \( p = 2 \). Repeat part (b). What is the effect of an increase in the output price?

5. First Welfare Theorem

There are two goods (\( x \) and \( y \)) and two agents (A and B). The agents’ utility functions are

\[
\begin{align*}
    u_A &= v_A(x) + y \\
    u_B &= v_B(x) + y
\end{align*}
\]

where \( v_A = 2 \ln(x) \) and \( v_B(x) = 4x^{1/2} \). The agents have incomes \( m_A = 10 \) and \( m_B = 10 \). The price of good \( y \) is \( p_y = 1 \); the price of good \( x \) is to be determined.

A single firm produces good \( x \). It has cost function \( c(q) = q^2/2 \).
(a) Show that \( p = 2 \) is an equilibrium price. Find the equilibrium allocations \((x_A, x_B, q)\).

(b) Suppose a social planner chooses \((x_A, x_B, q)\) to maximise the total surplus,
\[
v_A(x_A) + v_B(x_B) - c(q)
\]
subject to \( q = x_A + x_B \). Verify your allocations from part (a) satisfy the FOCs from this optimisation problem.

6. Shifts in Supply Functions and Elasticities

Suppose the supply function is \( q = p \), and the demand function is \( q = 10 - p \).

(a) Find the equilibrium price and quantity.

(b) Suppose the supply curve shifts up to \( q = p - 2 \), so each unit costs $2 more to produce. Derive the new price and quantity.

Suppose the supply function is \( q = p \), and the demand function is \( q = 6 - p/5 \).

(c) Find the equilibrium price and quantity.

(d) Suppose the supply curve shifts up to \( q = p - 2 \). Derive the new price and quantity.

Suppose the supply function is \( q = p \), and the demand function is \( q = 25 - 4p \).

(e) Find the equilibrium price and quantity.

(f) Suppose the supply curve shifts up to \( q = p - 2 \). Derive the new price and quantity.

(g) Given these results, explain how the elasticity of the demand curve affects the impact of a shift in the supply function.

7. Taxation

Suppose utility is quasilinear in good \( x \) and the demand function is \( q = 10 - p \). The supply function is \( q = p \).
(a) Solve for the equilibrium price. Solve for consumer and producer surplus.

(b) Suppose there is a $2 producer tax. What is the new equilibrium price? What price does the firm receive? What is the change in producer and consumer surplus? What is government revenue? What is the deadweight loss?

(c) Suppose there is a $2 consumer tax. What is the new equilibrium price? What price does the firm receive? What is the change in producer and consumer surplus? What is government revenue? What is the deadweight loss?

8. Imports

The demand for portable radios is given by: \( Q = 5000 - 100p \). The local supply curve is given by \( Q = 150p \).

a) Find the market equilibrium.

b) Suppose that radios can be imported at a price of $10 per unit. Find the market equilibrium and the amount of radios imported.

c) Suppose that the local producers convince the government to impose a tariff of $5 per radio. Find the market equilibrium, the total revenue of the tariff and the effect on the consumer and producer surplus, and the deadweight loss.