Economics 11: Homework 2

1. Utility Functions (4 points)

Define the utility functions and draw the indifference curves for each of the following cases:

a) “Every time I consume one unit of \( x_1 \), I want to consume 2 units of \( x_2 \).”

b) “If the price of \( x_1 \) is bigger than the price of \( x_2 \), I will only consume \( x_2 \). If the price of \( x_2 \) is bigger than the price of \( x_1 \), I will only consume \( x_1 \).”

c) “I don’t care about the prices of \( x_1 \) and \( x_2 \), I only want to consume \( x_2 \).”

2. Consumption with Perfect Substitutes (4 points)

Suppose John views butter and margarine as perfectly substitutable in the ratio of one to one for each other.

a) Draw a set of indifference curves that describes John’s preferences for butter and margarine.

b) If butter costs $2 per package, while margarine cost only $1, and John has a $20 budget to spend for the month, which butter-margarine combination will he choose?

(c) Can you show this graphically? What happens when the price of margarine increases to $3?

3. Convexity and Monotonicity (6 points)

For the following utility functions, sketch the indifference curves and explain whether the underlying preferences satisfy convexity and monotonicity. Throughout, assume \( x_1, x_2 \geq 0 \).

(a) \( u(x_1, x_2) = x_1 + x_2 \)

(b) \( u(x_1, x_2) = x_1^2 + x_2^2 \)

(c) \( u(x_1, x_2) = \min\{x_1 + x_2, 1\} \)
(d) $u(x_1, x_2) = I(x_1) + I(x_2)$, where $I(x)$ is the integer component of $x$. For example, $I(3.64) = 3$ and $I(5.2) = 5$.

(e) $u(x_1, x_2) = -(x_1 - 1)^2 - (x_2 - 1)^2$.

(f) $u(x_1, x_2) = \min\{x_1, x_2\}$.

4. Monotone Transformations of Utility Functions (6 points)

The agent consumes $x$ units of a pizza, where we assume that $0 \leq x \leq 4$. The agent receives utility

$$u(x) = \begin{cases} x & \text{if } 0 < x \leq 2 \\ 4 - x & \text{if } 2 < x \leq 4 \end{cases}$$

Which of the following utility functions represent the same preferences as above:

(a) $v(x) = 2x$ if $0 \leq x \leq 2$, and $v(x) = 4 - x$ if $2 < x \leq 4$.

(b) $v(x) = 2x$ if $0 \leq x \leq 2$, and $v(x) = 8 - 2x$ if $2 < x \leq 4$.

(c) $v(x) = 4 - (2 - x)^2$ if $0 \leq x \leq 4$.

(d) $v(x) = 4 - (2 - x)^3$ if $0 \leq x \leq 4$.

(e) $v(x) = 4 - (2 - x)^4$ if $0 \leq x \leq 4$.

(f) $v(x) = -(2 - x)^2$ if $0 \leq x \leq 4$. 