1. Expenditure Minimisation (5 points)

Suppose that an individual has utility \( u(x_1, x_2) = x_1(1 + x_2) \). Throughout assume income \( m \) is sufficiently high so there is an internal solution.

a) Find the Marshallian demand for \( x_1 \) and \( x_2 \).

b) Are \( x_1 \) and \( x_2 \) normal or inferior goods?

c) Find the indirect utility function.

d) Invert the indirect utility function to find the expenditure function.

e) What is the minimum expenditure necessary to achieve a utility level of \( u = 72 \) with \( p_1 = 4 \) and \( p_2 = 2 \)?

2. Calculating Elasticities (4 points)

An agent’s demand for good 1 is given by

\[
x_1^*(p_1, p_2, m) = (m + p_2) / 2p_1
\]

Compute the agent’s price elasticity, cross elasticity and income elasticity of \( x_1 \) when \( m = 200, \quad p_1 = 5 \) and \( p_2 = 10 \).

3. Complements and Substitutes (4 points)

Suppose \( u(x_1, x_2) = x_1x_2 \). Are \( x_1 \) and \( x_2 \) gross complements or substitutes? Are they net complements or substitutes?
4. Slutsky Equation (7 points)

An agent has utility \( u(x_1, x_2) = \left( x_1^{-1} + x_2^{-1} \right)^{-1} \) for goods \( x_1 \) and \( x_2 \). The prices of the goods are \( p_1 \) and \( p_2 \). The agent has income \( m \).

(a) Show preference are convex. You can do this graphically or by showing that MRS is decreasing in \( x_1 \).

(b) Solve for the agent’s optimal choice of \((x_1, x_2)\).

(c) Show the agent’s indirect utility function is given by

\[
v = \frac{m}{(p_1^{1/2} + p_2^{1/2})^2}\]

(d) Solve for the agent’s Hicksian demand.

(e) Solve for the expenditure function.

(f) Verify the Slutsky equation for good \( x_1 \). In particular, show that both sides are as follows:

\[
\frac{\partial x_1^*}{\partial p_1} = - \left[ p_1^{-1} + \frac{1}{2} p_1^{-3/2} \frac{p_1^{1/2}}{p_2^{1/2}} \right] \frac{m}{(p_1^{1/2} + p_2^{1/2})^2} = \frac{\partial h_1}{\partial p_1} - x_1^* \frac{\partial x_1^*}{\partial m}
\]