1. Production with One Input (4 points)

Digging clams by hand in Sunset Bay requires only labour input. The total number of clams obtained per hour \((q)\) is given by: \(q = 100L^{1/2}\) Where \(L\) is he labour input per hour.

a) Graph the relationship between \(q\) and \(L\)

b) Find the marginal and average product of labour.

2. Production Function (4 points)

A firm has one input, \(z\), and the following production function

\[
f(z) =
\begin{align*}
2x & \quad \text{if } 1 \geq x \\
2 & \quad \text{if } 2 \geq x \geq 1 \\
x & \quad \text{if } x \geq 2
\end{align*}
\]

(a) Is this production function concave?

(b) Does it exhibit decreasing, constant or increasing returns to scale (or neither)?

3. Cost Functions (4 points)

A firm has cost function \(c(q) = 100 - 10q + 5q^2\).

a) Find the fixed cost.\(^1\)

b) Find the variable cost.\(^2\)

\(^1\)Definition: The fixed cost is the cost that is independent of output.

\(^2\)Definition: The variable cost is the cost that varies with the level of output.
c) Find the average cost.

d) Find the marginal cost.

e) Draw the relationship between MC and AC. Prove that they always intersect at the minimum AC.

4. Cost Minimisation: Cobb Douglas (4 points)

Suppose that a firm production function is given by the Cobb-Douglas function: \( f(z_1, z_2) = z_1^\alpha z_2^\beta \). The cost of the inputs is \( z_1 \) and \( z_2 \).

a) Find marginal and average productivity of the two factors.

b) Does this production function have increasing, constant or decreasing returns to scale?

c) Show that cost minimisation requires \( \beta r_1 z_1 = \alpha r_2 z_2 \).

d) Suppose \( \alpha = \beta = 1/4 \). Find the cost function.

5. Cost Minimisation Problem (4 points)

A firm has production function

\[
 f(z_1, z_2) = z_1^{1/2}(z_2 - 1)^{1/2}
\]

The prices of the inputs are \( r_1 \) and \( r_2 \).

(a) Find \( MP_1, MP_2, \) and \( MRTS \).

(b) If \( z_2 \) is fixed at 5, what is the short-run cost function? Find the short-run marginal cost and average cost.

(c) What are the long-run input demand functions? What is the long-run cost function? Find the long-run marginal cost and average cost.

(d) Does the production function exhibit increasing, constant or decreasing returns to scale.