# Partial Equilibrium: Positive Analysis

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In this Chapter we consider consider the interaction between different agents and firms, and solve for equilibrium prices and quantities.

Section 1 introduces the idea of partial equilibrium. Section 2 looks at how we aggregate agent's demand curves and firm's supply curves to form market demand and market demand supply. Section 3 defines an equilibrium, and discusses basic properties thereof. Section 4 looks at short-run equilibrium, where entry and exit are not possible. Finally, Section 5 considers long-run equilibrium, where entry and exit are possible.

## 1 Partial Equilibrium

When studying partial equilibrium, we consider the equilibrium in one market, taking as exogenous prices in other markets and agents' incomes, as well as preferences and technology. The main advantage of this model is simplicity: the equilibrium price is found by equating supply and demand. The model can also be used for welfare analysis, evaluating the effect of tax changes or the introduction of tariffs. However, the assumption that we can analyse one market independently of others can be dubious in some cases.

As an alternative, economists sometimes study general equilibrium. In this model, we fix preferences and technology and suppose agents are endowed with goods and shares. We then

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solve for all prices simultaneously, equating supply and demand in each market. While this approach is far more general (hence the name), it is harder to analyse.

To illustrate the difference between partial and general equilibrium consider the worldwide market for cars.[1] A partial equilibrium analysis would add up the world's demand for cars to form a market demand curve. It would also add up the different firms' supply curves to form a market supply curve. The price of cars can then be found where demand equals supply. Intuitively, if the price were lower the would be excess demand and the price would be bid up; if the price were higher there would be excess supply and competition among suppliers would drive the price down. An exogenous increase in demand from China, due to Government construction of highways, would then shift up the demand curve, raising equilibrium price and quantity. In a general equilibrium model, there would be many other effects. First, the value of car firms would rise, increasing the income of their shareholders. Second, the increased demand for cars would push up the price of complements, such as oil. Third, there would be an increase in demand for inputs, such as steel, so commodity prices would rise. Ultimately, there is no single correct model: rather, there is a tradeoff between complexity and realism which depends on the markets at hand and the questions one is interested in.

## 1.1 Partial Equilibrium Model

We make the following assumptions:

1. We are interested in market 1. The price of this good, $p_1$, is to be determined.[2] We assume that all consumers and firms face the same prices (the law of one price), and that they are all price takers.

2. On the demand side, there are $J$ agents who desire good 1. Each agent $j$ has income $m^j$ and utility $u^j(x_1, \dots, x_N)$. The consumer spends her income on $N$ outputs which have prices $\{p_1, p_2 \dots, p_N\}$, where $\{p_2 \dots, p_N\}$ are exogenous. For simplicity, we often take $N = 2$.

3. On the supply side, there are $K$ firms who sell good 1. Each firm $k$ has a production technology $f^k(z_1, \dots, z_M)$, where the $M$ input prices are given exogenously by $\{r_1 \dots, r_M\}$. For simplicity, we often take $M = 2$.

[1] We assume that there is one type of car. This assumption is highly stylised but, recall, the purpose of a model is not to describe the real world exactly, but to make useful abstractions.

[2] For simplicity, we will sometimes denote the price of this good by $p$.

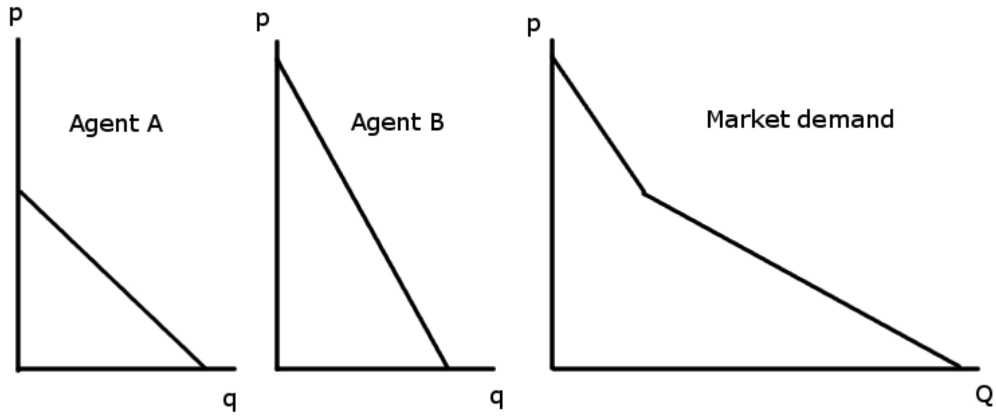


Figure 1: **Summing Demand.** Market demand is the horizontal sum of individual demands.

# 2 Market Demand and Supply

## 2.1 Market Demand

Solving the consumer's utility maximisation problem, we can derive agent $j$'s Marshallian demand for good 1,

$$x_1^j(p_1, \dots, p_N; m^j)$$

We can then form market demand by summing the individual agent's demands:

$$X_1(p_1, \dots, p_N; m^1, \dots, m^J) = \sum_{j=1}^{I} x_1^j(p_1, \dots, p_N; m^j)$$

Market demand depends on the tastes of the agents, the prices of goods, the number of agents and the distribution of income in the economy. This means that if we redistribute money from every agent to one special agent (think Bill Gates), then this will change the demand for high-value items like yachts.

Figure 1 shows that when we sum demand, we are effectively adding the demand curves horizontally.

### 2.2 Market Supply

Solving the firm's profit maximisation problem, we can derive firm $k$'s supply of good 1,

$$q^k(p_1, r_1 \ldots, r_M)$$

We can then form market supply by summing the individual firm's supply functions:

$$Q(p_1, r_1 \ldots, r_M) = \sum_{k=1}^{I} q^k(p_1, r_1 \ldots, r_M)$$

Note that market demand depends on the technologies of the firms, the price of good 1, the prices of the inputs and the number of firms. This means that if the number of firms increases then the market supply curve will shift out; similarly, if the number of firms decreases then the supply curve will shift in. We will see examples of this below.

## 3 Equilibrium

The equilibrium price of good 1, $p_1^*$, is found by equating supply and demand,

$$X_1(p_1^*, p_2; m^1, \ldots, m^I) = Q(p_1^*, r_1, r_2) \tag{3.1}$$

where, for simplicity, we assume there are 2 output goods and 2 input goods. Figure 2 shows the classic demand and supply picture.

The idea behind equation (3.1) is that if there is excess demand then consumers will bid the market price up, and if there is excess supply firms will compete for customers and reduce the market price.[3]

We now consider two general questions concerning the equilibrium price. First, does there exist a price $p_1^*$ that satisfies (3.1)? Second, is there one one price $p_1^*$ that satisfies (3.1) or many?

[3]This process is called tatonnement. One problem with this intuition is that it contradicts our assumption of price taking: how can firms undercut each other when they cannot affect the price? This suggests that the correct interpretation of price taking is not that firms cannot offer a different price, but that they have no incentive to do so.

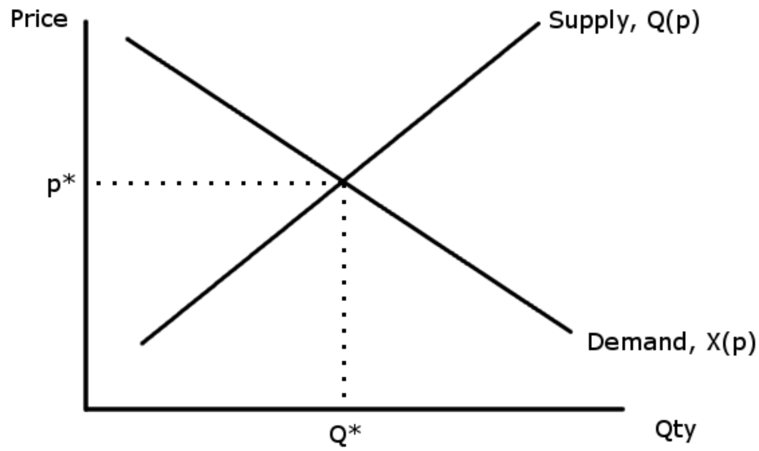


Figure 2: **Equilibrium.** In equilibrium, supply equals demand.

### 3.1 Existence

Does a price, $p_1^*$, exist that equates supply and demand? We can ensure existence if three conditions hold, all of which are satisfied in Figure 2.

1. For high prices, supply exceeds demand. This is trivially satisfied since demand must fall to zero as the price rises to infinity.
2. For low prices, demand exceeds supply. This is easy to satisfy since one would expect supply to fall to zero as prices converge to zero.
3. The supply and demand functions are continuous in $p_1$.

Figure 3 shows an example with one firm and one consumer where an equilibrium does not exist. The problem in this example is that at price $p'$ the consumer wants 2 units, while the firm wishes to supply either 0 or 3 units. This problem is caused by the fixed cost: if the firm's production function is concave then its supply curve will be continuous. However, even if the individual firms' supply functions contain jumps, this is not a problem if the market is sufficiently large. In the example, if there are 3 identical agents then we can have two firms producing $q = 3$. The total demand then equals $2 + 2 + 2 = 6$, while the total supply equals

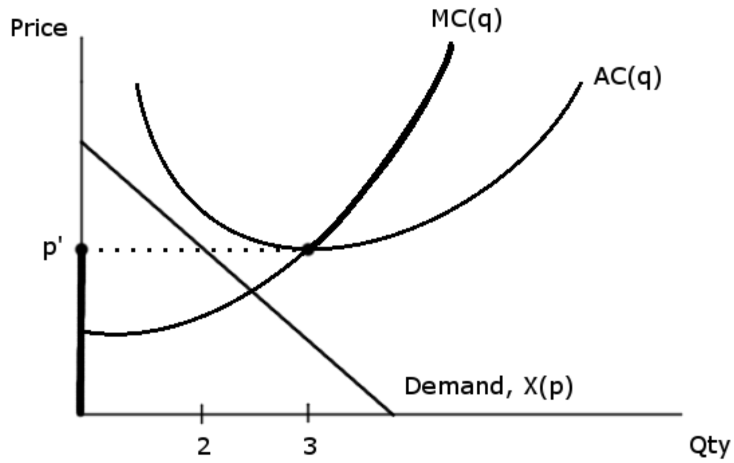


Figure 3: **Nonexistence of Equilibrium.** Due to a fixed cost, supply jumps at price $p'$. As a result, no equilibrium exists.

$3 + 3 = 6$, and we have an equilibrium.[4]

### 3.2 Uniqueness

In Figure 2 there is a unique equilibrium price. Can there be more? From the law of supply we know that the supply curve is upward sloping. If the good is ordinary for each agent then the market demand is downward sloping and there can at most be one equilibrium. In contrast, if the good is a Giffen good, then there may be multiple equilibria, as shown in Figure 4.

## 4 Short-Run Equilibrium

In Section 3 we discussed equilibrium in general. It is useful to differentiate between the following time periods:[5]

[4]Without fixed costs, equilibrium may also fail to exist if the marginal cost curve is first increasing, then decreasing, and increasing again. This causes a discontinuity in the supply function, as shown in the Production Chapter. Again, this is not a problem if the market is sufficiently large.

[5]In the Production Chapter, we considered a fourth case when some factors are flexible, but others are not. We omit this here for simplicity.

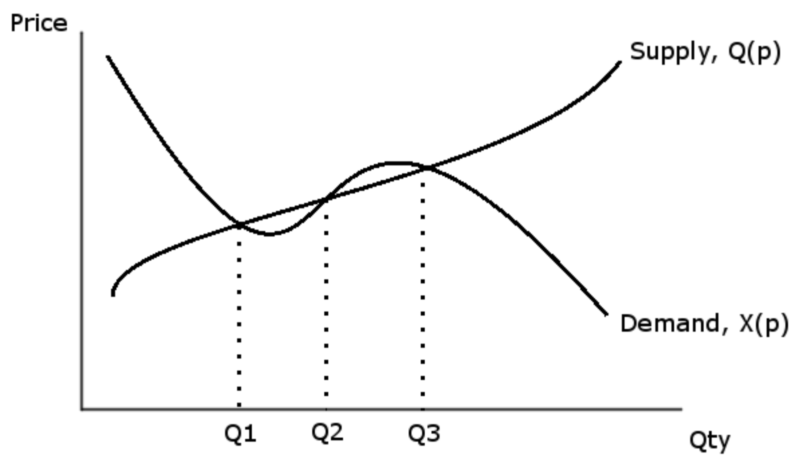


Figure 4: **Multiple Equilibria.** In this figure, the demand curve has an upward sloping component. As a result, there are three equilibria with quantity $Q_1$, $Q_2$ and $Q_3$ being sold.

1. In the **very short run** all the factors of production are fixed, and output is fixed.
2. In the **short run** all factors are flexible, but fixed costs are sunk. Firms cannot enter or exit.
3. In the **long run** all factors are flexible and fixed costs are not sunk. Hence firms can enter and exit freely.

In the very short-run, the analysis is straightforward since output is fixed. We solve a numerical example in Section 5.3.

In this Section we analyse short-run equilibrium, assuming the number of firms is fixed. In this case, each firm operates on its individual supply curve, and market supply equals the sum of individual firm supply.

### 4.1 Example

Suppose there are 15 agents, ten of whom have income $m_j = 10$ and five of whom have $m_j = 20$. Each agent has symmetric Cobb-Douglas utility,

$$u^j(x_1, x_2) = x_1 x_2$$

Solving the agent's utility maximisation problem, the demand function of each agent for good 1 is

$$x_1^j = \frac{m_j}{2p_1}$$

Summing over demand curves, market demand is given by

$$X_1 = \frac{200}{2p_1} = \frac{100}{p_1}$$

Next, suppose there are 9 firms with production function

$$f^k(z_1, z_2) = (z_1 - 1)^{1/3}(z_2 - 1)^{1/3}$$

Solving the firm's cost maximisation problem, its cost function is

$$c^k(q, r_1, r_2) = 2(r_1 r_2)^{1/2} q^{3/2} + (r_1 + r_2)$$

Solving the profit-maximisation problem, its supply function is given by

$$q^k = \frac{1}{9}\frac{p_1^2}{r_1 r_2}$$

Summing over firms, market supply is thus

$$Q = \frac{p_1^2}{r_1 r_2}$$

Equating supply and demand,

$$p_1^* = (100 r_1 r_2)^{1/3}$$

### 4.2 Shifts in Supply and Demand

We are often interested in how shifts in supply or demand affects prices and quantities. A shift in demand may be due to changes in income (e.g. a recession), changes in the price of substitutes or complements, changes in the quality of the goods, and so forth. Similarly, a shift in the supply curve may be due to changes in input prices, the number of producers or changes in technology.

Figure 5 shows the effect of an increase in demand. The left-hand side shows the market from the firm's perspective, while the right-hand side shows the entire market. The increase in demand causes the equilibrium price to rise and each firm to move up its supply function. As

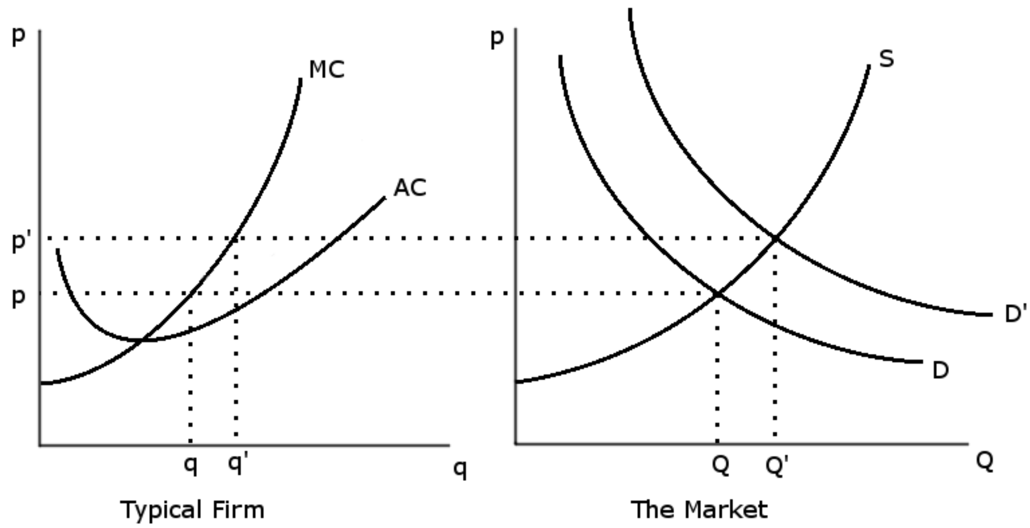


Figure 5: **Increase in Demand.** This figure shows the effect on an increase in demand from the firm's perspective (left), and the market's perspective (right). The original demand is $D$, with price $p$ and quantities $q$ and $Q$. When demand shifts up to $D'$, the price rises to $p'$ and quantities rise to $q'$ and $Q'$. Note that, in this picture, entry and exit are impossible so the firm's supply curve coincides with its marginal cost curve.

a result, each firm's profit increases.

Figure 6 shows the effect of a shift in demand for different supply functions. When supply is inelastic there is a small change in quantity and a large change in price. Conversely, when supply is elastic, there is a large change in quantity and a small change in price.

Exercise: How does a shift in the supply curve affect price and quantity when the demand curve is elastic/inelastic?

## 5 Long-Run Equilibrium

In the long run, we assume there are many identical potential entrants. This means that entry will occur if there exists a quantity $q$ such that $p > AC(q)$. Conversely, exit will occur if $p < AC(q)$. As a result price must be driven down to the minimum average cost

$$p = \min_{q \geq 0} AC(q).$$

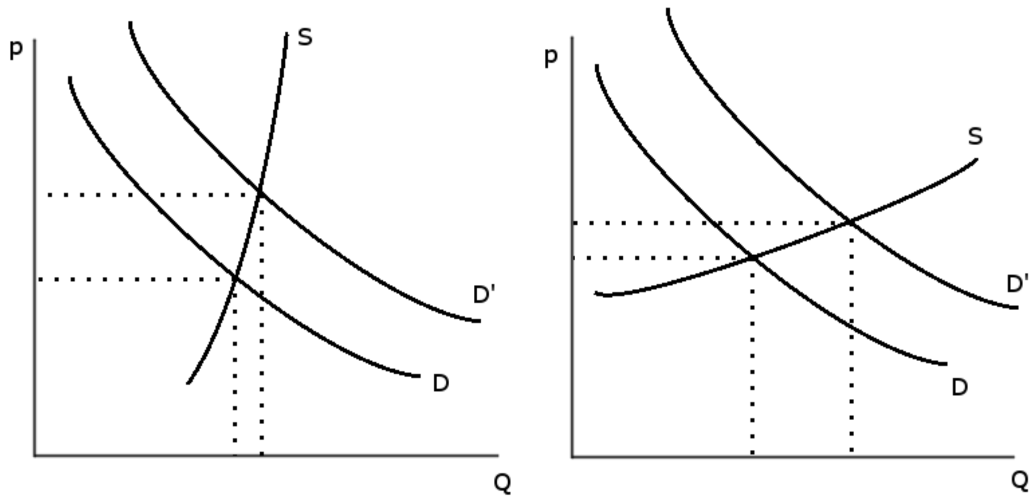


Figure 6: **Shifts in Demand.** This picture shows the effect of an increase in demand when supply is inelastic (left) or elastic (right). When supply is inelastic, the price rises substantially, but there is little extra output. When supply is inelastic, the price rises a little, and there is a lot of extra output.

We can thus think of the **long-run supply function** as a horizontal line equal to the minimum average cost. Note: this does not contradict the first-order condition of each firm since $AC(q) = MC(q)$ when $AC(q)$ is minimised. Hence we have $p = AC(q) = MC(q)$. See Figure 7.

In this class, we adopt a simple model of "the long-run", where there are an infinite number of identical firms. One could imagine an alternative model, where news firms are not as efficient as current firms, implying that the long-run supply function is upward sloping.[6] As an extension, one might suppose there is a difference between demand in the short-run and the long-run. For example, consider the market for oil. In the short-run, demand is very inelastic as consumers have fixed commutes. In the long-run consumers can change their cars, organise car-pools and even change jobs, in reaction to a change in the price of oil, making demand much more inelastic.

## 5.1 Example

Here we continue the example from Section 4.1. The algebra can get a little involved, so I urge you to work with example out, on your own, with $r_1 = r_2 = 1$.

[6] This would also occur if new firms bid up the price of common inputs.

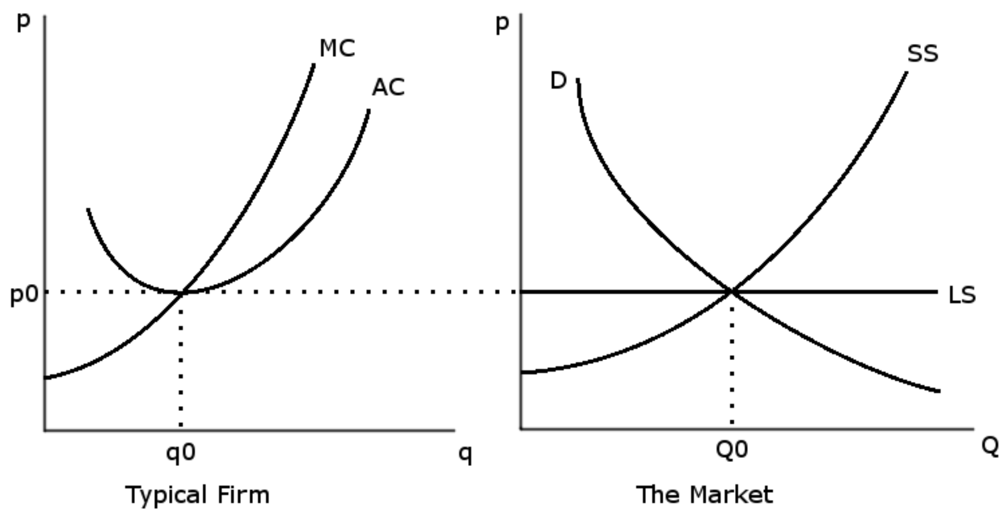


Figure 7: **Long-Run Equilibrium.** In the long-run equilibrium, each firm produces at the minimum average-cost, and the firms enter until aggregate supply equals aggregate demand. In this figure, LS is the long-run supply curve, while SS is the short-run supply corresponding to output $Q_0$.

The firm's cost function is

$$c(q, r_1, r_2) = 2(r_1 r_2)^{1/2} q^{3/2} + (r_1 + r_2)$$

Dividing by $q$, average cost is

$$AC(q) = c(q)/q = 2(r_1 r_2)^{1/2} q^{1/2} + (r_1 + r_2) q^{-1}$$

As argued above, average cost is minimised in the long-run. Differentiating,

$$\frac{dAC(q)}{dq} = (r_1 r_2)^{1/2} q^{-1/2} - (r_1 + r_2) q^{-2}$$

Setting this equal to zero and rearranging, the AC-minimising quantity is

$$q^* = (r_1 r_2)^{-1/3} (r_1 + r_2)^{2/3}$$

The long-run supply curve is a horizontal line at the minimum average cost. Substituting $q^*$ into the average cost, the price in the long-run is

$$p^* = AC(q^*) = 3(r_1 r_2)^{1/3} (r_1 + r_2)^{1/3}$$

We now switch to the market level. Since we know the price level, we can calculate the market demand. Using the demand function from Section 4.1,

$$X(p^*) = 100/p^* = \frac{100}{3}(r_1 r_2)^{-1/3}(r_1 + r_2)^{-1/3}$$

In equilibrium, aggregate supply equals aggregate demand. Since each firm produces $q^*$, we know the number of firms is

$$K^* = X(p^*)/q^* = \frac{100}{3}(r_1 + r_2)^{-1}$$

Note that $K^*$ may not be an integer, but we won't worry about this.

## 5.2 Shifts in Demand

Suppose we start in long-run equilibrium, as shown in figure 7. Suppose there in an increase in demand (e.g. due to government expenditure).

In the very short run, quantity is constant, and the market price rises a lot. This is shown in figure 8.

In the short run, firms increase their output in response to the increase in demand. Each firm moves up its individual supply curve (i.e. their marginal cost curve) and the market moves up its aggregate supply curve. As a result, firms now make positive profits, as shown in figure 9.

In the long-run the positive industry profits attracts entrants. This causes the industry short-run supply curve to shift out, and the price level to move back to the minimum average cost. The new aggregate quantity is given by the intersection of the long-run supply function and the demand function. Each individual firm returns to operating at minimum average cost, and making zero profits. See figure 10.

## 5.3 Example

Suppose market demand is linear:

$$X(p) = 1500 - 50p.$$

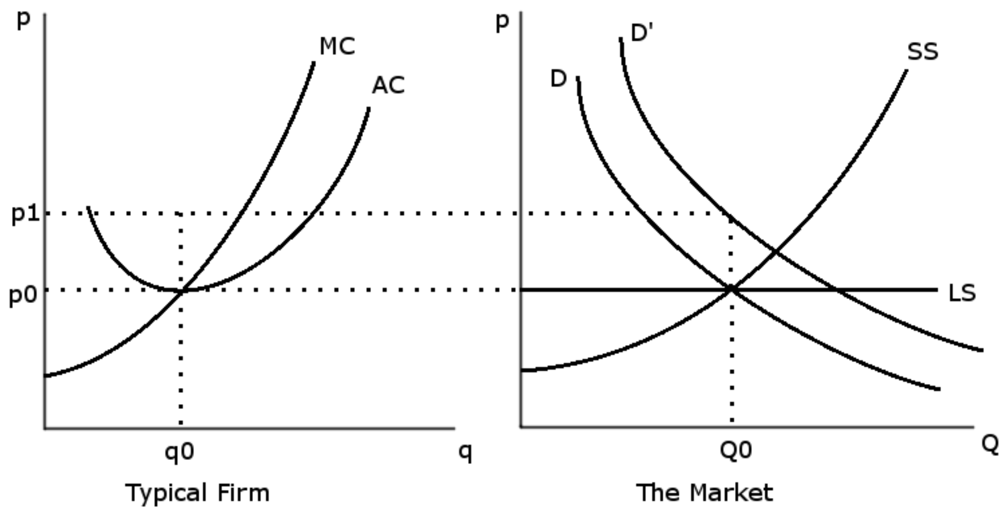


Figure 8: **Very-Short Run.** In the very-short run, each firm produces the same quantity $q_0$ and no firms enter, so market quantity is given by $Q_0$. The price rises to $p_1$ to clear the market.

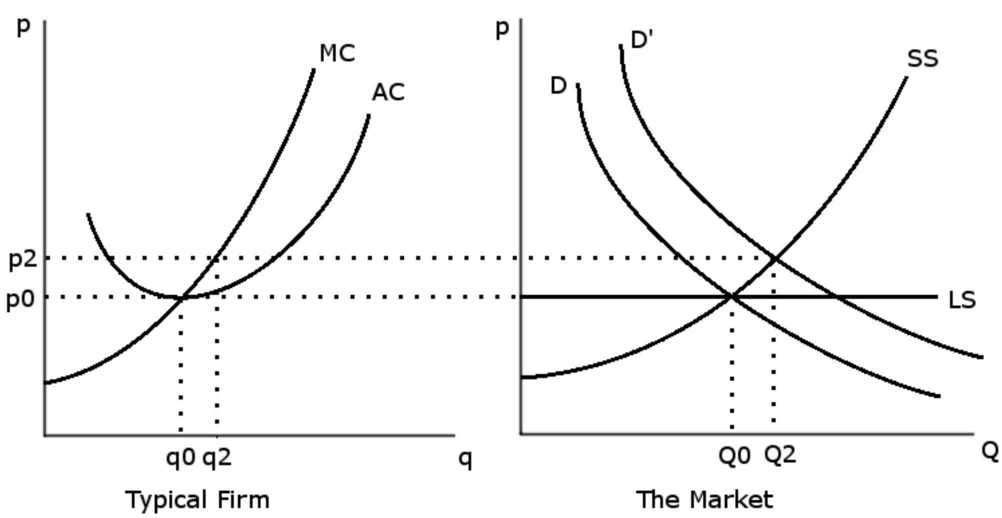


Figure 9: **Short Run.** In the short run, the individual firms increase their quantity to $q_2$. As a result, the market moves up its short-run supply curve and the equilibrium is given by $p_2$.

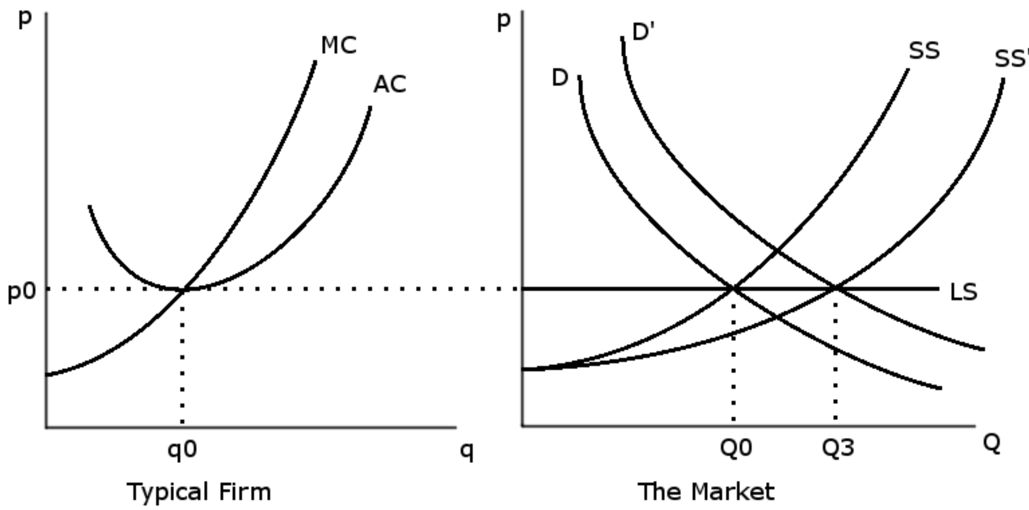


Figure 10: **Long Run.** In the long run, new firms enter, the industry reverts to the long run supply function and the short run supply function shifts to $SS'$. At this point, each firm again produces at long-run minimum average cost, $q_0$.

There are many potential firms, each with cost function

$$c(q) = 100 + q^2/4.$$

Let's calculate the initial long-run equilibrium. The average cost is

$$AC(q) = c(q)/q = 100q^{-1} + q/4$$

In the long-run equilibrium, firms operate at minimum average cost. Differentiating,

$$\frac{dAC(q)}{dq} = -100q^{-2} + 1/4 = 0$$

Rearranging, each firm produces

$$q^* = 20$$

Substituting into the average cost, the market price is then

$$p^* = AC(q^*) = 10$$

Market demand is

$$X(p^*) = 1500 - 50p^* = 1000$$

which equals market supply. Hence the number of firms is

$$K^* = X(p^*)/q^* = 1000/20 = 50.$$

Suppose demand falls (e.g. the price of a complement rises). In particular, new demand is given by

$$X(p) = 1200 - 50p.$$

In the **very short run**, each firm's output is fixed. Hence market output is fixed at 1000. Equating supply and demand,

$$1200 - 50p = 1000.$$

Hence the equilibrium price is $p^* = 4$.

In the **short run**, each firm reduces its output due to the reduction in demand. Each firm maximises profits:

$$\pi(q) = pq - c(q) = pq - 100 - q^2/4$$

Taking the first-order condition, the optimal output is $q^*(p) = 2p$. This gives this individual firm's supply curve. There are 50 firms, so the market supply curve is

$$Q(p) = 100p$$

Equating supply and demand,

$$1200 - 50p = 100p$$

Rearranging, the short-run equilibrium price is $p^* = 8$. Each firm produces $q = 2p^* = 16$, and makes profits

$$\pi = pq - c(q) = 8 \times 16 - 100 - 16^2/4 = -36.$$

In the **long run**, the negative profits cause firms to exit the industry. In the long-run, the price returns to $p^* = 10$, and each active firm produces $q^* = 20$. Given the price, market demand is

$$X(p^*) = 1200 - 50p^* = 700$$

which also equals market supply. Thus the number of firms is given by

$$K^* = X(p^*)/q^* = 700/20 = 35$$

which is less than the 50 firms we started with.