# Economics 11: Second Midterm

**Instructions:** The test is closed book/notes. Calculators are allowed. Please write your answers on this sheet. There are 100 points.

Name:

UCLA ID:

TA:

<table>
<thead>
<tr>
<th>Question</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Questions 1–4</td>
<td></td>
</tr>
<tr>
<td>Question 5</td>
<td></td>
</tr>
<tr>
<td>Question 6</td>
<td></td>
</tr>
<tr>
<td>Question 7</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
</tr>
</tbody>
</table>
Short Questions (25 points)

Question 1

The substitution effect is negative. Assuming there are two goods, this means that the Hicksian demand $h_1(p_1, p_2, \pi)$ is decreasing in $p_1$, and similarly for good 2. Show this is true, either graphically or mathematically.

[Hint: For the graphical proof, you should sketch the expenditure minimising choices, and consider the effect of a rise in $p_1$. For the mathematical proof, you should use Shephard’s Lemma, which states that $\partial e/\partial p_1 = h_1$, and the concavity of the expenditure function.]
Question 2

An agent chooses to consume in periods $t \in \{1, 2\}$. Her income is $m_1$ and $m_2$, while her consumption is $x_1$ and $x_2$. The consumer faces interest rate $r$.

In class we claimed that the agents budget is given by the equation

$$m_1 + \frac{1}{1 + r} m_2 \geq x_1 + \frac{1}{1 + r} x_2$$

Explain this equation.
Question 3

The cost function $c(r_1, r_2, q)$ is concave in the input price of good 1, $r_1$. Explain the idea behind this result, either in words or pictures (or both).
Question 4

A consumer’s demand for good 1 is given by

\[ x_1^*(p_1, p_2, m) = \frac{m}{p_1 + p_2} \]

She has income \( m = 8 \), and faces prices \( p_1 = 1 \) and \( p_2 = 1 \).

Calculate (a) the income elasticity of demand, (b) the own-price elasticity of demand, and (c) the cross-price elasticity of demand.
5. Consumer Surplus (25 points)

An agent consumes quantity \((x_1, x_2)\) of goods 1 and 2. She has utility

\[ u(x_1, x_2) = (1 + x_1)(1 + x_2) \]

The prices of the goods are \(p_1\) and \(p_2\). The consumer has income \(m\). [Assume \(m\) is sufficiently large so we obtain an interior solution.]

(a) Calculate the agent’s Marshallian demand. [Note: this problem is well behaved in that the FOCs are sufficient.]

(b) Are the goods (gross) complements or substitutes?

(c) Calculate the agent’s indirect utility function.

(d) Calculate the agent’s expenditure function.

(e) Suppose the consumer has income \(m = 8\) and faces prices \(p_1 = 1\) and \(p_2 = 1\). What is her utility?

(f) Suppose the price of good 1 increases to \(p'_1 = 4\). How much extra income does the agent need to compensate her for the price rise?
Space for Question 5.
6. Labour Supply (25 points)

An agent chooses leisure $x_1$, and consumption $x_2$ to maximise her utility. Her utility function is given by

$$u(x_1, x_2) = x_1^2 x_2$$

The agent is endowed with $T$ hours that she divides into leisure and work. The agent’s wage is given by $w$, while we normalise the price of good 2 to 1. As a result, the agent’s budget constraint is given by

$$w(T - x_1) \geq x_2$$

(a) Derive the agent’s Marshallian demand for $x_1$ and $x_2$.

(b) How does a change in the wage affect the demand for $x_1$? Interpret this result in terms of income and substitution effects.

(c) How does a change in the wage affect the demand for $x_2$? Interpret this result in terms of income and substitution effects.
Space for Question 6.
7. Cost Minimisation (25 points)

A firm has two inputs, \( z_1 \) and \( z_2 \). It has production function

\[
f(z_1, z_2) = (z_1 + z_2)^{1/2}
\]

The firm faces input price \( r_1 \) and \( r_2 \), where we assume \( r_1 < r_2 \).

(a) Does the production technology satisfy increasing/constant/decreasing returns to scale, or none of the above.

(b) The firm wishes to attain target output \( q \). Derive the cost minimising inputs \( z_1^*(r_1, r_2, q) \).

(c) Derive the cost function \( c(r_1, r_2, q) \).

(d) Derive the average cost and the marginal cost functions.

(e) Does the average cost increase or decrease in \( q \)? Intuitively, how does this result relate to your finding in part (a)?
Space for Question 7.
Space for Rough Work (Do not write your answers here)
Space for Rough Work (Do not write your answers here)