Short Questions (25 points)

**Question 1**

An agent consumes quantity \((x_1, x_2)\) of goods 1 and 2. She has utility

\[ u(x_1, x_2) = x_1^{1/2} x_2^{1/2} \]

(a) Derive the MRS.

(b) Show that preferences are convex.

**Question 2**

Given any two cars, I always prefer the one that is bigger and faster. Given this information, are my preferences transitive? Are they complete?

**Question 3**

Assume that preferences satisfy the usual axioms (transitivity, completeness and continuity). Also assume they satisfy monotonicity. Can an indifference curve cross itself? Explain your answer.

**Question 4**

(a) Define the expenditure function (either mathematically or in words).

(b) Intuitively explain why the expenditure function is concave in prices.

**Question 5**

(a) Define an inferior good.

(b) An agent consumes two goods and always spends all her income. Can both goods be inferior?
6. Basic Consumer Choice (25 points)

An agent consumes quantity \((x_1, x_2)\) of goods 1 and 2. She has utility

\[
u(x_1, x_2) = x_1^{1/3} + x_2^{2/3}\]

The prices of the goods are \(p_1 = 1\) and \(p_2 = 1\). The consumer has income \(m = 9\). Calculate the consumer’s optimal demand.

Note: For this question, you may find the quadratic formula useful. If \(ax^2 + bx + c = 0\) then

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

7. Hicksian Demand (25 points)

An agent consumes quantity \((x_1, x_2)\) of goods 1 and 2. She has utility

\[
u(x_1, x_2) = x_1x_2
\]

The prices of the goods are \((p_1, p_2)\).

(a) Set up the expenditure minimisation problem.

(b) Derive the agent’s Hicksian demands.

(c) Derive the agent’s expenditure function.

8. Choice with Satiation (25 points)

An agent consumes quantity \((x_1, x_2)\) of goods 1 and 2. She has utility

\[
u(x_1, x_2) = -\frac{1}{2}(x_1 - 10)^2 - \frac{1}{2}(x_2 - 10)^2
\]

The agent faces prices \(p_1 = 1\) and \(p_2 = 1\) and has income \(m\). [Hint: you may find it useful to plot the indifference curves].

(a) Suppose \(m = 10\). Solve for the agent’s optimal choice.
(b) Suppose $m = 30$. Solve for the agent’s optimal choice.

(c) Given the above prices, plot the agent’s income offer curve (income expansion path) and the Engel curve for good 1.\footnote{Reminder: The Engel curve shows how demand for a good varies with income.}