Economics 11: Solutions to Practice First Midterm - Version B

Short Questions (25 points)

Question 1

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$$

(a) Derive the MRS.

(b) Show that preferences are convex.

Solution

(a) The MRS is

$$\frac{MU_1}{MU_2} = \frac{\frac{1}{2}x_1^{-1/2}x_2^{1/2}}{\frac{1}{2}x_1^{1/2}x_2^{-1/2}} = \frac{x_2}{x_1}$$

(b) We can show preferences are convex by showing the MRS is decreasing in x_1 along the indifference curve. Along the indifference curve, $x_1^{1/2}x_2^{1/2} = k$. Rearranging, we get $x_2 = k^2/x_1$. Plugging into MRS,

$$MRS = \frac{k^2}{x_1^2}$$

This is clearly decreasing in x_1.

Question 2

Given any two cars, I always prefer the one that is bigger and faster. Given this information, are my preferences transitive? Are they complete?

Solution

They are transitive, but not complete. It's unclear how I rank two cars A and B, where A is faster and b is bigger.

Question 3

Assume that preferences satisfy the usual axioms (transitivity, completeness and continuity). Also assume they satisfy monotonicity. Can an indifference curve cross itself? Explain your answer.

Solution

No. This violates monotonicity.

Question 4

(a) Define the expenditure function (either mathematically or in words).

(b) Intuitively explain why the expenditure function is concave in prices.

Solution

(a) The expenditure function is the minimal expenditure needed to attain a target utility level.

(b) Suppose p_1 rises. If I keep my demands constant then I attain the same utility level and my expenditure rises linearly. I can do better than this by rebalancing my demands, introducing concavity.

More formally, fix p_2 and any allocations (x_1, x_2) that attain the target utility $\overline{u}$. Define the pseudo–expenditure function by

$$\eta_{x_1,x_2}(p_1) = p_1 x_1 + p_2 x_2$$

which is linear in p_1. The expenditure function is given by the lower envelope of

$$\{\eta_{x_1,x_2}(p_1) : u(x_1, x_2) = \overline{u}\}$$

Since the minimum of linear functions is concave, the expenditure function is therefore concave.

Question 5

(a) Define an inferior good.

(b) An agent consumes two goods and always spends all her income. Can both goods be inferior?

Solution

(a) Demand decreases as income increases.

(b) No: if the agent's income rises, her expenditure on at least one good must rise.

6. Basic Consumer Choice (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = x_1^{1/3} + x_2^{2/3}$$

The prices of the goods are $p_1 = 1$ and $p_2 = 1$. The consumer as income $m = 9$. Calculate the consumer's optimal demand.

Note: For this question, you may find the quadratic formula useful. If $ax^2 + bx + c = 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solution

The tangency condition states that

$$\frac{\frac{1}{3}x_1^{-2/3}}{\frac{2}{3}x_2^{-1/3}} = 1$$

Rearranging, $x_2 = 8x_1^2$. Using the budget constraint,

$$x_1^* = 1 \qquad \text{and} \qquad x_2^* = 8$$

7. Hicksian Demand (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = x_1 x_2^2$$

The prices of the goods are (p_1, p_2).

(a) Set up the expenditure minimisation problem.

(b) Derive the agent's Hicksian demands.

(c) Derive the agent's expenditure function.

Solution

(a) The agent minimises

$$\mathcal{L} = p_1 x_1 + p_2 x_2 + \lambda[\overline{u} - x_1 x_2^2]$$

(b) The FOCs are:

$$p_1 = \lambda x_2^2$$
$$p_2 = 2\lambda x_1 x_2$$

From these we find $2p_1 x_1 = p_2 x_2$. The constraint states that

$$x_1 x_2^2 = \overline{u}$$

Solving for the Hicksian demands

$$h_1 = \frac{1}{4^{1/3}} \overline{u}^{1/3} \left(\frac{p_2}{p_1} \right)^{2/3}$$

and

$$h_2 = 2^{1/3} \overline{u}^{1/3} \left(\frac{p_1}{p_2} \right)^{1/3} = \frac{2}{4^{1/3}} \overline{u}^{1/3} \left(\frac{p_1}{p_2} \right)^{1/3}$$

(c) The expenditure function is

$$e = p_1 h_1 + p_2 h_2 = \frac{3}{4^{1/3}} \overline{u}^{1/3} p_2^{2/3} p_1^{1/3}$$

8. Choice with Satiation (25 points)

An agent consumes quantity (x_1, x_2) of goods 1 and 2. She has utility

$$u(x_1, x_2) = -\frac{1}{2}(x_1 - 10)^2 - \frac{1}{2}(x_2 - 10)^2$$

The agent faces prices $p_1 = 1$ and $p_2 = 1$ and has income m. [Hint: you may find it useful to plot the indifference curves].

(a) Suppose $m = 10$. Solve for the agent's optimal choice.

(b) Suppose $m = 30$. Solve for the agent's optimal choice.

(c) Given the above prices, plot the agent's income offer curve (income expansion path) and the Engel curve for good 1.[1]

Solution

The key point here is to observe that $(10, 10)$ is a bliss point.

(a) The tangency condition says

$$\frac{x_1 - 10}{x_2 - 10} = 1$$

Rearranging, $x_1 = x_2$. Using the budget constraint,

$$x_1^* = 5 \qquad \text{and} \qquad x_2^* = 5$$

(b) The agent can afford the bliss point $(10, 10)$, and so will consume this.

(c) Demands are given by

$$x_1^* = \min\{m/2, 10\} \qquad \text{and} \qquad x_2^* = \min\{m/2, 10\}$$

[1]Reminder: The Engel curve shows how demand for a good varies with income.

The income offer curve is a 45^0 slope, and then stops at $(10, 10)$. The Engel curve rises at slope $1/2$ and stops at 10.